

# Energy ranking preservation in an $N$ -body cosmological simulation

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## ABSTRACT

In this paper, we present a study of the cosmic flow from the point of view of how clusterings at different dynamical regimes in an expanding universe evolve according to a ‘coarse-grained’ partitioning of their ranked energy distribution. By analysing a lambda cold dark matter cosmological simulation from the Virgo Project, we find that cosmic flows evolve in an orderly sense, when tracked from their coarse-grained energy cells, even when non-linearities are already developed. We show that it is possible to characterize scaling laws for the pairwise velocity distribution in terms of the energy cells, generally valid at the linear and non-linear clustering regimes.

**Key words:** stellar dynamics – methods:  $N$ -body simulations – large-scale structure of Universe.

## 1 INTRODUCTION

Cosmological  $N$ -body simulations play a crucial role in the study of the formation of cosmic structure. These structures are believed to have grown from weak density fluctuations present in the otherwise homogeneous and rapidly expanding early universe. These fluctuations were amplified by gravity, eventually turning into the rich structure that we see around us today (Springel et al. 2005). Cosmological  $N$ -body simulations usually start from initial conditions set at some early epoch, then follow the later evolution of the dark matter (and, if included, other matter components) into the non-linear regime where it can be compared with the large-scale structure in galaxy surveys. In these simulations, the dominant mass component, the cold dark matter (CDM), is assumed to be made of elementary particles that interact only gravitationally, so the collisionless dark matter fluid can be represented by a set of discrete point particles.

Gravitational, long-range interactions are peculiar because: (i) they are inherently negative specific heat systems (i.e. taking energy away from it heats it up; see e.g. Lynden-Bell & Lynden-Bell 1977; Padmanabhan 1990; Lynden-Bell 2001), or, equivalently, internal temperature gradients are enhanced instead of being erased (El-Zant 1998); (ii) the potential energy is in a *superextensive* quantity, which leads to an intrinsic instability in the case of virialized systems when their mass is increased (e.g. Heggie & Hut 2003). The difficulty of characterizing the dynamical evolution and final equilibrium state of collisionless systems is deepened by the fact that the classical statistical mechanics and thermodynamics have been little developed for gravitational problems due to complications posed by the long-range, unshielded nature of the gravitational force (e.g. Padmanabhan 1990, 2005). Here we focus on also peculiar, but

largely overlooked property of gravitational  $N$ -body systems, the *energy ranking preservation* phenomenon (ERP). The ERP was first noted by Quinn & Zurek (1988); it re-emerged later in the literature, as a subsidiary result, in different contexts such as the evolution of simulated galaxy mergers and collapses (Kandrup, Mahon & Smith 1993), the numerical study of ‘violent relaxation’ processes (e.g. Funato, Makino & Ebisuzaki 1992a), the applicability of the secondary infall model to collapsing cosmological protohaloes (Zaroubi, Naim & Hoffman 1996), or the study of properties of dark matter haloes (Henriksen & Widrow 1999; Dantas et al. 2003; Dantas & Ramos 2006).

A gravitational  $N$ -body system displays ERP when the relative ordination or ranking of sufficiently large groups of particles with respect to their mean energy is preserved along their possibly intricate trajectories in the phase space. In this context, ERP means that there is a ‘coarse-grained’ sense in which the ranking relative to the mean one-particle, mechanical energy of given collections of elementary particles is strictly *not* violated during the gravitational evolution of the system. As remarked by Kandrup et al. (1993), ‘mesoscopic constraints’ seem to be operative at the level of collections of particles when a large  $N$ -body system evolves gravitationally towards equilibrium.

This work is part of a project first motivated from a study of the properties of elliptical galaxies in the context of the so-called ‘fundamental plane’ (Dantas et al. 2003). We have subsequently analysed the ERP in individual haloes in a cosmological simulation and have found some preliminary results on the behaviour of the coarse-grained level necessary for the establishment of the ERP (Dantas & Ramos 2006). In this paper, we aim to address the ERP effect within a large-scale  $N$ -body cosmological simulation in a more general context (as opposed to the restricted ‘individual haloes’ approach of previous studies). We investigate the problem in a more global sense (all particles of the simulation, inside and outside haloes

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will be considered) and at different dynamical regimes and scales, namely, at the linear regime, where structures are not yet virialized, and at the small-scale non-linear regime, where particles probably are part of virialized or quasi-virialized structures.

In fact, the non-Hubble component of a galaxy velocity through the universe (its peculiar velocity), due to the acceleration caused by clumps in the matter distribution, is an important quantity in cosmology (e.g. Juszkiewicz, Springel & Durrer 1999; Fukushige & Suto 2001; Landy 2002; Feldman et al. 2003). Here, we will be interested in studying the *pairwise velocity distribution* (PVD) from the point of view of the ERP phenomenon. In other words, we analyse a cosmological  $N$ -body simulation in the linear and non-linear regimes as a function of different energy partitionings, under the perspective of characterizing the behaviour of the underlying cosmic flow.

This paper is organized as follows. In Section 2, we present the methodology and illustrate the ERP. In Section 3, we analyse the behaviour of the PVD. In Section 4, we offer a bird's eye view in the literature to contextualize the ERP phenomenon within our current understanding of the gravitational collapse and virialization mechanisms. In Section 5, we present the summary and final conclusions.

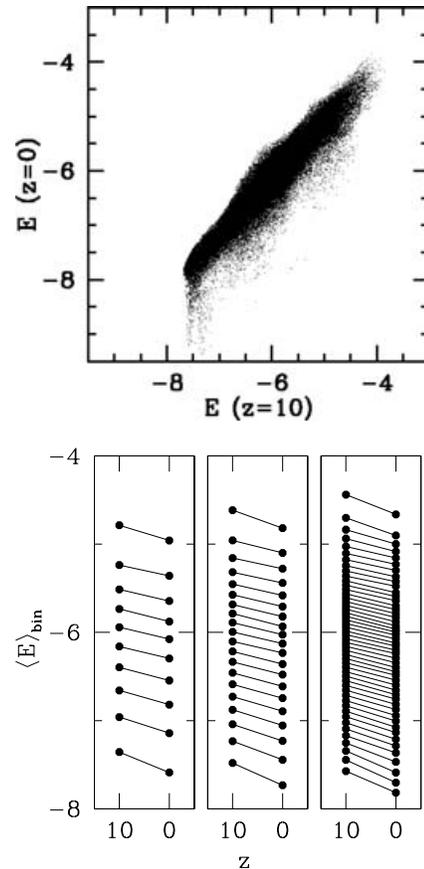
## 2 METHODOLOGY AND THE ERP PHENOMENON

In this work, we used in our analysis a  $\Lambda$ CDM  $N$ -body simulation of the Virgo Consortium (the  $\Lambda$ CDM model is currently the most observationally favoured cosmological scenario, e.g. Spergel et al. 2003, 2006).

The simulation box has  $239.5 \text{ Mpc } h^{-1}$  ( $h = 0.7$ ) of size, with  $256^3$  particles, each with mass of  $6.86 \times 10^{10} M_{\odot} h^{-1}$ , therefore representing approximately galaxy-sized objects. We have selected a random fraction of the particles (one out of 100) in order that our analysis becomes computationally feasible. We have also restricted our analysis to the initial ( $z = 10$ ) and final ( $z = 0$ ) outputs of this cosmological run. Particles of a given initial model are sorted according to their mechanical energies. The energy space is then partitioned into few (e.g. 10) cells or bins of equal number of particles and for each of these bins, the mean energy is calculated. The mean energies of these *same* collections of particles are then recalculated for the final model and compared with their initial values. As we will show below, generally these mean energy cells do not cross each other along the evolution (ranking is preserved), although particles individually gain or lose energy as the system virializes.

In Fig. 1, we present two panels illustrating the global characteristics of the energy space in the Virgo's  $\Lambda$ CDM simulation. In the top panel, we show a scatter plot of the initial ( $z = 10$ ) versus final ( $z = 0$ ) energies of the particles in the simulation. The mechanical comoving one-particle energy of a particle  $i$  was calculated classically from  $E_i = (1/2 m v_i^2) - 1/2 \sum_{j \neq i} G m^2 / |x_i - x_j|$ , with comoving position  $x_i$  and peculiar velocity  $v_i$ . Units used were Mpc for length, Gyr for time and  $M_{\odot}$  for mass. It represents the energy with respect to the comoving reference frame (i.e. the frame which moves with the cosmological expansion of the simulation box). The energy associated to the dynamics of expansion of the cosmological box does not enter into the above computations.

The straight  $\sim 45^\circ$  line pattern exhibited in the top panel of Fig. 1 highlights the strong memory preservation of the initial condition state along the evolution of the system in the energy space. We remark, however, that there is a significant spread around the correlation, which indicates individual gain/loss of energy ('mixing')



**Figure 1.** Top panel: a scatter plot of the initial (redshift  $z = 10$ ) versus final ( $z = 0$ ) energies of the particles in the cosmological simulation. Bottom panel (the ERP effect): the ordering of the mean energies of collections of particles are mesoscopically maintained for various levels of energy partitioning ( $N_{\text{bins}} = 10, 20$  and  $50$ ; see text for details). Only the initial and final energy bins are shown here, connected by line segments.

during the evolution. In the bottom panel of Fig. 1, we present the ERP effect for different choices of energy partitioning ( $N_{\text{bins}} = 10, 20$ , and  $50$ ; we have fixed this choice of partitioning throughout this work). The energy cells do not cross, meaning that the most (least) energetic particles at the initial state are still the most (least) energetic particles at the final state. Note that the energy ordination is observed despite the fact that particles individually have mixed in energy space. This unexpected orderly evolution contrasts with the highly complex structure of the actual phase space of the system, which displays sensitivity to changes in their initial conditions which are characteristic of chaotic dynamical systems (Kandrup & Smith 1991, 1992; Goodman, Heggie & Hut 1993; Merritt & Valluri 1996; El-Zant 1998; Kandrup, Vass & Sideris 2003). Hence, clearly, memory is preserved only in a coarse-grained, mesoscopic sense (bottom panel of Fig. 1).

Also note (cf. top panel of Fig. 1) that some of the most bound particles tend to become at the end state even more gravitationally bound, as evidenced by a tenuous 'tail' at the bottom/left-hand side of the scatter plot (see also fig. 13 of Quinn & Zurek 1988; and fig. 9 of Henriksen & Widrow 1999). The final mean coarse-grained energies are distributed in a smaller range than their initial counterparts. This would not be expected for energies evaluated at a fixed background. But in this case, the energies are calculated with respect to the comoving frame (for further details, see Dantas & Ramos 2006).

### 3 THE PAIRWISE VELOCITY DISTRIBUTION

We will proceed our analysis considering the PVD from an ‘energy-space’ perspective. Here, we consider a pair of galaxies lying at a comoving separation vector  $\mathbf{R}$ . We define the pairwise peculiar velocity as

$$\delta v_{\parallel}(R) \equiv \frac{1}{R}(\mathbf{v}_i - \mathbf{v}_j) \cdot \mathbf{R}, \quad (1)$$

where  $\mathbf{v}_i$  and  $\mathbf{v}_j$  are the proper peculiar velocities of a given pair of galaxies in the system. The PVD is the distribution of all particles  $\delta v_{\parallel}(R)$  for a given scale  $R$  probed. The importance of this quantity cannot be underestimated. Note that, separated at cosmological distances, a given galaxy pair will be approximately following the uniform Hubble flow. In other words, it is expected that the ratio  $\delta v_{\parallel}(R)/(HR)$ , where  $H$  is the Hubble parameter, tends to zero in this case. On the other hand, objects at much smaller scales evolving under non-linear regimes will have their dynamics supersede the Hubble flow.<sup>1</sup>

Hence, the properties of the PVD, evaluated at different comoving scales, are intimately related to linear/non-linear regimes. Our major concern here is to describe the behaviour of the PVD considering the fact that there is an ordering preservation of the mesoscopic particle energy cells during the cosmological evolution. How does the non-linear evolution of the pairwise motions behave under such an energy constraint?

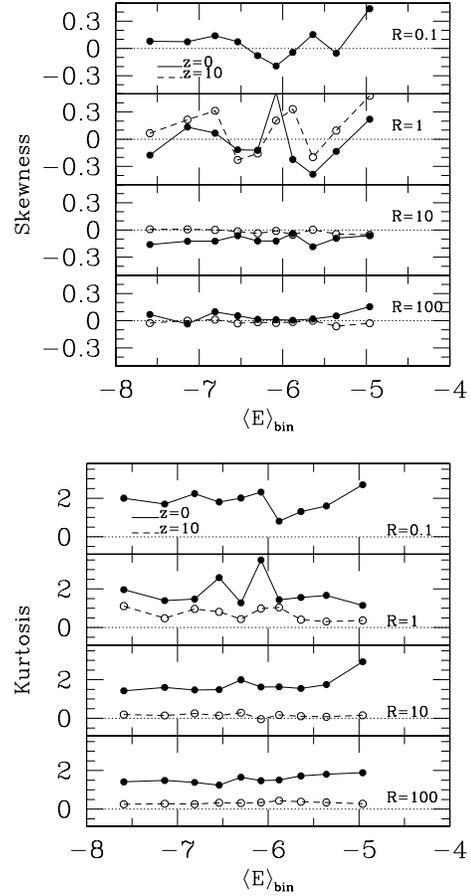
Fig. 2 is an illustration ( $N_{\text{bins}}$  fixed to 10) of the typical results of skewness and kurtosis<sup>2</sup> of the PVDs for every energy cell separately. All PVDs at  $z = 0$  show significant departures from gaussianity. Although this is a behaviour well known in *unconstrained* PVDs, and modelled analytically in previous works (e.g. Diaferio & Geller 1996; Sheth 1996; Seto & Yokoyama 1998), it is not obvious why this feature should remain in an energy-constrained PVD. In fluid turbulence, for instance, energy-constrained PVDs recover gaussianity (Naert et al. 1998), indicating that the power-law tails apparent in the velocity histograms stem directly from the spatially intermittent nature of the underlying energy cascade that drives the system (Frisch 1995). This is clearly not the case here.

More unexpectedly in Fig. 2, is the evident correlation between early and late epoch statistics, for certain scales ( $R = 1$  and 10 Mpc). Note that scales below 10 Mpc are precisely those that probe the non-linear regime, where peculiar velocities dominate the Hubble flow and particles are already part of virialized or quasi-virialized clusters. On the other hand, on larger scales ( $R \geq 100$  Mpc), where there is little discernible structure in cosmological box and the mass distribution appears homogeneous and isotropic, there is no evidence of memory on the skewness and kurtosis of the energy-constrained PVDs.

In Fig. 3, we plot the dispersion  $\langle(\delta v_{\parallel}(R) - \bar{v}_{\parallel})^2\rangle$  of the PVD as a function of the mean energy cell for the initial ( $z = 10$ , to the left-hand side, circles) and final ( $z = 0$ , to the right-hand side, triangles)

<sup>1</sup>Davis & Peebles (1977) have argued that for an isolated system in virial equilibrium, and therefore highly non-linear, the physical (proper) separation of particle pairs within the system is expected to remain constant on average (the so-called ‘stable clustering’ hypothesis). In the universe, however, no system is exactly isolated, and continuous merging and accretion are expected to occur in a hierarchical fashion. Despite this complication, the stable clustering hypothesis has been used as a model to the power spectrum of density fluctuations encompassing non-linear structures (Hamilton et al. 1991), and has been extensively investigated for haloes resulting from cosmological simulations (e.g. Jing 2001).

<sup>2</sup>The kurtosis, as usually defined, was decreased by 3.

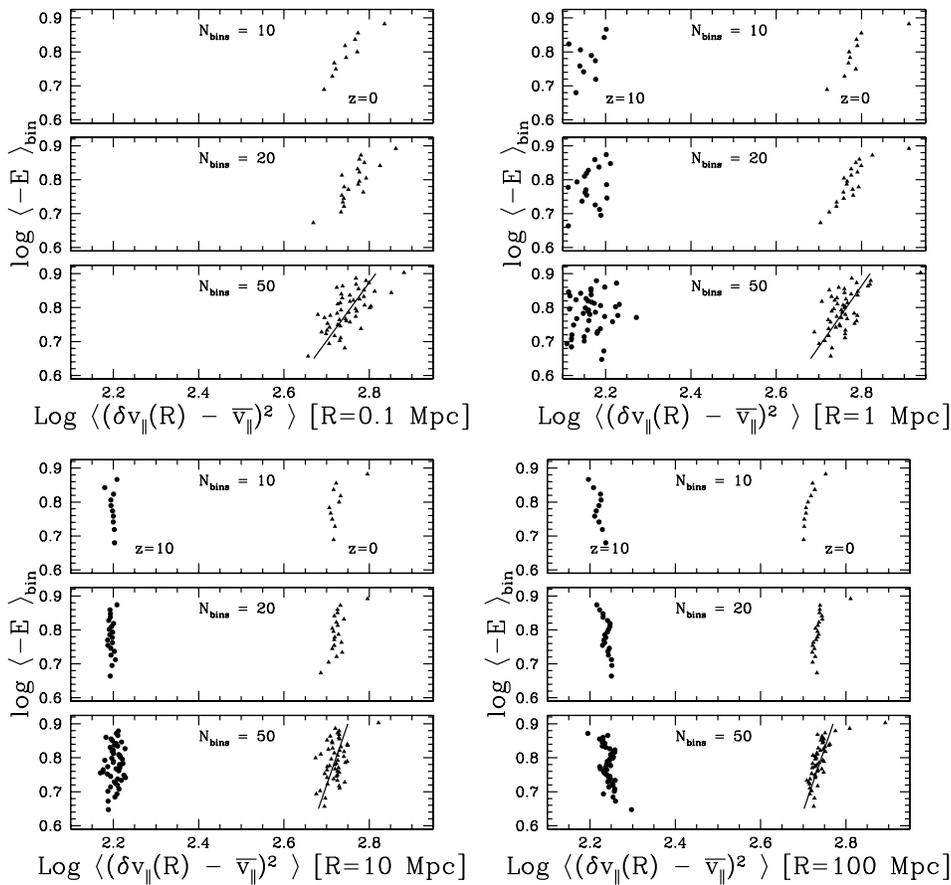


**Figure 2.** Skewness (top panel) and kurtosis (bottom panel) of the PVDs obtained separately for every energy cell as a function of the mean energy of the cell for  $z = 0$  (solid lines) and  $z = 10$  (dashed lines).  $N_{\text{bins}}$  fixed to 10 in all these plots. Dotted lines indicate the expectations for a perfectly gaussian distribution. The absence of data for  $z = 10$  in the first top panels of the figures is due to lack of statistics in this scale ( $R = 0.1$ ).

states. At this point, let us review what has actually been measured and its meaning: (i) we measure the radial component, peculiar, velocity difference of each pair of particles, for each mesoscopic energy bin, to obtain the energy-constrained PVD (equation 1). This results in a distribution of velocity differences that is well known to deviate from gaussianity in the energy-unconstrained case (Diaferio & Geller 1996; Sheth 1996; Seto & Yokoyama 1998); and (ii) we measure the dispersion of the resulting energy-constrained PVD. Then, note that what is being considered is *not* the velocity dispersion of the particles of a given energy cell. It is the dispersion of the PVD for each energy cell. In other words, the former and the latter are different physical measures.

Overall, we observe that the more (negative) energetic the cell, the larger the corresponding dispersion. Moreover, the relation between dispersion and energy follows a power law in which the characteristic exponent increases with the probed scale (cf. Fig. 3 and corresponding legend). Note, however, that probing smaller scales does *not* necessarily mean that one is also probing more (negative) energetic cells of central halo regions, which possess higher velocity dispersions, since, as already mentioned, the *velocity dispersion and the dispersion of the PVD are different physical measures*.

In the case of less (negative) energetic cells (fix the triangle dots mostly at the bottom of each fitted line in Fig. 3), for redshift



**Figure 3.** The coarse-grained mean energies as a function of the related dispersion of the PVDs (see text for details). This mosaic shows the results for every scale  $R$  probed by the estimator; top panels: results for an energy partition of  $N_{\text{bins}} = 10$ ; intermediate panels:  $N_{\text{bins}} = 20$ ; lower panels:  $N_{\text{bins}} = 50$ . Circles: results for the initial state ( $z = 10$ ); triangles: results for the final state ( $z = 0$ ). The absence of points in the first upper panel is due to lack of statistics in this scale for  $z = 10$ . Straight lines are fits to the data points, at  $z = 0$  and  $N_{\text{bins}} = 50$ , without considering the points corresponding to the most negative energetic cells. The corresponding slopes ( $\alpha$ ) for these fits (according to scale  $R$ ) are:  $(\alpha, R) = (1.130, 0.1), (1.562, 1), (3.215, 10)$  and  $(4.167, 100)$ .

$z = 0$ , moving to smaller scales, in fact *decreases* the dispersion of the PVD (the characteristic exponent decreases for smaller scales). Clearly, these less (negative) energetic cells, even at small scales, are probing, most of the time, not yet virialized motions, still in the linear regime. This result is in agreement with those of Diaferio & Geller (1996), where it is found that unrelaxed systems tend to have more centrally peaked PVD than gaussian distributions. Also, some of the particles composing these less (negative) energetic cells, at small scales, can also make part of the population of the outermost parts of dark matter haloes, which are not yet virialized. In any case, all energy cells follow the same simple scaling law, regardless of the linear or non-linear clustering regime.

#### 4 VIRIALIZATION AND THE ERP PHENOMENON

Up to this point, we have shown that the ERP is a phenomenon that occurs embodying all particles of a  $\Lambda$ CDM cosmological simulation, regardless of the virialization condition of the immediate regions to which the particles are associated with. Also, it is reasonable to consider that the energy-constrained PVDs scaling laws found in this work are a reflection of a remarkably smooth dynamical evolution of the large-scale motions resulting from the clustering process taking place in the simulation. In this section, we allow for

a brief digression and review the relevant literature on the virialization of dissipationless gravitational systems in order to bring the ERP effect into this context.

##### 4.1 Dissipationless gravitational systems

In a first approximation, if the dissipation and evolution of the baryonic component of a gravitational system can be disregarded, the problem of virialization can be studied by the means of numerical experiments that attempt to reproduce the purely gravitational evolution of a set of  $N$  point masses.<sup>3</sup> Well before such experiments could properly be performed, the theoretical work by Chandrasekhar in 1943 (viz. Binney & Tremaine 1987) already indicated the existence of different dynamical regimes, namely: if  $(8 \ln N)/N \ll 1$ , the system is collisionless in the sense that each particle responds to the average gravitational field of the system; otherwise, the system is collisional or marginally collisional.

In the collisional case, local fluctuations or granularities of the gravitational potential are important to the overall dynamics. The accumulative effect of the energy exchange between stars as they

<sup>3</sup>Dissipational effects tend to contribute to the formation of deep central potentials (e.g. Carlberg 1986).

deflect each other during an encounter results that, at the gravitational two-body relaxation time-scale, the ‘memory’ of the initial orbits of the stars have been erased. Objects like globular clusters are considered examples of such systems. In the collisionless case, however, the corresponding relaxation time-scale is longer than the age of the universe; hence the system cannot reach a thermal equilibrium state from binary collisions. Therefore, it can only relax to a quasi-equilibrium state through some other mechanism, which also must operate rapidly enough (see detailed discussion and references in the next subsection). Elliptical galaxies and dark matter haloes are believed to be examples of such systems.

#### 4.2 Brief review on virialization of gravitational systems

The first attempts to provide some statistical/thermodynamical description of gravitational stellar dynamics and collisionless relaxation started in the 1960s with the pioneering work of Antonov (see e.g. Lynden-Bell 2001 and references therein), Ogorodnikov (1965), Saslaw (1968, 1969) and Lynden-Bell (1967). Lynden-Bell attempted to answer the problem of how a collisionless system would be able rapidly to relax to a quasi-equilibrium state, since the equipartition of energy cannot proceed through two-body encounters. The resulting theory (‘violent relaxation’) stated that *if* rapid fluctuations in the mean gravitational potential energy field ( $\langle\phi\rangle$ ) of a system far from equilibrium persisted long enough before damping, these oscillations would promote the mechanism for changing the energy per unit mass ( $E_*$ ) of each star in the system ( $dE_*/dt = \partial\langle\phi\rangle/\partial t$ ). Note that  $E_*$  can change by a large amount under this hypothesis, that is,  $|\Delta E_*| \sim |E_*|$ , so that the effect would be as significant as in the two-body relaxation case.

Soon after Lynden-Bell’s publication, several one-dimensional  $N$ -body numerical experiments were performed to test the outcome of gravitational collapse against the predictions of ‘violent relaxation’, but the results were mostly inconclusive (e.g. Hohl & Feix 1967; Hénon 1968; Hohl & Campbell 1968; Cuperman, Goldstein & Lecar 1969; Goldstein, Cuperman & Lecar 1969; Lecar & Cohen 1971; see also more recent developments along these lines in Tanekusa 1987; Mineau, Feix & Rouet 1990). These simulations generally started with a given initial volume of phase space uniformly occupied by all particles of the system, and the resulting end state object usually presented a degenerated core and a distinct halo population containing most of the mass (see also Bouvier & Janin 1970; Gott 1973), which could only be partially fit to Lynden-Bell’s distribution. These results indicated that ‘violent relaxation’ would probably be an incomplete process, in the sense that the fluctuations of the gravitational potential would damp too rapidly and not all phase-space cells dynamically accessible to the system would end up occupied. Consequently, the final equilibrium product generally resulted in a partially relaxed object with a stationary potential (except for small statistical fluctuations) that would not necessarily represent the configuration that maximized the total entropy of the system.

Note that the general idea behind most of the approaches to derive a theory of collisionless relaxation is to assume that the final equilibrium state will be the most probable state of the system. Such a state could be predicted from first principles by maximizing the entropy of the system under the appropriate constraints (usually, the total mass and energy), taking into account the fact that the phase-space density must evolve under the collisionless Boltzmann equation. The standard result turns out to be, however, that the entropy is extremized if and only if the distribution function is of the

isothermal sphere. But the energy and mass of an isothermal sphere diverge for large radii, so this leads to a contradiction with the initial assumptions of the problem. The general conclusion is that *no* distribution function compatible with fixed mass and energy maximize the entropy: arbitrarily large entropy configurations can be obtained by an arrangement of the particles of the system (e.g. Binney & Tremaine 1987).<sup>4</sup> For instance, the entropy can increase without bound by confining a small fraction of the mass in a compact core and arranging the remainder in a diffuse halo. Although the existence of such a core-halo configuration is a natural consequence of the relaxation of collisional systems, like globular clusters, it does not seem to be a physically accessible one to collisionless systems, since elliptical galaxies and dark matter haloes are not observed to have such structures.

Lynden-Bell was aware that in real systems ‘violent relaxation’ would probably not proceed to completion (see section 6 of his 1967 paper). Hence, the theory would only represent a qualitative idea that needed to be complemented by other mechanisms and/or superseded by a more robust theory. This line of thought was strengthened more and more from different fronts, in particular from theoretical and observational studies of the scaling laws in elliptical galaxies (e.g. Binney 1982; Tremaine, Hénon & Lynden-Bell 1986; White & Narayan 1987; Richstone & Tremaine 1988; Hjorth & Madsen 1991, 1993, 1995; Trenti & Bertin 2005; Trenti, Bertin & van Albada 2005), and from more elaborate three-dimensional  $N$ -body simulations (e.g. van Albada 1982; May & van Albada 1984; McGlynn 1984; Villumsen 1984; Aguilar 1988; Palmer, Papaloizou & Allen 1990; Funato, Makino & Ebisuzaki 1992b). These simulations showed that not all initial conditions would lead to the same final end state; in special, only clumpy, ‘cold’ initial states would preferentially lead to objects more closely resembling the profiles of elliptical galaxies.

Further extensions and/or elaborations over Lynden-Bell’s idea (e.g. Severne & Luwel 1980; Gurzadian & Savvidy 1986; Spergel & Hernquist 1992; Kull, Treumann & Boehringer 1997), as well as other developments towards a more consistent theory of gravitational relaxation (e.g. Shu 1978; Wiechen, Ziegler & Schindler 1988; Ziegler & Wiechen 1989, 1990; Chavanis, Sommeria & Robert 1996; Nakamura 2000), were subsequently carried out. The main difference between several of these approaches lies in the adopted definition of entropy and the constraints under which it is maximized. Even though there is now a large body of work towards a theory of the gravitational relaxation phenomenon, it remains a challenging endeavour, and in fact a deeper evaluation of some of these theories lead to severe inconsistencies (*viz.*, Arad & Johansson 2005; Arad & Lynden-Bell 2005).<sup>5</sup>

<sup>4</sup>This fact leads to several investigations on the properties of *truncated* isothermal spheres, that is, constrained to a finite spherical shell and truncated at a given energy. Although such a procedure seems too arbitrary, it can be formally justified observing that the external parts of a gravitational  $N$ -body system behave just like the boundary conditions artificially introduced by a fixed shell. Such an approach has been proved useful to the physical modelling of the problem, since the type of shell introduced (and consequently the type of physical quantity that is exchanged with the external reservoir) describes the system as a member of a specific kind of *ensemble* (e.g. Padmanabhan 1990).

<sup>5</sup>Note at this point that another physical mechanism that is known to contribute to the collisionless relaxation of gravitational systems is ‘phase mixing’, in which the phase-space microscopic cells are continually stretched into thinner and thinner filaments as relaxation proceeds, until the coarse-grained phase-space density no longer changes, and the system is said to have

Parallel to the developments in stellar dynamics,  $N$ -body cosmological simulations of hierarchically clustering model universes started to address, at increasingly higher spatial and temporal resolutions, the issue of the dynamical and structural properties of virialized dark matter haloes. For instance, the importance of mergers of smaller subsystems – as opposed to simple collapses – to the formation of elliptical galaxies and dark matter haloes started to become clear important issues (e.g. Frenk et al. 1985; Zhang et al. 2002). The main result, however, is the existence of a universal halo profile (Navarro, Frenk & White 1997; Navarro et al. 2004), to which an explanation in terms of the dynamical evolution and expected final equilibrium state is still being searched for (e.g. Austin et al. 2005; Barnes et al. 2005; Dehnen & McLaughlin 2005; Lu et al. 2006). Note that, as previously mentioned; simulations of isolated collapses (and mergers) tend to produce different end states depending on the initial conditions. So the apparent universality of dark matter halo profiles in cosmological simulations could be interpreted as the fact that the underlying relaxation process in such scenarios produce systems in a state close to the most probable one, as derived according to some more fundamental relaxation principle, yet to be uncovered.

### 4.3 How does the ERP phenomenon fit in?

Clearly, ‘violent relaxation’ is routinely observed in  $N$ -body experiments, but it rapidly damps out at about the free-fall time-scale of the system. Gravitational potential fluctuations can be violent enough to mix the one-particle phase-space structure, but not so violent as to remove all ‘memory’ of the initial coarse-grained, mean particle energy ordering.

Dissipationless relaxation must therefore be described as a process in which collections of particles closely spaced in energy space can efficiently mix in phase space, but underlying such a mixing, there is a restriction to the exchange of energy, which must proceed within some upper and lower bounds. Such bounds can only change linearly and orderly with respect to all other energy cells of the system, or at least must be restored subsequently to the major potential fluctuations. This suggests that a line of investigation to the ERP effect would be a formulation based on a maximum entropy principle in which a further constraint, in addition to the total mass and energy, must be imposed, namely, a ‘mesoscopic constraint’, operative only at the level of a coarse-grained distribution function (Kandrup et al. 1993).

Some clues on how this theory must be accomplished, at our present stage of understanding, are outlined as follows.

- (1) The development of a physically well-motivated, incomplete treatment of the ‘violent relaxation’ process will be an indispensable step to the formulation (see e.g. Hjorth & Madsen 1991).
- (2) The underlying ‘mesoscopic’ constraint must hold even under large fluctuations of the gravitational potential field (Kandrup et al. 1993).
- (3) Some heuristic inputs must be taken into account, which we collect from this and previous work, namely as listed below.

reached a coarse-grained steady-state configuration (Binney & Tremaine 1987). However, ‘phase mixing’ and ‘violent relaxation’ (whatever the given potentials admit or not the coexistence of regular and chaotic orbits) are, in fact special cases of a more general relaxation process of collisionless gravitational systems (Ziegler, Wiechen & Arendt 1994; Kandrup 1998).

(i) Evidences that the ERP depends on halo mass, in the sense that more-massive haloes show more rank preservation than less-massive ones (Dantas & Ramos 2006).

(ii) Some reasonable validity of Arnold’s theorem when applied to individual haloes, showing that the ‘mesoscopic’ constraint acts at the level where collections of particles behave dynamically as an individual particle with a characteristic frequency (or alternatively, energy) in the mean potential field of the halo (Dantas & Ramos 2006).

(iii) The fact that cosmic flows evolve in an orderly sense, when tracked from their coarse-grained energy cells, even when non-linearities are already developed (Fig. 3).

(iv) Other ‘memory’-like effects, such as those evident in higher order statistics of the PVDs, for scales  $R \leq 10$  Mpc (Fig. 2).

(v) Other scaling correlations associated with the ERP phenomenon, namely, that simulations of galactic mergers produce ‘fundamental plane’-like objects and some energy rank violation; whereas simple collapses are homologous systems and show clear ERP (Dantas et al. 2003).

## 5 CONCLUSION

Summarizing in this paper, we have analysed the behaviour of a cosmological  $N$ -body simulation in the linear and non-linear regimes as a function of different energy partitionings. We have found evidences that the ERP effect, already observed in different, more restricted contexts, is also present in a large-scale, realistic cosmological  $N$ -body simulation from the Virgo Consortium. We have observed that ERP-like ‘memory’ effects are also evident in higher order statistics of the PVDs, for scales  $R \leq 10$  Mpc. Finally, we have noted that there is a general scaling law relating the energy cells to the energy-constrained PVD velocity dispersion that appears to hold regardless the dynamical regime being probed.

The expansion of the cosmological background brings more complexity to the virialization problem when compared with the ‘simpler’ studies of statistical mechanics of isolated gravitating systems (Padmanabhan & Engineer 1998). There are, however, common effects that persist in both scenarios, like the ERP phenomenon investigated here. Although several insights can be obtained from the bulk of current knowledge on the non-linear gravitational evolution, a complete theory is yet to be derived (Padmanabhan 2005). For instance, when viewed from the energy-space perspective, the cosmic motions and clusterings, at several stages of development (from the linear throughout the non-linear phase), do display a certain degree of simplicity and an unexpected level of order. The ERP phenomenon is certainly an important ingredient to the outstanding open problem of the nature of the collisionless relaxation process.

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