

# Dempster-Shafer Theory as an Inference Method for Corresponding Geometric Distorted Images

José Demisio Simões da Silva<sup>1,2</sup>, Paulo Ouverá Simoni<sup>2,3</sup>

<sup>1</sup>Instituto Nacional de Pesquisas Espaciais - INPE, Laboratório Associado de Computação e Matemática Aplicada - LAC Av. dos Astronautas, 1758, São José dos Campos, 12227010, SP, Brazil

<sup>2</sup>Univerisdade Braz Cubas - UBC, Av. Francisco Rodrigues Filho, 1233, Mogi das Cruzes, SP, Brazil

<sup>3</sup>Universidade de Guarulhos - UnG, Praça Tereza Cristina, 1, Guarulhos, SP, Brazil  
[demisio@lac.inpe.br](mailto:demisio@lac.inpe.br), [paulosimoni@ig.com.br](mailto:paulosimoni@ig.com.br)

***Abstract.** This paper presents further results of the application of Dempster-Shafer Theory for uncertainty reasoning as a computation model to correspond distorted images in Computer Vision. In previous work (Silva and Simoni, 2002), the model was applied to correspond images that presented differences in brightness and contrast, as an extension of the work in (Silva and Simoni, 2001b). The results showed the effectiveness and robustness of the method in corresponding non-equalized pairs of images and led to new experiments in which the model is applied to distorted images, that is, pairs of images in which one is rotated in relation to the other. The right image was rotated in 7 different angles (0.5; 1; 1.5; 2; 2.5; 3; and 3.5 degrees). The model was applied to correspond only point in the images and it successfully established the correspondence for the rotated images, except for the 3.5 degrees rotation. The correspondence is based on contextual and structural features of a point, treated as corresponding evidences whose combination is performed by a modified version of Dempster's rule of combination. A search process maximizes the Belief on the combined evidences.*

## 1 Introduction

Spatially separated cameras may provide images from which one may infer 3D information of the environment. A number of researches have been conducted in trying to solve the biggest problem in this task, that is, the correspondence of the images. Anadan (1985), Barnard and Thompson (1980), and Jones (1997) report different methods to approach the correspondence problem. New algorithms have recently been proposed. However a general solution has not been achieved due to the ill-posed nature of the problem, resulting from the existence of occlusion, geometric distortions, etc. from one image to the other. Adaptive approaches include neural networks, regularization, learning strategies, optimization techniques (Jones, 1997), and genetic algorithms (Saito and Mori, 1995; Silva and Simoni, 2000a; Silva and Simoni, 2000b; Silva and Simoni, 2001a).

The algorithms to solve the correspondence problem require the establishment of reference features that can be easily identified in both images. Such reference features are image elements based within a window from which it is possible to gather area token information. Thus, the algorithms are area-based, token-based or a hybridization of both. In area-based algorithms the size of the window has to be appropriately determined to include relevant information to be used as correspondence evidence. Token-based algorithms depend on token extraction and definition, thus requiring image preprocessing to detect the edges that may be a time consuming task. Hybrid algorithms combine both area and token approaches and may inherit the advantages and disadvantages of both.

Once reference features have been established in one image, a search has to be conducted to find similar features in the other images. A large number of candidates may result, from which a unique correspondence has to be picked to satisfy the problem constraints.

In corresponding the extracted features, comparisons are performed by measuring feature similarity by using Cross-Correlation, Sum of Squared Differences, Euclidean Distance, Hamming Distance, or any other metrics that can be taken as similarity or dissimilarity metrics.

New algorithms have been proposed attempting to introduce new aspects and possibilities in the correspondence of image. Kanade and Okutomi (1994) proposed an algorithm in which adaptive window sizes are searched to satisfy the constraints for the problem, that is, compatibility, uniqueness, and map continuity (Marr, 1982). The work was further improved by Saito and Mori (1995), which proposed a search based on genetic algorithms to come out with the optimal window size to provide the closest disparity map in relation to its statistical model. This GA approach searches for a disparity map that optimizes both the compatibility of the corresponding points and the map continuity.

However, the algorithms generally fail in establishing the correspondence under existing image distortions, such as occlusion from one image to the other, illumination changes, contrast changes, geometric distortions, or/and the size and shape of the windows used.

In Silva and Simoni (2001) we proposed a point wise hybrid approach for the correspondence problem, in which the Island Model Parallel Genetic Algorithm performed the search. The individuals in the population represented possible correspondences, whose fitness was associated to the combination of the corresponding evidences by Dempster-Shafer's rule of combination for uncertainty reasoning. The model was applied to simultaneously establish multiple correspondences. Area features were restricted to a window around the point leading to the *contextual* features. The tokens were related to *structural* features of a point within the window, directly associated to existing edges within the window. The correspondence was hierarchically developed (Silva and Simoni, 2000b): a low-level point correspondence *satisfying the uniqueness and point compatibility constraints*; and a high-level correspondence of the polygonal regions, *satisfying the structural coherence constraint related to the map continuity constraint*. The model was sensible to the number of candidate points for the reference points, the complexity of the geometric features of the polygonal regions, and

the existence of occlusion in the images. The parallel GA addressed the problem related to the large number of candidates.

The model was further applied to correspond images that presented illumination and contrast changes (Silva and Simoni, 2002). The motivation was the fact one may reason that, even in the presence of adverse conditions, it is possible to establish partial correspondences between images, meaning image parts may still be corresponded despite the non-existence of a formal way to estimate the distortions in the images. The conducted experiments used artificially distorted images and led to important results that showed the robustness of the model.

In this paper, we further explore the uncertainty-reasoning model by applying it to pairs of images that present geometric differences from one image to the other. The aim is to check the robustness of the method when applied to pairs of images in which one image is rotated in relation to the other. As in (Silva and Simoni, 2002), the model is applied to establish the correspondence between a pair of points.

In Section 2, we briefly review the methodology for extracting corresponding evidences in both images. In Section 3, we comment on Dempster-Shafer theory when applied to the correspondence problem. In section 4 we describe the conducted experiments and their results. Finally, in Section 5 we present some conclusions.

## 2 Corresponding Evidences

As in Silva and Simoni (2001) and Silva and Simoni (2002), the correspondence model presented in this paper is based on existing *contextual* and *structural* evidences of the reference and candidate points in both images. Such evidences are related to the size of the window around the point (*contextual*) and to binary edge elements within the window as described in Figure 2, whose extraction was performed by Perceptron neuron networks and in Figure 1.

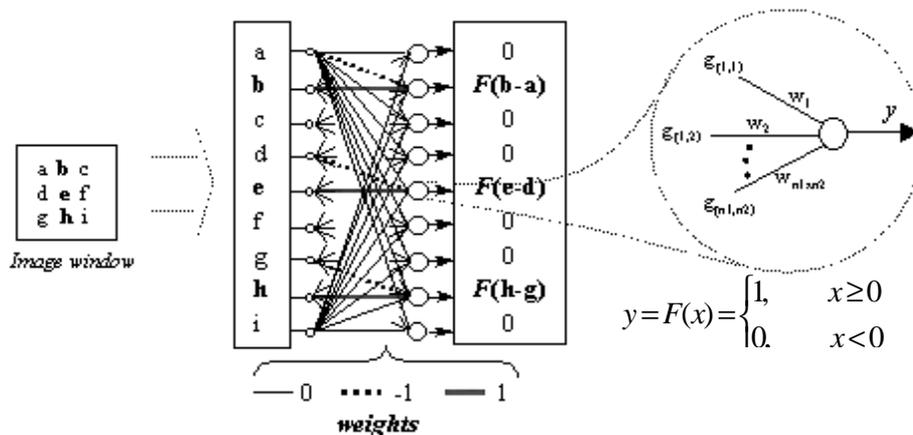


Figure 1. The vertical line structure in a 3 x 3 window. Weights are inserted as shown.

By extending the definition of connectivity to  $\left[\frac{(n-1)}{2}\right]_4$  and  $\left[\frac{(n-1)}{2}\right]_8$  connectivity ( $n$  is the size of the window,  $n=3, 5, 7, 9, 11, \dots$ ) we proposed two additional structural evidences: the *pattern of differences* and the *predominant structure* (Silva and Simoni, 2001). For the  $\left[\frac{(3-1)}{2}\right]_8$ -connected  $3 \times 3$  image window in Figure 1, the pattern of difference  $D$  is given by equation (1) and it is implemented by a Perceptron neural network as in Figure 1.

$$D=[F(b-e), F(c-e), F(f-e), F(i-e), F(h-e), F(g-e), F(d-e), F(a-e)] \quad (1)$$

( $F$  is the threshold function).

The *predominant structure* results from the computation of the energy of the pixels inside the window, in pre-defined directions given by morphological kernels as in Figure 2 for a  $3 \times 3$  window (different window sizes require redefinition of such kernels). The kernel with higher energy is chosen as the *predominant structure*. In Figure 2, 20 binary morphological kernels are presented for a  $3 \times 3$ -image window. Larger the windows and larger number of structural features lead to more complex binary structures become that may increase the likelihood of the uniqueness constraint to be satisfied.

0 0 0	0 1 0	0 1 0	0 0 0	0 0 0	0 1 0	1 0 0	0 0 1	1 0 0	0 0 1
1 1 1	0 1 0	0 1 1	0 1 1	1 1 0	1 1 0	0 1 0	0 1 0	0 1 0	0 1 0
0 0 0	0 1 0	0 0 0	0 1 0	0 1 0	0 0 0	0 0 1	1 0 0	1 0 0	0 0 1
1 0 1	0 0 0	0 1 0	0 1 0	0 0 1	1 0 0	0 0 0	0 0 1	0 0 0	1 0 0
0 1 0	0 1 0	0 1 0	0 1 0	0 1 0	0 1 0	1 1 0	1 1 0	0 1 1	0 1 1
0 0 0	1 0 1	1 0 0	0 0 1	0 1 0	0 1 0	0 0 1	0 0 0	1 0 0	0 0 0

**Figure 2. Basic binary morphological structures within a  $3 \times 3$  window.**

A pair of points (reference and a candidate points) correspond if both lie within similar contexts and belong to similar structures. Thus, the correspondence may intuitively be established by comparison of *contextual* and *structural features* of the points. Such comparison require the use similarity criteria, such as: correlation between the *Micro areas* ( $C_{micro}$ ); correlation between the *Macro areas* ( $C_{macro}$ ); Hamming distance among binary structures ( $N_1 N_2 N_3 N_4 N_5 N_6 N_7 N_8 N_9 N_{10}$ ); Absolute difference between gray levels ( $G$ ) (Silva and Simoni, 2001).

The candidate points are then assigned a vector with similarity measurements for 13 different matching criteria, together with the point line and pixel coordinates ( $l_j, p_j$ ).

$$Q_i=[l_j p_j C_{macro} N_1 N_2 N_3 N_4 N_5 N_6 N_7 N_8 N_9 N_{10} G C_{micro}] \quad (2)$$

The evidence extraction procedures were first proposed in Silva and Simoni (2000a) and used in Silva and Simoni (2000b), (2001), (2002), and in this paper.

### 3 Dempster-Shafer Theory in the Correspondence Problem

In Dempster-Shafer (DS) Theory, requires a *Universe of Discourse* (or *Frame of Discernment*)  $U$  consisting of mutually exclusive alternatives, corresponding to an attribute value domain (Giarratano and Riley, 1994). For instance, in satellite image classification the set  $U$  may consist of all possible classes of interest.

Each subset  $S \subseteq U$  is assigned a basic probability  $m(S)$ , a belief  $Bel(S)$ , and a plausible belief (or plausibility)  $Pls(S)$  so that:

$$m(S), Bel(S), Pls(S) \in [0,1] \text{ and } Pls(S) \geq Bel(S) \quad (3)$$

The basic probability  $m$  represents the strength of an evidence. Thus, a group of pixels (for example) that belong to a certain class, may be assigned a value  $m$  representing the effect of the pixels as representative of the class.  $Bel(S)$  summarizes all the reasons to believe  $S$ .  $Pls(S)$  expresses how much one should believe in  $S$  if all currently unknown facts were to support  $S$ . The true belief in  $S$  is somewhere in the *belief interval*  $[Bel(S), Pls(S)]$ .

The basic Probability Assignment  $m$  is defined as the function

$$m : 2^U \rightarrow [0,1] \quad (4)$$

where  $m(\emptyset) = 0$  and the sum of  $m$  over all subsets of  $U$  is 1 ( $\sum_{S \subseteq U} m(S) = 1$ ). For a given basic probability assignment  $m$ , the belief ( $Bel$ ) of a subset  $A$  of  $U$  is the sum of  $m(B)$  for all subsets  $B$  of  $A$ , and the plausibility ( $Pls$ ) of a subset  $A$  of  $U$  is

$$Pls(A) = 1 - Bel(A') \quad (5)$$

where  $A'$  is the complement of  $A$  in  $U$ .

The rule of combination states that two basic probability assignments  $m_1$  and  $m_2$  are combined into a third basic probability assignment by the normalized orthogonal sum  $m_1 \oplus m_2$  defined as:

$$m_1 \oplus m_2(A) = \frac{\sum_{X \cap Y = A} m_1(X)m_2(Y)}{1 - k}, \quad k = \sum_{X \cap Y = \emptyset} m_1(X)m_2(Y) \quad (6)$$

The original rule of combination if equation (6) is computationally expensive. A faster alternative is given in Haddawy (1987) as may be seen in equations (7) and (8) that directly combine beliefs and plausibilities, directly assigned to the existing corresponding evidences.

$$Bel(S) = 1 - \frac{(1 - Bel_1(S))(1 - Bel_2(S))}{1 - [Bel_1(S)(1 - Pls_2(S)) + (1 - Pls_1(S))Bel_2(S)]} \quad (7)$$

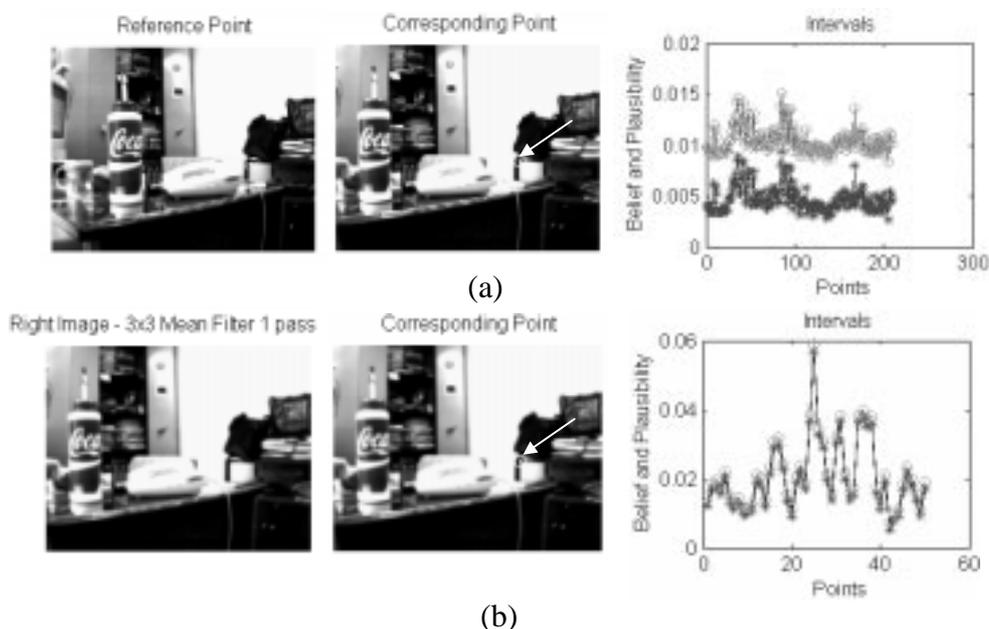
$$Pls(S) = \frac{Pls_1(S)Pls_2(S)}{1 - [Bel_1(S)(1 - Pls_2(S)) + (1 - Pls_1(S))Bel_2(S)]} \quad (8)$$

The Dempster-Shafer evidence combination is preceded by evidence (similarity measurements) extraction. A reference point is set in an image, for which *context* and *structural* features are computed. Several points (*candidates*) are picked that lie in similar *context* and belong to similar structures. The *candidate* points form the *Universe of Discourse* that is dynamically established for each new reference point. In any case, the subsets of possible solutions are singletons, that is, one element set.

The existence of possible occlusion, illumination differences, and image distortions is modeled by an *uncertain factor* assigned to the problem. Beliefs are directly assigned to the available information. The Dempster-Shafer calculus combines the available evidences resulting in a belief and a plausibility in the combined evidence that represents a consensus on the correspondence. The model maximizes the belief in the combined evidences.

#### 4 Implementation and Results

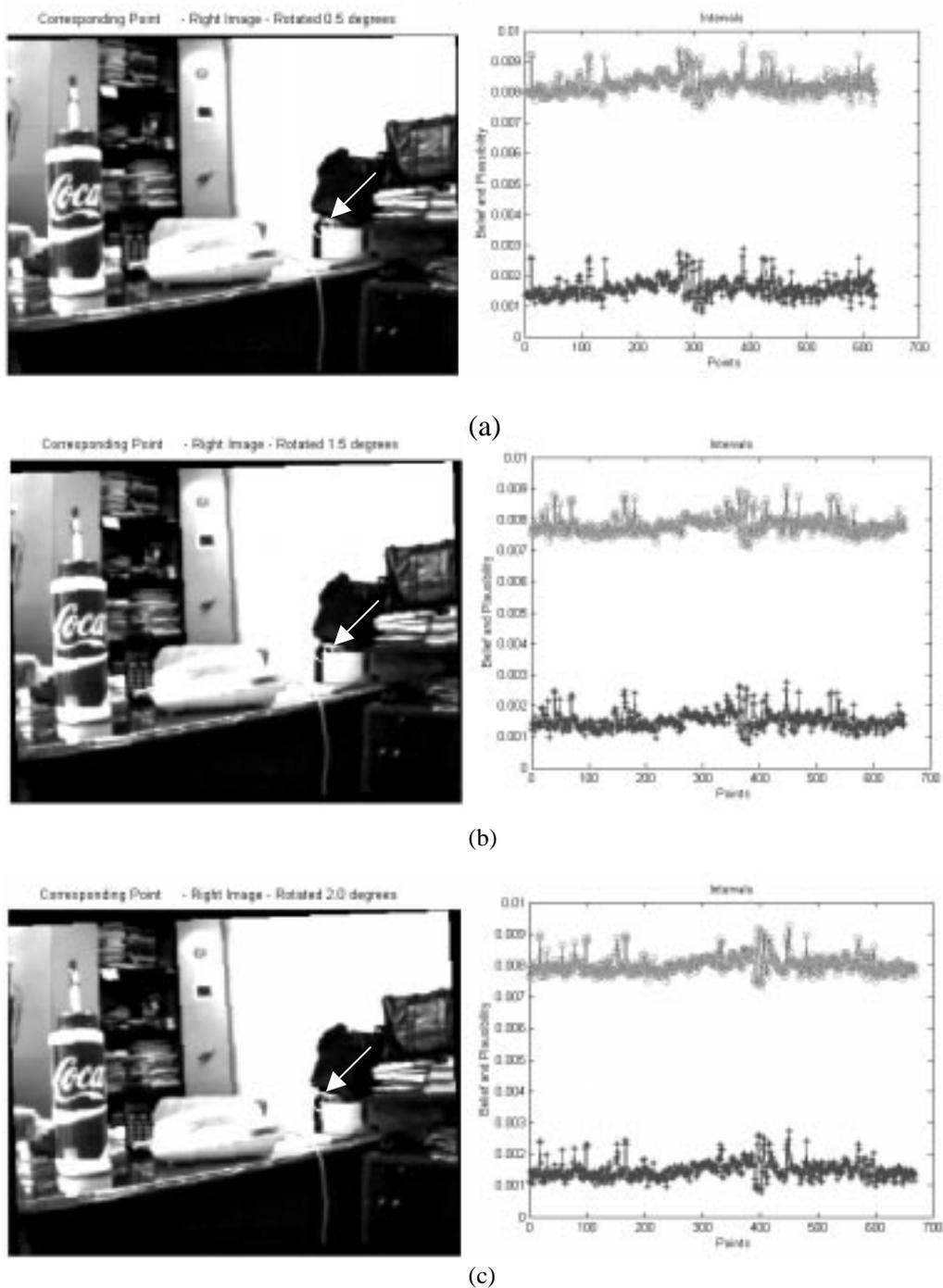
In previous works we showed the adequacy and robustness of the uncertainty reasoning approach to the correspondence problem (Figures 3.a and 3.b) (Silva and Simoni, 2001; Silva and Simoni, 2002). In this paper we investigate the robustness of the model when applied to images that present geometric distortions from one image to the other.



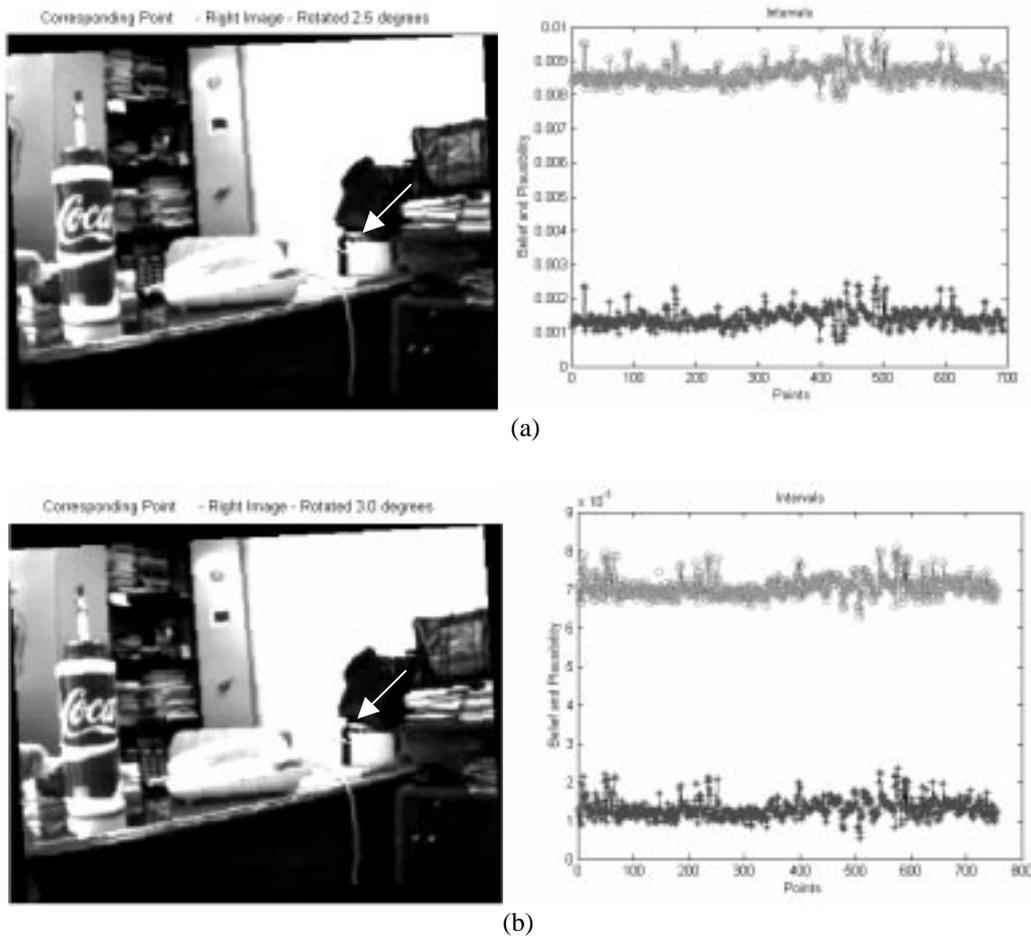
**Figure 3 - (a) Correspondence found between images with similar brightness [From Silva and Simoni (2001)]. (b) Correspondence found - Right image filtered (1 pass) with the 3x3 low pass mean filter [From Silva and Simoni (2002)].**

Rotating the right image by 0.5, 1.0, 1.5, 2.0, 2.5, 3.0 and 3.5 degrees simulated the effects of geometric distortions. Contrast and illumination were not changed in the images, because the aim is to check the robustness of the model for rotated images. Distorted images may result from the use of uncalibrated. Such may have many

geometric differences for which an inverse problem of determining camera coordinates is needed to geometrically relate on image to the other. In this paper we propose the application of the Dempster-Shafer approach in an attempt to reduce the dependence of preprocessing that may be time consuming. The idea is to check the limits of the model to establish the correspondence for raw images, that is, for non-preprocessed images.



**Figure 4 - (a) Correspondences found - Right image rotated of 0.5 degrees; (b) Right image rotated of 1.5 degrees; c) Right image rotated of 2.0 degrees.**



**Figure 5 - (a) Correspondences found - Right image rotated of 2.5 degrees; (b) Right image rotated of 3.0 degrees.**

**Table 1 - Line and Pixel coordinates of the corresponding point found in the conducted experiments.**

Experiment	line	pixel
<i>Results in Silva and Simoni (2001) - Figure 3.a</i>	137	224
<i>Results for a 3x3 Mean filter (1 pass) in Silva and Simoni (2002) - Figure 3.b</i>	137	224
Right image rotated 0.5 degrees	140	226
Right image rotated 1.0 degrees	140	227
Right image rotated 1.5 degrees	142	228
Right image rotated 2.0 degrees	142	229
Right image rotated 2.5 degrees	143	230
Right image rotated 3.0 degrees	144	231
Right image rotated 3.5 degrees	145	116

In the results presented in Figures 4.a), 4.b), 4.c), 5.a) and 5.b) the Dempster-Shafer model was applied to the images when the right image was rotated in 0.5, 1.5,

2.0, 2.5, and 3.0 degrees. Table 1 presents the coordinates of the points found in the experiments described in Figures 4 and 5. Visual analysis of Figures 4 and 5 show the model was able to approximate the corresponding point with minimum error, for the rotation angles of 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0. The rotation angle of 3.5 degrees was found to be the limit for the experiments conducted.

## 5 Conclusion

In this paper, we present further results of an ongoing research that investigates the use of an uncertainty reasoning and genetic algorithm based approach to the correspondence problem in Computer Vision. The robustness of the method previously developed (Silva and Simoni, 2001) is investigated when it is applied to geometrically distorted images. The corresponding evidences are the contextual and structural evidences. Perceptron neural networks are used to extract features in the neighborhood of the points.

From Table 1 and visual analyses of Figures 4 and 5, one may infer a certain degree of robustness of the approach, when applied to images that are slightly rotated. The experiments showed high angles values lead to big errors if the same parameters are used (such the size of the window). However, the success achieved by applying the methodology to the distorted images, shows the importance of inserting a degree of uncertainty in the process.

In order to produce the results in this paper, the original right image was artificially rotated of predefined angles. Besides, only one point was corresponded as in Silva and Simoni (2002).

Future work will be conducted to verify the robustness of the uncertainty reasoning methodology to correspond simultaneous points in real world images that present geometric distortions as discussed in this paper. Also, the method will be tested with images that simultaneously present illumination and/or contrast changes, and geometric distortions. A specific geometric distortion will be related to differences in scale from one image to the other.

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