MULTI-OBJECTIVE OPTIMIZATION APPLIED TO A SATELLITE CONSTELLATION CONSIDERING UNCERTAINTIES IN THE POSITION MEASURES AND ERRORS IN THE ORBITAL MANEUVERS

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In this work the problem of orbital maintenance of a symmetrical constellation of satellites, with minimum fuel consumption, is studied using bi-impulsive maneuvers with time constraint. When all the satellites are considered it is almost impossible to determine the optimal maneuver strategy that minimizes the fuel consumption. Therefore, the goal of this work is to formulate and to study maneuver strategies that make possible to obtain solutions with small fuel consumption considering all the satellites in the constellation. The problem can be formulated as a multi-objective problem due to the nature of the station-keeping of a satellite constellation. Thus, the multi-objective problem was defined and a new multi-objective optimization method was applied. This method can consider n conflicting objectives simultaneously without reducing the problem to an optimization of only one objective, as occurs with most of the methods found in the literature. This method, called the Smallest Loss Criterion, was presented in Rocco et al. (2003). It was compared with other existent methods and it was verified that it may supply better results to the problem of the station-keeping of satellite constellations. In this work, some applications of the Smallest Loss Criterion, presented in Rocco et al. (2003) and Rocco et al. (2005), are reproduced considering now that the satellite positions are determined using GPS measures and that there are errors in the magnitude and direction of the impulses applied in the orbital maneuvers. So, these measures of the satellite positions contain uncertainties that are filtered by a Kalman filter.

Keywords. *Astrodynamics, Orbital Transfer, Satellite Constellation, Maneuvers Optimization, Kalman Filter.*

1. Introduction

In this work the problem of orbital maintenance of symmetrical constellations of satellites, with minimum fuel consumption, is studied using impulsive maneuvers with time constraint. To perform the station keeping of a constellation the problem of optimizing simultaneously the maneuvers for *n* satellites must be studied. When all the satellites are considered it is not simple to determine the optimal maneuver strategy that minimizes the fuel consumption with time constraint. The problem can be formulated as a multi-objective problem due to the nature of the station-keeping of a satellite constellation. Thus, the multi-objective problem applied to satellite constellations was defined and a new multi-objective optimization method was applied. This method can consider *n* conflicting objectives simultaneously without reducing the problem to an optimization of only one objective, as occurs with most of the methods found in the literature. This method, called The Smallest Loss Criterion, was presented in Rocco (2002), Rocco et al. (2003) and Rocco et al. (2005). In this work, some applications of the Smallest Loss Criterion, are reproduced considering now that the satellite positions are determined using GPS measures. These measures of the satellite positions contains uncertainties that are filtered by a Kalman filter. Another extension considered by this work is the introduction of errors in the magnitude and direction of the impulses applied in the orbital maneuvers. The uncertainty of the position and the maneuver errors turn the problem similar to the real case, so the application to real missions becomes more accurate.

It was considered that the orbital maintenance of the satellites is made in an autonomous way, that is, the satellites can communicate with each other in order to obtain de position of each, and determine the best set of maneuver to be applied. Therefore, the problem is not to define which satellite will be maneuver, but how to simultaneously maneuver all the satellites. However the autonomous maneuver strategy, that was adopted, is not the main goal of this work. It was used as a form of eliminating the dependence of a control center and a ground tracking station. Thus, the effort was concentrated on the resolution of the multi-objective problem relative to the choice of the best set of maneuvers and not in problems of visibility and connection with the ground.

2. The Multi-Objective Problem

According to Cohon (1978), the static optimization of one objective problem can be defined in the following way:

$$\begin{aligned} &\text{Maximize} \quad \mathrm{Z}(\mathbf{x}) \ \text{ with relation of } \ \mathbf{x} \in \mathbf{R}^n \\ &\text{Subject to} \quad \mathrm{g_i}(\mathbf{x}) \le 0 \qquad i = 1, 2, \ldots, m \\ &\qquad\qquad\quad \mathbf{x} \ge 0 \\ &\text{Given} \qquad \mathrm{Z}(\cdot),\ g_i(\cdot) \end{aligned} \tag{1}$$

or

$$\begin{aligned} &\text{Maximize} \quad \mathrm{Z}(\mathbf{x}) \ \text{ with relation to } \ \mathbf{x} \in \mathbf{R}^n \\ &\text{Subject to} \quad \mathbf{x} \in \mathbf{F}_d \\ &\text{Given} \qquad \mathrm{Z}(.),\ \mathbf{F}_d \end{aligned} \tag{2}$$

where $\mathbf{F}_d$ is the feasible area of the decision space, defined by:

$$\mathbf{F}_d = \left\{ \mathbf{x} \in \mathbf{R}^n \mid \mathrm{g_i}(\mathbf{x}) \le 0, i = 1, 2, \ldots, m; \mathbf{x} \ge 0 \right\} \tag{3}$$

The multi-objective problem can be defined by:

$$\begin{aligned} &\text{Maximize} \quad \mathbf{Z}(\mathbf{x}) = \left[\mathrm{Z}_1(\mathbf{x}), \mathrm{Z}_2(\mathbf{x}), \ldots, \mathrm{Z}_p(\mathbf{x}) \right] \\ &\text{Subject to} \quad \mathbf{x} \in \mathbf{F}_d \end{aligned} \tag{4}$$

Therefore, in this case, the objective function, is a vector with dimension p. Thus, the multi-objective optimization consists of the minimization (or maximization) of a vector of objectives $\mathbf{Z}(\mathbf{x})$ that may be subject to constraints or bounds. This kind of problem is very common in a station keeping of satellite constellations because during the maneuvers many parameters must be optimized at the same time. In problems of unidimensional optimization (when we have one objective), the possible solutions ($\mathbf{x} \in \mathbf{F}_d$) can be compared by means of the objective function, that is, given two solutions $\mathbf{x}^1$ and $\mathbf{x}^2$ we can compare $Z(\mathbf{x}^1)$ with $Z(\mathbf{x}^2)$ and determine the optimal solution so that $\mathbf{x} \in \mathbf{F}_d$ doesn't exist such as $Z(\mathbf{x}) > Z(\mathbf{x}^*)$. In problems of multi-dimensional optimization (multi-objective problem), in general, it is not possible to compare all the possible solutions because the comparison on the basis of one objective can be contradicted with the comparison based on another objective. Namely, supposing that:

$$\mathbf{Z}(\mathbf{x}^1) = \left[\mathrm{Z}_1(\mathbf{x}^1), \mathrm{Z}_2(\mathbf{x}^1) \right] \quad \text{and} \quad \mathbf{Z}(\mathbf{x}^2) = \left[\mathrm{Z}_1(\mathbf{x}^2), \mathrm{Z}_2(\mathbf{x}^2) \right] \tag{5}$$

$\mathbf{x}^1$ is better than $\mathbf{x}^2$ if and only if:

$$\mathrm{Z}_1(\mathbf{x}^1) > \mathrm{Z}_1(\mathbf{x}^2) \text{ and } \mathrm{Z}_2(\mathbf{x}^1) \ge \mathrm{Z}_2(\mathbf{x}^2) \quad \text{or} \quad \mathrm{Z}_1(\mathbf{x}^1) \ge \mathrm{Z}_1(\mathbf{x}^2) \text{ and } \mathrm{Z}_2(\mathbf{x}^1) > \mathrm{Z}_2(\mathbf{x}^2) \tag{6}$$

If $\mathrm{Z}_1(\mathbf{x}^1) > \mathrm{Z}_1(\mathbf{x}^2)$ and $\mathrm{Z}_2(\mathbf{x}^1) < \mathrm{Z}_2(\mathbf{x}^2)$ we cannot conclude anything regarding $\mathbf{x}^1$ and $\mathbf{x}^2$.

3. The Samallest Loss Criterion

As previously shown in Rocco et al. (2003), there are several multi-objective optimization methods. The methodologies found in the literature generally start the problem with a multi-objective approach but end reducing the problem to the unidimensional case, by means of simplifications or influence factors. Or, when the approach was really multi-objective, the result found was a group of solutions candidates to the optimal solution, and in this case, for the choice of the optimal solution other external approaches to the problem should be used. In practical applications, it would be convenient to apply a methodology capable to find the solution that assists all the objectives.

The Smallest Loss Criterion, presented in Rocco (2002) and Rocco et al. (2003), may be enunciated in the following way: In a problem with *n* conflicting objectives, where the intention is to optimize the *n* objectives simultaneously, privileging none of them, the solution should be the one which results in the smallest loss for each one of the objectives. Because there is no solution which optimizes simultaneously the *n* objectives individually.

An attempt to reach this solution would be to find the barycenter of a normalized *n*-dimensional figure, where the vertexes are the optimal solution for each objective. Therefore, for problems with three objectives the solution would be in the center of a normalized triangle, for *n* objectives the solution would be in the center of a normalized *n*-dimensional figure. Figure 1 shows an example of this criterion applied to a problem with three conflicting objectives. In this example, *S*1, *S*2 and *S*3 are the optimal solutions for each one of the objectives, considered separately. *B* is the barycenter of the triangle formed by the segments $\overline{S1S2}$, $\overline{S2S3}$ and $\overline{S3S1}$. By the barycenter definition, the distance from *B* to the vertexes of the triangle represented by the solutions *S*1, *S*2 and *S*3 is the same. So, if the barycenter *B* is adopted as a solution for the multi-objective problem, the segment $\overline{S1B}$ represents the loss in relation to objective 1, and in the same way, the segments $\overline{S2B}$ and $\overline{S3B}$ represent the loss in relation to objectives 2 and 3 respectively. Thus, from Figure 1 it can be concluded that if the three objectives are equally considered, the best solution is that one which coincides with the barycenter of the triangle.

4. The Satellite Position Measures

In this work a constellation composed by three satellites with circular and equatorial nominal orbits is used. Therefore the orbital nominal elements of the satellites are given by:

$$e = 0.00000000 \qquad i = 0.00000000$$
$$l = 7010.00000000 \text{ km} \qquad \omega = 0.00000000$$
$$a = 7010.00000000 \text{ km} \qquad \Omega = 0.00000000$$

where e is the eccentricity, l is the semi-latus rectum, ω is the argument of perigee, a is the semi-major axis, i is the inclination and Ω is the right ascension of the ascending node.

It was considered that to assist the specifications of the mission, the satellites should be positioned in such a way that the difference among the true longitudes of the satellites (θ_1, θ_2 and θ_3) should be equal to 120°. With the actual true longitudes, the position constraints $\delta\theta_1$, $\delta\theta_2$, $\delta\theta_3$ and $\delta\theta$ which represent the position error of the satellites, can be calculated, as shown in Figure 2.

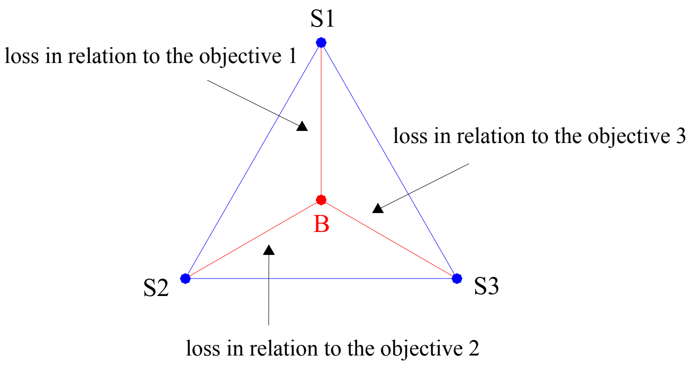

Fig. 1 – The Smallest Loss Criterion.

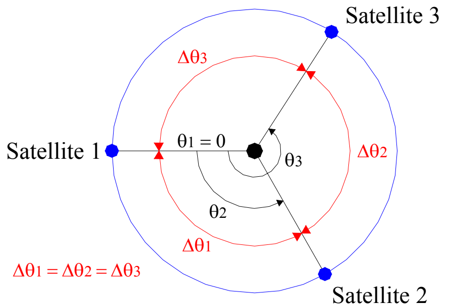

Fig. 2 – Nominal position of the satellites.

$$\theta_1 \le \theta_2: \quad \delta\theta_1 = (\theta_2 - \theta_1) - \frac{2\pi}{3} \tag{7}$$

$$\theta_2 < \theta_1: \quad \delta\theta_1 = (2\pi - \theta_1 + \theta_2) - \frac{2\pi}{3} \tag{8}$$

$$\theta_2 \le \theta_3: \quad \delta\theta_2 = (\theta_3 - \theta_2) - \frac{2\pi}{3} \tag{9}$$

$$\theta_3 < \theta_2: \quad \delta\theta_2 = (2\pi - \theta_2 + \theta_3) - \frac{2\pi}{3} \tag{10}$$

$$\theta_3 \le \theta_1: \quad \delta\theta_3 = (\theta_1 - \theta_3) - \frac{2\pi}{3} \tag{11}$$

$$\theta_1 < \theta_3: \quad \delta\theta_3 = (2\pi - \theta_3 + \theta_1) - \frac{2\pi}{3} \tag{12}$$

$$\delta\theta = \frac{|\delta\theta_1| + |\delta\theta_2| + |\delta\theta_3|}{3} \tag{13}$$

But in this work, it was considered that the position of the satellites (θ_1, θ_2 and θ_3) was obtained using a GPS constellation (Global Positioning System). Thus, the measures were corrupted by an error caused by the uncertainties, modeled by an aleatory error of average zero (Oliveira et al. 2005). Beyond this, to consider the propulsion errors, in magnitude and in direction to the impulses during the orbital maneuvers, the magnitude of these errors was amplified. Therefore, the measures must be filtered by a Kalman filter.

4.1. The Kalman Filter

The Kalman filter can be considered the solution for the linear-quadratic-gaussian problem, which is the problem of instantaneous state estimation of a dynamic linear system disturbed by a gaussian white noise. The filter is an estimator with characteristic of real time, that may incorporate a dynamic noise in the model of the dynamics. For that reason, it is said that this filter is the optimal solution of minimum variance (Maybeck, 1979).

The Kalman filter is generally used for linear dynamics, therefore it can be also used in non-linear dynamics, as it may be seen in Grawal and Andrews (1993) and Maybeck (1979), by the use of the extended Kalman filter. The filter consists of two stages: propagation or prediction (time-update); and actualization or correction (measurement-update).

In the propagation phase, the state $\mathbf{x}$ and the covariance $\mathbf{P}$ are propagated, from the instant t_{k-1} to t_k. The discrete linear model is given by the following equation.

$$\mathbf{x}_{k+1} = \boldsymbol{\varphi}_{k+1,k}\mathbf{x}_k + \boldsymbol{\Gamma}_k\boldsymbol{\omega}_k \tag{14}$$

where $\mathbf{x}$ is the state to be estimated, $\boldsymbol{\varphi}$ is the state transition matrix, $\boldsymbol{\Gamma}$ is a matrix which relates the dynamic noise to the state, and $\boldsymbol{\omega}$ is the vector of dynamic noise modeled by a white noise defined by:

$$\boldsymbol{\omega}_k = \mathrm{N}(0, \mathbf{Q}_k) \tag{15}$$

The formal equations for the propagation stage are the following ones:

$$\overline{\mathbf{x}}_k = \boldsymbol{\varphi}_{k,k-1}\hat{\mathbf{x}}_{k-1} \tag{16}$$

$$\overline{\mathbf{P}}_k = \boldsymbol{\varphi}_{k,k-1}\hat{\mathbf{P}}_{k-1}\boldsymbol{\varphi}^t_{k,k-1} + \boldsymbol{\Gamma}_k\mathbf{Q}_k\boldsymbol{\Gamma}^t_k \tag{17}$$

where $\overline{\mathbf{x}}_k$ and $\overline{\mathbf{P}}_k$ are the state and the covariance propagated to the instant *k*.

The actualization phase is used to correct the state and the covariance in the instant *k*, considering the measures $\mathbf{y}_k$ using the observation model given by:

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{x}_k + \mathbf{v}_k \tag{18}$$

where $\mathbf{H}$ is the matrix, which relates the state to the measures, and $\mathbf{v}$ is the noise vector of measures modeled by a white noise defined by:

$$\mathbf{v}_k = \mathrm{N}(0, \mathbf{R}_k) \tag{19}$$

The measures in the instant *k*, supplies information to correct the state and the covariance. This phase, simply incorporates this information to the estimates. The equations are the same of the Kalman form.

$$\mathbf{K}_k = \overline{\mathbf{P}}_k\mathbf{H}^t_k\left(\mathbf{H}_k\overline{\mathbf{P}}_k\mathbf{H}^t_k + \mathbf{R}_k\right) \tag{20}$$

$$\hat{\mathbf{P}}_k = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\overline{\mathbf{P}}_k \tag{21}$$

$$\hat{\mathbf{x}}_k = \overline{\mathbf{x}}_k + \mathbf{K}_k\left(\mathbf{y}_k - \mathbf{H}_k\overline{\mathbf{x}}_k\right) \tag{22}$$

where $\mathbf{K}_k$ is the Kalman gain, and $\hat{\mathbf{x}}_k$ and $\hat{\mathbf{P}}_k$ are the state and the covariance updated to the instant *k*. The updating phase corrects the state and the covariance for the instant t_k using the measured $\mathbf{y}_k$.

5. Determination of the Maneuvers to be Apllied

The orbital maneuver method used in this work was implemented by Rocco (1997). It was based in the equations presented by Eckel and Vinh (1984). The method provides the transfer orbit with minimum fuel and fixed time transfer.

Considering only one satellite and that the maneuver is bi-impulsive, the total velocity increment is:

$$V = \Delta v_1 + \Delta v_2 = \mathrm{F}(\mathbf{x}) \tag{23}$$

If we consider the case of minimum fuel and fixed time we have the constraint relation:

$$T - T_0 = 0 \tag{24}$$

and the performance index is:

$$J = V + k(T - T_0) \tag{25}$$

Eckel and Vinh (1984) showed that the solution of the problem depends on three variables: the semi-latus rectum p of the transfer orbit and the true anomalies α_1 and α_2 that define the position of the impulses in the initial and final orbits. Therefore, the necessary conditions can be defined such as:

$$\frac{\partial V}{\partial p} + k\frac{\partial T}{\partial p} = 0 \qquad \frac{\partial V}{\partial \alpha_1} + k\frac{\partial T}{\partial \alpha_1} = 0 \qquad \frac{\partial V}{\partial \alpha_2} + k\frac{\partial T}{\partial \alpha_2} = 0 \tag{26}$$

The constraints of the satellite dynamic equations were shown in Rocco (2000a and 2000b).

The set of two equations are obtained by eliminating the Lagrange's multiplier k from Eq. (26):

$$\frac{\partial V}{\partial p}\frac{\partial T}{\partial \alpha_1} - \frac{\partial V}{\partial \alpha_1}\frac{\partial T}{\partial p} = 0 \qquad \frac{\partial V}{\partial p}\frac{\partial T}{\partial \alpha_2} - \frac{\partial V}{\partial \alpha_2}\frac{\partial T}{\partial p} = 0 \tag{27}$$

The two final optimal conditions are obtained evaluating the partial derivatives in these equations and doing some simplifications, as shown in Rocco et al. (1999, 2000a and 2000b). Thus, a system of equations composed by the two final optimal conditions (Eq. 27) and the time constraint equation (Eq. 24) must be solved to obtain the transfer orbit that performs the maneuver spending a minimum fuel consumption but with a specific time.

It may be assumed that in the initial instant the orbital elements of the satellites are given by (angles in radians):

Satellite 1:

a_1	=	7000.000000000	Ω_1	=	0.00000000	u_1	=	0.00000000
e_1	=	0.00000000	ω_1	=	0.00000000	f_1	=	0.00000000
i_1	=	0.00000000	M_1	=	0.00000000	θ_1	=	0.00000000

Satellite 2:

a_2	=	7002.00000000	Ω_2	=	0.00000000	u_2	=	2.09439510
e_2	=	0.00000000	ω_2	=	0.00000000	f_2	=	2. 09439510
i_2	=	0.00000000	M_2	=	2.09439510	θ_2	=	2. 09439510

Satellite 3:

a_3	=	7005.00000000	Ω_3	=	0.00000000	u_3	=	4.18879020
e_3	=	0.00000000	ω_3	=	0.00000000	f_3	=	4.18879020
i_3	=	0.00000000	M_3	=	4.18879020	θ_3	=	4.18879020

where M is the mean anomaly, u is the eccentric anomaly, f it is the true anomaly and θ is the true longitude.

Nine maneuvers were calculated for each satellite varying the semi-major axis of the final orbit, increasing a few kilometers above the nominal altitude. Each maneuver has different values of time spent and velocity increment and generates different value of position constraint (Eq. 13).

Satellite 1: Maneuver 1: $a_{final} = a_{nominal} = 7010.00$ km

a = 7006.23867081 e = 0.01326311 ω = 3.26866839 Δv = 0. 19987923 t = 100.00000000

Maneuver 2: $a_{final} = 1.0001\, a_{nominal}$ = 7010.701 km

a = 7006.63796680 e = 0. 01351801 ω = 2.42997588 Δv = 0.20369289 t = 105.00000000

Maneuver 3: $a_{final} = 1.0002\, a_{nominal}$ = 7011.402 km

a = 7007.44799235 e = 0.01575290 ω = 4.90124628 Δv = 0.23740951 t = 96.00000000

Maneuver 4: $a_{final} = 1.0003\, a_{nominal}$ = 7012.103 km

a = 7007.55340389 e = 0.01459562 ω = 2.20184554 Δv = 0.21989383 t = 110.00000000

Maneuver 5: $a_{final} = 1.0004\, a_{nominal}$ = 7012.804 km

a = 7008.02403816 e = 0.01516596 ω = 1.32249657 Δv = 0.22846977 t = 112.00000000

Maneuver 6: $a_{final} = 1.0005\, a_{nominal}$ = 7013.505 km

a = 3836.77670075 e = 0.01739282 ω = 0.29288430 Δv = 0.26206514 t = 103.00000000

Maneuver 7: $a_{final} = 1.0006\, a_{nominal}$ = 7014.206 km

a = 7010.12115853 e = 0.02070642 ω = 4.35673109 Δv = 0.31206379 t = 91.00000000

Maneuver 8: $a_{final} = 1.0007\, a_{nominal}$ = 7014.907 km

a = 7009.54050148 e = 0.01719814 ω = 3.30253352 Δv = 0.25904536 t = 115.00000000

Maneuver 9: $a_{final} = 1.0008\, a_{nominal}$ = 7015.608 km

a = 7010.17329534 e = 0.01832564 ω = 4.28868863 Δv = 0.27603504 t = 113.00000000

Satellite 2: Maneuver 1: $a_{final} = a_{nominal} = 7010.00$ km

a = 7009.89878708 e = 0.02357146 ω = 2.66210297 Δv = 0.35551209 t = 45.00000000

Maneuver 2: $a_{final} = 1.0001\, a_{nominal}$ = 7010.701 km

a = 7010.08756172 e = 0.02307433 ω = 0.40841488 Δv = 0.34798748 t = 50.00000000

Maneuver 3: $a_{final} = 1.0002\, a_{nominal}$ = 7011.402 km

a = 7013.51891464 e = 0.03116608 ω = 5.97325255 Δv = 0.47005461 t = 40.00000000

Maneuver 4: $a_{final} = 1.0003\, a_{nominal}$ = 7012.103 km

a = 7012.09184248 e = 0.02679366 ω = 5.40831462 Δv = 0.40405904 t = 50.00000000

Maneuver 5: $a_{final} = 1.0004\, a_{nominal}$ = 7012.804 km

a = 7014.21291716 e = 0.03114449 ω = 1.00151843 Δv = 0.46967916 t = 46.00000000

Maneuver 6: $a_{final} = 1.0005\, a_{nominal}$ = 7013.505 km

a = 7014.84750776 e = 0.03178445 ω = 2.36898083 Δv = 0.47930837 t = 48.00000000

Maneuver 7: $a_{final} = 1.0006\, a_{nominal}$ = 7014.206 km

a = 7018.53624118 e = 0.03853833 ω = 5.85432126 Δv = 0.58117626 t = 42.00000000

Maneuver 8: $a_{final} = 1.0007\, a_{nominal}$ = 7014.907 km

a = 7017.77136611 e = 0.03641804 ω = 3.58576902 Δv = 0.54916088 t = 47.00000000

Maneuver 9: $a_{final} = 1.0008\, a_{nominal} = $ 7015.608 km

a = 7019.61949065 e = 0.03923146 ω = 1.26072063 Δv = 0.59157668 t = 46.00000000

Satellite 3: Maneuver 1: $a_{final} = a_{nominal} = 7010.00$ km

a = 7008.73401092 e = 0.01326067 ω = 4.77507241 Δv = 0.35551209 t = 50.00000000

Maneuver 2: $a_{final} = 1.0001\, a_{nominal} = $ 7010.701 km

a = 7009.30604746 e = 0.01440036 ω = 1.04053440 Δv = 0.21714484 t = 52.00000000

Maneuver 3: $a_{final} = 1.0002\, a_{nominal} = $ 7011.402 km

a = 7010.39664945 e = 0.01768713 ω = 2.88363429 Δv = 0.26671294 t = 48.00000000

Maneuver 4: $a_{final} = 1.0003\, a_{nominal} = $ 7012.103 km

a = 7010.61115869 e = 0.01712727 ω = 4.99224585 Δv = 0.25824379 t = 55.00000000

Maneuver 5: $a_{final} = 1.0004\, a_{nominal} = $ 7012.804 km

a = 7010.63003574 e = 0.01568271 ω = 2.62158502 Δv = 0.23642274 t = 66.00000000

Maneuver 6: $a_{final} = 1.0005\, a_{nominal} = $ 7013.505 km

a = 7012.19041979 e = 0.02045313 ω = 2.45798715 Δv = 0.30837499 t = 55.15000000

Maneuver 7: $a_{final} = 1.0006\, a_{nominal} = $ 7014.206 km

a = 7014.76064778 e = 0.02710172 ω = 0.67385816 Δv = 0.40865145 t = 45.05000000

Maneuver 8: $a_{final} = 1.0007\, a_{nominal} = $ 7014.907 km

a = 7013.62236513 e = 0.02285244 ω = 1.63272865 Δv = 0.34452264 t = 57.50000000

Maneuver 9: $a_{final} = 1.0008\, a_{nominal} = $ 7015.608 km

a = 7014.66121992 e = 0.02490301 ω = 1.53540684 Δv = 0.37543220 t = 56.50000000

When all satellites of the constellation are considered, there are 729 combination of maneuvers. The position constraint of each combination must be calculated considering the maneuvers of all satellites. However, each maneuver of each combination spends a certain time. Thus, the satellite orbits must be propagated so that the position of each satellite, after all of them have been maneuvered, can be determined. In the orbit propagation, the effect of atmospheric drag and the Earth gravity potential perturbation were considered, using equations 28 to 33 (Rocco, 1999).

$$\dot{\Omega} = -\frac{3}{2}\frac{J_2 R_T^2}{p^2}\bar{n}\cos i \tag{28}$$

$$\dot{\omega} = \frac{3}{2}\bar{n}\left\{\frac{J_2 R_T^2}{p^2}\left(2 - \frac{5}{2}\operatorname{sen}^2 i\right) - \frac{J_3 R_T^3}{p^3}\frac{\operatorname{sen}\omega}{e \operatorname{sen} i}\left[\left(\frac{5}{2}\operatorname{sen}^2 i - 2\right)\operatorname{sen}^2 i + e^2\left(2 - \frac{35}{2}\operatorname{sen}^2 i \cos^2 i\right)\right]\right\} + \ldots \tag{30}$$

$$\dot{e} = \frac{3}{2}\frac{J_3 R_T}{p^3}\bar{n}\left(1 - e^2\right)\operatorname{sen} i \cos\omega\left(\frac{5}{2}\operatorname{sen}^2 i - 2\right) \tag{31}$$

$$\bar{n} = \sqrt{\frac{\mu}{a_0^3}}\left[1 + \frac{3}{2}\frac{J_2 R_T^2}{p^2}\left(1 - \frac{3}{2}\operatorname{sen}^2 i\right)\left(1 - e^2\right)^{1/2}\right] \tag{32}$$

$$\dot{a} = -C_D \frac{A}{m}\rho\frac{a^2 V^3}{\mu} \tag{33}$$

where: J_n = harmonic coefficients;
C_D = atmospheric drag coefficient;
ρ = atmosphere density at that altitude;
A = effective cross-section area.

The time spent and the velocity increment of each combination were obtained, respectively, by the sum of the times and the velocity of each maneuver in the combination. The multi-objective approach was applied after the calculation of the time spent, the velocity increment and the position constraint of each combination. The Smallest Loss Criterion (Rocco et al., 2003) was utilized to find the best combination of maneuvers. The non-dominated solution (Pareto non-inferiority, Pareto, 1909) are the combination shown in Tab. 1. These solutions are the extreme solutions (solutions that present the minimum value of time, velocity or position constraint). Other non-dominated solutions could be obtained, but this work is a discrete study, so the Pareto frontier is not known. To determine the entire Pareto frontier it would be necessary to solve the constrained equations (objective functions) for a large number of inputs. This becomes impracticable when these equations are of difficult solution, because the numerical methods utilized to solve the equations can not easily obtain the solution, and depending of the initial values the method may not converge. Thus, in this case, only the extreme solutions can be defined as non-dominated.

Table 1. Non-dominated solutions.

Maneuvers Combination	Sat. 1 - Maneuver7 Sat. 2 - Maneuver3 Sat. 3 - Maneuver7	Sat. 1 - Maneuver 1 Sat. 2 - Maneuver 2 Sat. 3 - Maneuver 1	Sat. 1 - Maneuver 3 Sat. 2 - Maneuver 7 Sat. 3 - Maneuver 4
Time (s)	176.05000000	200.00000000	193.00000000
Velocity Increment (km/s)	1.19076985	0.74783655	1.07682956
Position Constraint (rad)	1.17056204	3.32572560	0.14963693

The barycenter, obtained considering normalized measures, is shown in Tab. 2.

Table 2. Barycenter

	Normalized	Not Normalized
Time	0.82113997	189.68333333 (s)
Velocity Increment	0.76594647	1.00514532 (km/s)
Position Constraint	0.43977775	1.54864152 (rad)

From the Smallest Loss Criterion, the best solution seems to be that one where the parameters are the closest to the barycenter. Calculating the distances between the barycenter and the points that represent each combination of maneuvers the best combination may be obtained. The first nine better combinations are shown in Table 3. The nine worse combinations are shown in Table 4.

Table 3. Better combinations.

Maneuvers Combination	Sat. 1 - Maneuver 7 Sat. 2 - Maneuver 2 Sat. 3 - Maneuver 7	Sat. 1 - Maneuver 3 Sat. 2 - Maneuver 8 Sat. 3 - Maneuver 3	Sat. 1 - Maneuver 7 Sat. 2 - Maneuver 5 Sat. 3 - Maneuver 4
Distance (normalized measures)	1st 0.05460029	2nd 0.06459413	3rd 0.08509774
Maneuvers Combination	Sat. 1 - Maneuver 7 Sat. 2 - Maneuver 9 Sat. 3 - Maneuver 4	Sat. 1 - Maneuver 2 Sat. 2 - Maneuver 9 Sat. 3 - Maneuver 3	Sat. 1 - Maneuver 6 Sat. 2 - Maneuver 8 Sat. 3 - Maneuver 3
Distance (normalized measures)	4th 0.09158683	5th 0.09716632	6th 0.09837753
Maneuvers Combination	Sat. 1 - Maneuver 5 Sat. 2 - Maneuver 3 Sat. 3 - Maneuver 7	Sat. 1 - Maneuver 6 Sat. 2 - Maneuver 7 Sat. 3 - Maneuver 2	Sat. 1 - Maneuver 7 Sat. 2 -Maneuver 5 Sat. 3 - Maneuver 7
Distance (normalized measures)	7th 0.09908425	8th 0.09996529	9th 0.10024376

Table 4. Worse combinations.

Maneuvers Combination	Sat. 1 - Maneuver 2 Sat. 2 - Maneuver 2 Sat. 3 - Maneuver 4	Sat. 1 - Maneuver 4 Sat. 2 - Maneuver 2 Sat. 3 - Maneuver 4	Sat. 1 - Maneuver 4 Sat. 2 - Maneuver 2 Sat. 3 - Maneuver 1
Distance (normalized measures)	729th 0.61313767	728th 0.58074133	727th 0.57700448
Maneuvers Combination	Sat. 1 - Maneuver 2 Sat. 2 - Maneuver 2 Sat. 3 - Maneuver 1	Sat. 1 - Maneuver 2 Sat. 2 - Maneuver 5 Sat. 3 - Maneuver 4	Sat. 1 - Maneuver 1 Sat. 2 - Maneuver 2 Sat. 3 - Maneuver 1
Distance (normalized measures)	726th 0.57632537	725th 0.57582367	724th 0.57251976
Maneuvers Combination	Sat. 1 - Maneuver 9 Sat. 2 - Maneuver 2 Sat. 3 - Maneuver 1	Sat. 1 - Maneuver 4 Sat. 2 - Maneuver 5 Sat. 3 - Maneuver 4	Sat. 1 - Maneuver 4 Sat. 2 - Maneuver 5 Sat. 3 - Maneuver 1
Distance (normalized measures)	723rd 0.56487069	722nd 0.54326794	721st 0.54138989

It can be concluded analyzing Table 3 that combination 727 is the best combination of maneuvers to be applied. And, analyzing Table 4 it can be concluded that combination 224 is the worst. The parameters of the best and the worst combinations are shown in Table 5.

Table 5. Best and worst combination.

Maneuvers Combination	Best Combination Sat. 1 - Maneuver7 Sat. 2 – Maneuver2 Sat. 3 - Maneuver7	Worst Combination Sat. 1 - Maneuver 2 Sat. 2 - Maneuver 2 Sat. 3 - Maneuver 4	Differences	
Time (s)	186.05000000	210.00000000	26.95000000	12.87 %
Velocity Increment (km/s)	1.06870272	0.80992416	-0.25877856	-31.95 %
Position Constraint (rad)	1.41792099	3.52141868	2.10349769	148.35 %

6. Conclusion

In this work, which is an extension of the work developed by Rocco (2002), the problem of orbital station keeping of satellite constellations was studied as a problem of multi-objective optimization. The multi-objectives methodologies found in the literature (Cohon, 1978), generally start the problem with an multi-objective approach but ended reducing the problem to the mono-objective case, this is done by means of simplifications or influence factors, or, when the approach was really multi-objective, the result found was a group of solutions candidates to the optimal solution, in this case, for the choice of the optimal solution we should use other external approaches to the problem. The best methodology found in the literature, that is based on Pareto (1909), presents this deficiency, as shown in Rocco et al. (2003). Therefore, it seems that doesn't exist in the revised literature any method really capable to accomplish the multi-objective optimization, considering all the objectives of the problem equally. Thus, a methodology that at least considers equally all the objectives was used. This methodology is based on what it was called Smallest Loss Criterion. It was considered in the example presented in this work three objectives, but the Smallest Loss Criterion allows to consider so many objectives as necessary. The constellation considered here was composed by $n = 3$ satellites. However, the multi-objective optimization method and the developed software for the control of the constellation allow to consider more than 3 satellites easily. The concept for the problem for $n = 3$ is identical to the problem for $n > 3$, but in this case, we would have a larger computer effort.

In Table 5 it can be seen that the best combination of maneuvers, determined by the software, may be considered superior than the worst combination because the time spent and the position constraint are better for the combination 727. The velocity increment of the combination 224 is smaller than the velocity increment of the combination 727. However, the position constraint of combination 224 is 148.35 % bigger than the position constraint of the other combination. Therefore, an increase in the velocity may be compensate by a great decrease in the position constraint. If in the calculation of the maneuvers the semi-major axis were varying alternately to a few kilometers above and a few kilometers below the nominal altitude, perhaps it would be possible to obtain better results in all three objectives, as occurred in Rocco et al. (2005)) where the application of the multi-objective approach turned possible to obtain great improvement in all objectives. Even so, the result obtained in this work is very similar to the result obtained by Rocco et al. (2005), but now it was considered all the possible combination of the maneuvers and the uncertainties in the measures, generated by the propulsion errors and measure errors. As it was expected, due to this uncertainties, the results wasn't so good as the results obtained when the position of the satellites are precisely determined. But the uncertainties turned the problem more real. The fact of that all combination of maneuvers were considered, contributed significantly in the search of the best set of maneuvers to be applied in the constellation. However, it increase drastically the computer effort. Calculating nine maneuvers for each satellite, considering a constellation with three satellite, there are 729 combination of maneuvers to analyze. If twenty maneuvers were calculated there would be 8000 combinations considering three satellite, 160000 combinations for four satellites and 3200000 for five. So it was decided to calculate only nine maneuvers for each one of the three satellites, but the methodology defined in this work could be adopted without modifications for any number of satellites or quantities of maneuvers calculated.

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