APPLICATION OF THE HÉNON'S ORBIT TRANSFER PROBLEM TO MANEUVER A SATELLITE IN A CONSTELLATION

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Abstract. The main objective of the present paper is to study minimum fuel maneuvers to change the position of a spacecraft that belongs to a constellation. The control used is a bi-impulsive maneuver, where the first impulse is applied in the initial position of the satellite to send it to a transfer orbit that will cross the desired final position of the spacecraft. Both initial and final position of the satellite belongs to the same Keplerian orbit. The goal is to find the transfer that has the minimum total increment in velocity and that perform the desired maneuver.

Keywords. Astrodynamics, Orbital Maneuvers, Satellite constellation.

1. Introduction

In this paper, the problem of transfer orbits from one body back to the same body (known in the literature as the Hénon’s problem) is used to study maneuvers that has the goal of changing the position of a satellite in a constellation, in the sense of going to a different point (true anomaly) of the same orbit. The net result is a relocation of the satellite in the same orbit. The problem of transfer orbits from one body back to the same body has been under investigation for a long time. Hénon (1968) originally developed a timing condition for orbits that allow a spacecraft to leave a massless body M₂, go in an orbit around the primary M₁ and meet M₂ again, after a certain time. This was treated as the problem of consecutive collision orbits in the restricted three body problem. Several authors then worked on improvements of this problem. Hitzl (1977) and Hitzl and Hénon (1977a and 1977b) studied stability and critical orbits. Perko (1974) derived a proof of existence and a timing condition for what was shown later to be a special case of Hénon’s work. Results for the perturbed case $\mu$ (mass of M₂ divided by the mass of M₁) > 0 (where M₂ has non-negligible mass and perturbs the orbit of M₃ around M₁) also appeared in the literature. Some examples are the papers published by Gomez and Ollé (1991a and 1991b) and Bruno (1981). Howell (1987) and Howell and Marsh (1991) extended Hénon's results for the case where the orbit of M₂ is elliptic.

In the present research this problem is formulated as that of an orbit transfer, as done previously in Prado (1993), which can be solved with Gooding's implementation of the Lambert's problem (Gooding, 1990). In the approach used here, the second body M₂ is a fixed point in the orbit of the spacecraft and not a real body, but this nomenclature is used to facilitate the comparison with the results obtained from the consecutive collision orbits problem approach. Both cases, with the circular or elliptic orbits for the spacecraft are considered in the present research. The implementation developed here is generic with respect to the angle that the spacecraft has to be shifted. These transfer orbits are studied in terms of the $\Delta V$ and the time required for the transfer. The $\Delta V$s are plotted against the transfer time for several cases and a family of transfer orbits with very small $\Delta V$ (on the order of 0.001 in canonical units, a system of units where the gravitational constant of M₁, the angular velocity of the spacecraft and the distance between M₁ and the spacecraft are all unity) is shown to exist in almost all cases studied. These orbits are studied in detail. They consist of a family of slightly different orbits (when compared to the orbit of M₂) that meet all the requirements to provide the transfer desired. A relocation of a geostationary satellite is shown as an example of a practical application of this theory.

2. Formulation of the Problem

Let M₁ be the main body of the system (the Earth, in the example used here) and M₂ be a fixed point in a circular or elliptic orbit around M₁. The massless spacecraft M₃ leaves the point M₂ from a position denoted by P $t = -\tau$, follows an orbit around M₁ and meets again with M₂ at a point Q $t = \tau$. The basic equations of the Kepler problem apply. The canonical system of units is used. Figure 1 shows a sketch of the transfer.

The solution to be found is the coordinate of the point P as a function of the transfer time. The solution is not unique, and a graph including many solutions was published by Hénon (1968). He plotted $\eta/\pi$ (where $\eta$ is the redefined "eccentric anomaly" of the point P) against $\tau/\pi$ (where $\tau$ is half of the transfer time). Another problem that is considered in the present research is the calculation of the $\Delta V$ and the time required for each of these transfers, in a search for transfer orbits with small $\Delta V$. The solution consists of plots of the $\Delta V$ against the time required for the transfer (both in canonical units). A detailed study of the transfer orbits with small $\Delta V$ is included.
2.1. Lambert's Problem Formulation

A different approach used in the present research formulates Hénon's problem as a Lambert's problem. The Lambert's problem can be defined as (Gooding, 1990):

"An (unperturbed) orbit, about a given inverse-square-law center of force is to be found connecting two given points, P and Q, with a flight time $\Delta t = (t_2-t_1)$ that has been specified. The problem must always have at least one solution and the actual number, which is denoted by N, depends on the geometry of the problem - it is assumed, for convenience and with no loss of generality, that $t > 0$.

Using this formulation, Hénon's problem can be defined in the following way: "Find an unperturbed orbit for $M_3$, around $M_1$, which leaves the point $P$ at $t = -\tau$ and goes to point $Q$ at $t = \tau". Since $M_2$ is assumed to have zero mass, it has no participation in the equations of motion of the system. Its only use is to relate the time $\tau$ with the eccentric anomaly $\eta$, in such way that $M_3$ has the same position as $M_2$ at $P$ and $Q$ at the times $t = -\tau$ and $t = \tau$, respectively.

3. Mathematical Formulation

In terms of mathematical formulation, Hénon's problem formulated as a Lambert's problem can be described as follows. The following information is available:

1. The position of $M_3$ at $t = -\tau$ (point $P$). It can be specified by the radius vector $R_1$ and the angle $-\tau$. $R_1$ can be related to $-\tau$ by using the equation $R_1 = a(1-e^2)/(1+e\cos(-\tau))$ for the orbit of $M_2$, since $M_2$ and $M_3$ occupy the same position at $t = -\tau$;

2. The position of $M_3$ at $t = \tau$ (point $Q$). It can be specified by the radius vector $R_2$ and the angle $\tau$. $R_2$ can be related to $\tau$ by using the same equation used in the above paragraph;

3. The total time for the transfer, $\Delta t = 2\tau$. Remember that the angular velocity of the system is unity, so $\tau$ can be considered to be the time as well as the angle;

4. The total angle the spacecraft must travel to go from P to Q, that is called $\phi$. For the case where the orbit of $M_3$ is elliptic this variable has several possible values. First of all, there are two possible choices for the transfer: the one that uses the direction of the shortest possible angle between P and Q (that is called the "short way"), and the one that uses the direction of the longest possible angle between these two points (that is called the "long way"). Which one is the shortest or the longest depends on the value of $\tau$. After considering these two choices, it is also necessary to consider the possibilities of multi-revolution transfers. In this case, the spacecraft leaves P, makes one or more complete revolutions around $M_1$, and then goes to Q. Then, by combining these two factors, the possible values for $\phi$ are: $2\tau + 2m\pi$ and $2(\pi-\tau)+2m\pi$, where $m$ is an integer that represents the number of complete revolutions during the transfer. There is no upper limit for $m$, and this problem has an infinite number of solutions. In the case where the orbit of $M_3$ is parabolic or hyperbolic, $\phi$ has a unique value. The multi-revolution transfer does not exist anymore (the orbit is not closed), and the only direction of transfer that has a solution is the one that makes the spacecraft goes in a retrograde orbit passing by periapse at $t = 0$.

The information needed (the solution of the Lambert's problem) is the Keplerian orbit that contains the points P and Q and requires the given transfer time $\Delta t = 2\pi$ for a spacecraft to travel between these two points. This solution can be specified in several ways. The velocity vectors at P or Q are two possible choices, since the corresponding position vectors are available. The Keplerian elements of the transfer orbit is also another possible set of coordinates to express...
the solution of this problem. In the implementation developed here, all three sets of coordinates are obtained, since all of them are useful later.

To obtain the $\Delta V$s, the following steps are taken:

1. Find the radial and transverse velocity components of $M_2$ at P and Q. They are also the velocity components of $M_3$ just before the first impulse and just after the second impulse, respectively, since they match their orbits at these points. They are obtained from the equations (Danby, 1988):

\[
V_r = \frac{e \sin(\nu)}{\sqrt{a(1-e^2)}} \quad V_t = \frac{1+e \cos(\nu)}{\sqrt{a(1-e^2)}},
\]

where $V_r$ and $V_t$ are the radial and transverse components of the velocity vector, $a$ and $e$ are the semi-major axis and the eccentricity of the transfer orbit and $\nu$ is the true anomaly of the spacecraft.

2. Find an unperturbed orbit for $M_3$ that allows it to leave the point P at $t = -\tau$ and arrive at point Q at $t = \tau$. This orbit is found by solving the associate Lambert's problem, as explained in the next section. At this point the total time for this transfer, $2\tau$, is already known.

3. Find the velocity components at these points (P and Q) in the transfer orbit determined in ii). They are the velocity components for $M_3$ just after the first impulse and just before the second impulse. They are provided by Gooding's Lambert routine (Gooding, 1990).

4. With the velocity components just after and just before both impulses it is possible to calculate the magnitude of both impulses ($\Delta V_1$ and $\Delta V_2$) and add them together to get the total impulse required ($\Delta V$) for the transfer.

4. **Gooding's Implementation of the Lambert's Problem**

   The solution of the Lambert's problem, as defined in the previous paragraphs, has been under investigation for a long time. The approach to solve this problem is to set up a set of non-linear equations (from the two-body problem) and start an iterative process to find an orbit that satisfies all the requirements. There is no closed-form solution available for this problem. The major difficulty is to choose the best set of equations and parameters for iterations to guarantee that convergence occurs in all cases. The routine used in this research is due to Gooding (1990). He chooses $\pm \sqrt{1-s/2a}$ as the parameter for convergence, where "a" is the semi-major axis of the transfer orbit and "s" the semi-perimeter of the triangle formed by P, Q and $M_1$. He also makes several substitutions of variables, trying to find the best set of equations to guarantee convergence in all cases. His implementation is able to find all the possible solutions of the Lambert's problem, including "long way", "short way" and "multi-revolution" transfers. He gives the velocity vectors at P and Q and the Keplerian elements of the transfer orbit in his solution. More detail can be found in Gooding, 1990.

   Including all phases of the present research, Gooding's routine has been called about 3 million times with no failure detected.

5. **Results**

   In this section some results are shown in the problem of finding the $\Delta V$s required for the transfers to be able to get the transfers with the minimum consumption. Plots of ($\Delta V$)$\times(\tau/\pi)$ were made for thousands of possible transfer orbits. Five orbits for $M_2$ around $M_1$ are used:

   1-) The circular orbit with $a = 1$.
   2-) The elliptic orbit with $e = 0.4$ and $a = 1$, with $M_2$ passing by periapse at $t = 0$.
   3-) The elliptic orbit with $e = 0.4$ and $a = 1$, with $M_2$ passing by apoapse at $t = 0$.
   4-) The elliptic orbit with $e = 0.97$ and $a = 1$, with $M_2$ passing by periapse at $t = 0$.
   5-) The elliptic orbit with $e = 0.97$ and $a = 1$, with $M_2$ passing by apoapse at $t = 0$.

   The results for orbits 1, 2 and 4 are shown in Fig. 2. The vertical axis shows the total $\Delta V$ in canonical units and the horizontal axis shows $\tau/\pi$, where $\tau$ is half of the transfer time. Only elliptic transfer orbits are included in these plots, since the hyperbolic or parabolic transfer orbits are too expensive, in terms of $\Delta V$ (always more than 1.6), to be useful. In these figures, $\tau/\pi$ varies from 0 to 14 and the maximum number of complete revolutions allowed for $M_3$, while in its transfer orbit, is also 14. This means that we restrict ourselves to the orbits contained in a square region with side 14 (0 $\leq \tau/\pi \leq 14$ and 0 $\leq \Delta V/\pi \leq 14$).

   An examination of those figures shows the existence of points (orbits) with very small $\Delta V$. They appear in several locations in the plot and they reveal a whole family of small $\Delta V$ transfer orbits. In all cases studied in this research, this family appears in the "short transfer time" part of the graph (small $\tau$). A more detailed plot of ($\Delta V$)$\times(\tau/\pi)$ is shown in Fig. 3. It includes only the orbits where $\Delta V < 0.5$ and it is restricted to orbit 1 (circular orbit) only. Plots for the orbits 3 and 5 are similar to the plots for orbits 2 and 4, respectively, and are omitted in the present text to save space. It is

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possible to see that the local minimums increase with time after \[\frac{\tau}{\pi} = 6.\] An investigation for \[\frac{\tau}{\pi}\] varying from zero to 200 (and with the maximum number of complete revolutions for \(M_3\) equal to 200) was done, and no more orbits with \(\Delta V < 0.1\) were found.

Figure 2a. \((\Delta V)\) vs \((\tau/\pi)\) for Orbit 1 for \(M_2\).

Figure 2b. \((\Delta V)\) vs \((\tau/\pi)\) for Orbit 2 for \(M_2\).
Table 1 shows the main characteristics of the orbits with $\Delta V < 0.1$ found in the circular and elliptic cases. It is interesting to see that for the circular case (see the part $e = 0$ in table 1) most of the orbits appear in pairs, with almost identical values of $\tau/\pi$. A good example is the pair formed by the first two orbits in Table 1: $\tau/\pi = 1.400$ and $\tau/\pi = 1.410$. In each pair one orbit has the periapse in a positive abscissa and the other one has the periapse in a negative abscissa. In this Table the orbit of $M_2$ is assumed to be elliptic with several values for the eccentricity. Both cases, $M_2$ at periapse at $t = 0$ and $M_2$ at apoapse at $t = 0$ are considered. Fig. 4 shows some of those orbits.

Table 1 and Fig. 4 show the mechanism of the majority of these transfer orbits. They consist of orbits with slightly different semi-major axis and eccentricity (compared with the orbit of $M_2$) and they have a periapse coincident with the periapse of the orbit of $M_2$. They have mean angular velocity ($n$) such that $2\tau (1-n)= \pm 2\pi$. Then, after $M_3$ makes $m$ complete revolutions in its transfer orbit, $M_2$ makes $m+1$ or $m-1$ complete revolutions in its own orbit and they can meet each other at the common periapse, after the time $2\tau$. 

Figure 2c. $(\Delta V)$ vs $(\tau/\pi)$ for Orbit 4 for $M_2$.

Figure 3. $(\Delta V)$ vs $(\tau/\pi)$ for $\Delta V < 0.5$ (Orbit 1 for $M_2$).
6. Practical Application

To show one possible practical application for these orbits, this theory is applied in a transfer for a satellite from one point in a circular geostationary orbit to another point in the same orbit (a point 180 degrees ahead of the initial point is used as an example, but the scheme proposed here can be used for any transfer angle desired). This problem is very important nowadays. Its solution can be used to transfer a geosynchronous satellite, to use it above a point with different longitude on Earth. Fig. 5 shows this situation.

Figure 4. Some Transfer Orbits with Small $\Delta V$.

Figure 5. Orbit Transfer for a Geosynchronous Satellite.
Fig. 6 shows the $(\Delta V) vs (\tau/\pi)$ for elliptic transfer orbits. Hyperbolic transfer orbits are also available, but they have $\Delta V$ too large to be useful. It is assumed that the change in longitude desired for the satellite is 180 degrees. Table 2 shows the whole family of small $\Delta V$ orbits. Under the assumption that the orbital velocity of the satellite is 3075 m/s (Wertz and Larson, 1991) and its orbital period is 1 day, Table 2 shows the real values of $\Delta V$ and $2\tau$ (total time required for the transfer). The mechanism used by these transfers is to insert $M_3$ in an elliptic transfer orbit that have a periapse coincident with the periapse of the orbit of $M_2$. These transfer orbits have a mean angular velocity ($n$) smaller than 1, such that $(1-n)2\tau = \pi$. Then, in the same time that $M_3$ makes $m$ revolutions in its transfer orbit, $M_2$ makes $m+(1/2)$ revolutions in its own orbit and $M_3$ meets with a point 180 degrees ahead of its initial point at Q. The same comment about other multi-revolution possible transfer orbits with a lower $\Delta V$ made in the previous cases are valid here. In this case $M_2$ does not exist as a real body. It is only a reference point in orbit and, in consequence, its mass is really zero. For this reason, this example fits very well the model used and the results found here are expected to be in close agreement with the real world.

7. Conclusions

The problem previously called "consecutive collision orbits" in the three-body problem is formulated as a problem of transfer orbits from one body back to the same body. Using this approach, Henon's problem became a special case of the Lambert's problem.

Gooding's implementation of the Lambert's problem (Gooding, 1990) is used to solve this problem with great success.

The $\Delta V$s and the transfer time required for these transfers are calculated. Among a large number of transfer orbits, a small family is found, such that the $\Delta V$ required for the transfer is very small. These orbits and their properties are shown in detail.

A practical applications for these orbits are studied in detail: a transfer for a satellite from a point in a circular geosynchronous orbit to another point in this same orbit, 180 degrees ahead of its initial point.

The possibilities of transfers like this one is open for several types of missions and the algorithm developed here can be used to relocate a satellite to a different position in one orbit.
Table 1. Transfer orbits with $\Delta V < 0.1$ for the circular and elliptic case

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where: $\tau$ = Half of the transfer time in canonical units, $\nu$ = Redefined true anomaly, $\eta$ = Redefined eccentric anomaly, $a$ = Semi-major axis of the transfer orbit, $e$ = Eccentricity of the transfer orbit, $S3 = 1$ if $M2$ is at periapse at $t = 0$ and -1 if it is at apoapse, $L = 1$ for "short way" transfer, 0 for "long way" transfer, $P = 1$ if periapse is in a positive abscissa, 0 if in a negative abscissa, $S = 1$ if transfer is direct, 0 if transfer is retrograde, $A = 1$ if $M3$ pass by the periapse at $t = 0$, 0 if it pass by the apoapse, $\Delta V$ = Velocity increment in meters/second.
Table 2. Transfer orbits with $\Delta V < 0.1$ for the transfer in the geosynchronous orbit

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The symbols are the same ones used in the previous tables.

8. Acknowledgment

The author is grateful to the Foundation to Support Research in the São Paulo State (FAPESP) for the research grant received under Contract 2000/14769-4 and to CNPq (National Council for Scientific and Technological Development) - Brazil for the contract 300221/95-9.

9. References