Macroscopic instabilities in lightning

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X Latin American Workshop on Plasma Physics combined with the 7th Brazilian Meeting on Plasma Physics
São Pedro, SP, Brazil, 30 November - 5 December, 2003

Characteristics of lightning discharges
Typically, a lightning stroke lowers negative charge to earth from a thundercloud. The peak current in the return stroke is 10\(^{10}\)A/km, and the discharge may last for 0.5s extending over a distance of 5km.

The first return stroke in a lightning discharge is preceded by a stepped leader wave called a stepped leader. Subsequent strokes in a multiple-stroke discharge are preceded by a fast moving, continuous leader wave called a dart leader. About 2/3 of natural lightning discharges present, on the average, 3 to 5 subsequent strokes.

Typical return stroke current pulse
The current in the main return stroke attains the median value of 30-40kA in ~1s and decays with a time constant 30-60s to a continuous value ~100A during pauses between successive strokes. The current peaks in subsequent strokes are usually smaller than the main peak, with rise times <1s. The time interval between successive strokes is ~30-40ms.

The figure shows the model current-time curve for a typical return stroke. The high current associated with successive strokes leads to the development of a shock wave and the associated hydrodynamical instability discussed in the following.

Decay of the lightning channel
Near each current peak the discharge attains its maximum temperature of 30,000-40,000K, which rapidly decays to 10,000-8,000K during pauses, when the pressure in the channel falls to nearly the ambient pressure.

Simulations (N.L. Alexandrov, E.M. Bazelyan and M.N. Strelner, Russian Phys. Rep. 26, 993, (2000)) show a slow contraction of the channel in the final phase of the current decay. The pressure and density gradients are oppositely directed giving rise to the Rayleigh-Taylor instability.

Artificially triggered lightning
Lightning can be triggered by launching small rockets carrying a thin copper wire connected to the launching platform. In this way, thirteen flashes were artificially produced during the past four years in the International Center for Triggered and Natural Lightning in Caico's Town, Holguin, Cuba. A fast digital camera (1,000 frames per second) was used for the first time to obtain detailed images of the slow cooling stage of the discharge channel.

The above images show a sequence of 1ms time intervals corresponding to the last two strokes of a 10-stroke flash. The peak current was 45kA and the visible channel had a radius of 0.5m estimated from the size of the launching platform. The last return stroke ended on the tip of a Pringles potato chip, a lighting rod that can be seen on the right side of the sequence. The images show the slow evolution of the beaded structure usually observed during pauses in triggered lightning experiments.

Macroscopic instabilities
The purpose of this poster is to examine the role of the hydromagnetic and Rayleigh-Taylor instabilities in the formation of the beads. The figure shows a cylindrical discharge deformed by the m0, m1, and combined modes.

Since the magnetic pressure is much smaller than the kinetic pressure (\(u^2/8\)), the hydromagnetic instabilities are much weaker than the hydrodynamic instability while this one lasts.

Actually, it will be shown that the beaded structure can be explained solely in terms of the hydrodynamic Rayleigh-Taylor instability in a cylindrical discharge with anomalous viscosity.

Viscous fluid equations of motion
Mass conservation
\[ \nabla \cdot \mathbf{v} = 0 \]
Equation of state
\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \]
Faraday's law + Ohm's law (perfectly conductive fluid)
\[ \mathbf{v} = \nabla \times \mathbf{A} \]
Ampère's law
\[ \mathbf{B} = \mathbf{B}_0 + \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} \]
Navier-Stokes equation
\[ \rho \frac{\partial \mathbf{v}}{\partial t} = \rho \mathbf{v} \cdot \nabla \mathbf{v} = \nabla \cdot (\mu \nabla \mathbf{v}) + \nabla (\tau) \]
Magnetostatic equilibrium
Equations without time dependence and with vanishing fluid velocity
\[ \mathbf{v} = 0 \]
\[ \mathbf{B} = \mathbf{B}_0 \]
\[ \mathbf{E} = 0 \]
External axial magnetic field in a cylindrical discharge (surface current)
\[ \mathbf{E} = -\frac{1}{2} \mu_0 \frac{\partial \mathbf{B}}{\partial t} \]
Pressure balance for a narrow acceleration profile at the edge
\[ p_L \approx \mu_0 (\mathbf{v} \cdot \nabla) \mathbf{v} \approx \frac{\mu_0}{2} \frac{\partial \mathbf{v}}{\partial t} \]
Maximum current density
\[ J_{\text{max}} = 2 \sqrt{2} \mu_0 \frac{d}{a} \]
Stability analysis
Perturbation in the fluid velocity in the Lagrangian description
\[ \delta w = e_z \frac{d}{a} \delta \mathbf{A} \]
Linearized equation of motion
\[ -\rho_0 \frac{\partial^2 \delta w}{\partial t^2} + \nabla \cdot (\rho_0 \mathbf{v} \delta w) + \frac{\rho_0}{\rho} \frac{\partial \mathbf{v}}{\partial t} \cdot \nabla \delta w = 0 \]
Stress continuity
\[ \mathbf{K} \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\rho_0 \nabla \mathbf{p} \]
Boundary conditions
Continuity of the magnetic field in the presence of the surface current
\[ \mathbf{B}_0 = 0 \quad \text{for} \quad \mathbf{K} = 0 \]
Fluid continuity and no-slip condition
\[ \mathbf{v} = 0 \quad \text{at} \quad \mathbf{K} = 0 \]
Stress continuity
\[ p_L \left( \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\rho_0 \nabla \mathbf{p} \]
Boundary conditions for the perturbations
Continuity of the magnetic field in the presence of the surface current
\[ \mathbf{K} = 0 \quad \delta \mathbf{A} = \delta \mathbf{A} \quad \text{at} \quad \mathbf{K} = 0 \]
Fluid continuity and no-slip condition
\[ \delta \mathbf{v} = 0 \quad \text{at} \quad \mathbf{K} = 0 \]
Stress continuity
\[ p_L \left( \frac{\partial \delta \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \delta \mathbf{v} \right) = -\rho_0 \nabla \delta \mathbf{p} \]
Differential equations for the perturbations
Magnetic scalar potential outside the discharge
\[ \delta \mathbf{B} = -i \mu_0 \delta \mathbf{A} \]
Equilibrium magnetic field
\[ \delta \mathbf{B} = 0 \quad \text{at} \quad \mathbf{K} = 0 \]
Differential equations (weak viscosity and low-frequency)
\[ -\rho_0 \frac{\partial^2 \delta \mathbf{v}}{\partial t^2} + \nabla \cdot (\rho_0 \mathbf{v} \delta \mathbf{v}) = 0 \]
Cylindrical symmetry
\[ \rho_0 \frac{\partial^2 \delta \mathbf{v}}{\partial t^2} + \nabla \cdot (\rho_0 \mathbf{v} \delta \mathbf{v}) = 0 \]
Dispersion relation
Normal pressure - \( \rho_0 \nabla \cdot \mathbf{v} + \rho_0 \mathbf{v} \cdot \nabla \mathbf{v} \)
\[ \frac{\partial}{\partial t} \left( \rho_0 \delta \mathbf{v} \right) + \nabla \cdot (\rho_0 \delta \mathbf{v} \mathbf{v}) + \rho_0 \mathbf{v} \cdot \nabla \mathbf{v} = 0 \]
Approximate dispersion relation
Vorticity balance
\[ \rho_0 \frac{\partial^2 \delta \mathbf{v}}{\partial t^2} = 0 \]
Interchange instability for negligible viscosity
\[ \rho_0 \frac{\partial^2 \delta \mathbf{v}}{\partial t^2} = 0 \]
Rayleigh-Taylor instability for negligible magnetic field
\[ \rho_0 \frac{\partial^2 \delta \mathbf{v}}{\partial t^2} = 0 \]
Maximum growth-rate
\[ \gamma = \sqrt{\frac{\rho_0 g}{\mu_0}} \]
Viscosity effects
Balance between inertial and viscous forces
\[ \rho_0 \frac{\partial^2 \delta \mathbf{v}}{\partial t^2} = \rho_0 v_0 \frac{\partial^2 \delta \mathbf{v}}{\partial t^2} \]
Reynolds number for anomalous viscosity
\[ R_e = \frac{\rho_0 v_0 a}{\mu_0} \]
Initial conditions
\[ \rho_0 \frac{\partial^2 \delta \mathbf{v}}{\partial t^2} = 0 \]
Fully developed turbulence
\[ \rho_0 \frac{\partial^2 \delta \mathbf{v}}{\partial t^2} = 0 \]
Scaling for the Rayleigh-Taylor instability
Maximum growth rate
\[ \gamma = \sqrt{\frac{\rho_0 g}{\mu_0}} \]
Comparing with the previous estimate
\[ \gamma = \sqrt{\frac{\rho_0 g}{\mu_0}} \]
Application to atmospheric discharges
Lightning parameters
\[ t = 10s \quad \mathcal{L} = 100m \quad I_0 = 20mA \quad \text{dashed line: } 4m \text{ (kV)} \]
Conclusions
In the beginning of the contracting stage the “gravitational” acceleration is relatively strong, the Rayleigh-Taylor instability rises very fast and the turbulence sets in on the small scale length \( L \). During the contraction the turbulence develops, the large scale fluctuations fill the arc channel and the viscosity becomes strongly anomalous, shifting the wavelength of the most unstable modes to values of the order of and larger than the channel radius. This process takes about 1ms, before the instability weakens and the spatial structure becomes frozen. From this point on one may conjecture that the discharge channel diffuses for a few milliseconds before the turbulence decays in the absence of a driving energy source. The visible pictures correspond to a diffuse channel showing the frozen spatial structure of the instabilities during the history of their evolution.