Superconductor-Insulator Transition and Universal Resistance in Josephson-Junction Arrays in a Magnetic Field

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The zero-temperature superconducting transition in a Josephson-junction array with half of a flux quantum per plaquette is considered. Its critical behavior is described by an effective free energy in three dimensions with a transition in a new universality class. From this we infer the critical behavior and the value of the universal resistance at the transition. Relevance of these results to recent experiments on very-small-capacitance arrays is discussed.

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The possibility of fabricating Josephson-junction arrays with well controlled parameters has made this system a very active field of research in recent years. As well as being of interest in its own right as a tool to investigate the effects of disorder, dissipation, and charging effects, it has also been used as a simplified model of more complicated systems such as high-T_c materials and granular films.[1] Very recently, some interesting experiments have been performed on two-dimensional arrays of junctions with very small capacitance.[2] A zero-temperature superconducting-to-insulator transition is observed when $\alpha = E_c/E_J$, the ratio of the charging energy E_c of the grains to the Josephson coupling energy E_J, exceeds a critical value. This transition was originally predicted to occur in granular superconductors and has been studied extensively over the years. More recently, the effects of dissipation have been included.[3] In the absence of disorder and of resistive shunting of the junctions, the $T=0$ critical behavior in zero magnetic field has been shown to belong to the universality class of a $(d+1)$-dimensional classical XY model provided the Coulomb interaction between the charged grains is short ranged.[4] Long-range Coulomb interactions can drive the transition to first order but it has been shown that there is a range of parameters for which the $1/r$ part of the interaction is irrelevant and the transition is continuous.[5]

In the experiments, the behavior of the sheet resistance near the transition in zero field is very similar to that observed in very thin homogeneous superconducting films.[6-8] It has been found that, right at the transition between the superconductor and insulator, the resistance per square is $R \approx h/4e^2 = 6.5$ kΩ, the quantum unit of resistance for pairs. This has also been predicted theoretically,[8] provided the transition to a Mott insulator is continuous, but experiments on junction arrays do not rule out a first-order transition. It was also noticed that an array in a superconducting state but close to the transition becomes an insulator when an external field is applied corresponding to $f = Ha_0/\Phi_0 = 1/2$, where a_0 is the area of an elementary plaquette and $\Phi_0 = hc/2e$ is a quantum of flux.[2] It is well known that such a magnetic field leads to frustration effects changing the ground state and the universality class of the transition.[9-12] However, the experiments also imply that the resistance at the transition is considerably smaller for fully frustrated arrays than for unfrustrated ones.[2]

In this paper, we analyze the zero-temperature critical behavior of a fully frustrated array in the absence of disorder and resistive shunting. This should provide a reasonable description of a real array except asymptotically close to the transition. We show that the transition between the superconducting and Mott insulating phases is continuous for some range of parameters and that the resistance at the transition is universal but considerably smaller than that for the unfrustrated case. In fact, in an exactly soluble limit, the critical resistance is exactly half that in zero magnetic field. We do not expect this ratio to be exact for a real array but certainly of this order and experimentally observable.

Within a mean-field approximation,[13] it can be shown that, for a frustrated array with $f=\frac{1}{2}$, the critical value of α is considerably smaller than for $f=0$. However, the evaluation of the resistance at the transition requires an analysis of the critical behavior. We point out that the transition in a two-dimensional array of junctions on a square or triangular lattice with $f=\frac{1}{2}$ can be described by a classical Ginzburg-Landau-Wilson effective free energy with two coupled complex fields in $d=2+1$ dimensions. This is not surprising in view of the fact that the finite-temperature transition can be described by a similar effective free energy in $d=2$ and the zero-temperature critical behavior of the quantum model is usually in the same universality class as a $d+1$ classical model.[14]

The finite-temperature $f=\frac{1}{2}$ system has not been susceptible to analytic analysis in $d=2$ but at $T=0$ an expansion about $d=3+1$ is likely to describe the critical behavior quite well. In fact, a $4-\epsilon$ expansion indicates the existence of a stable fixed point in a new universality class.[15] Monte Carlo simulations on a layered triangular antiferromagnetic XY model,[16] which is in the same universality class as an $f=\frac{1}{2}$ array, is consistent with these ideas. Invoking the universality hypothesis and using the results from these investigations one can predict that the superfluid density is $\rho_s \approx \delta^{(d-1)\nu}$, where $\delta = \alpha_c - \alpha$ and $\nu \approx 0.53$ in $d=2$. The corresponding result for the unfrustrated case is $\nu \approx 0.67$. Similarly, the Mott gap and the crossover temperature to classical behavior vanish as δ^{ν}. The resistance R at the transition is given by $1/R = \lim_{w\to 0}(2e)^2\rho_s(-imw)/(-iw) = Ch/4e^2$ in $d=2$, with C a universal number which depends on the nature of the transition.[8] This is not easy to calculate but in the limit where the number of components of each complex field is infinite, we find that $C(f=1/2)=4/\pi$ which is to be compared with $C(f=0)=8/\pi$ first obtained in Ref. 8.

Assuming that the transition is continuous and that any long-ranged Coulomb interaction renormalizes to zero, we can analyze the system in terms of a self-charging model of a regular array of junctions described by the Hamiltonian[17]

$$H = -\frac{E_c}{2}\sum_i \left(\frac{d}{d\theta_i}\right)^2 - E_J \sum_{\langle ij\rangle} \cos(\theta_i - \theta_j - A_{ij}) , \qquad (1)$$

where $A_{ij} = (2\pi/\Phi_0)\int_i^j \mathbf{A}\cdot d\mathbf{l}$ with $\mathbf{A}$ the vector potential due to the external field $\mathbf{B}$ and the gauge-invariant sum around a plaquette $\sum A_{ij} = 2\pi f$. In this model, $E_c = 4e^2/C$, where C is the capacitance of a grain and E_J is the Josephson coupling energy. A more general system with a small intergrain capacitance may also be considered but the critical behavior will be the same as that of Eq. (1) since it depends on the small-wave-number behavior of the capacitance matrix. Effects of quasiparticle tunneling,[3] which renormalizes the intergrain capacitance, can then be included in the same generalization. Direct dissipation mechanisms, such as resistive shunting, are not included in the model as they seem to be unimportant in the system of Ref. 2. It is easy to see that the Hamiltonian of Eq. (1) is periodic under $f \to f+1$ provided one ignores effects such as reduction of E_J due to flux penetration of individual junctions and also disorder in the array.[18]

To derive a free-energy functional for a two-dimensional array, we first write the partition function in a path-integral representation and then use a Hubbard-Stratonovich transformation[4,10] to decouple the interaction term by introducing an auxiliary field $\Psi_i(\tau)$ coupling linear to $\exp(i\theta_i)$. This yields a partition function $Z = Z_0 \int D\Psi \exp(-S[\Psi])$, where

$$S = \int d\tau \tfrac{1}{2} \sum_{i,j} \Psi_i^* J_{ij}^{-1} \Psi_j - \ln\left\langle \exp\left[\tfrac{1}{2}\int d\tau \sum_i e^{i\theta_i(\tau)}\Psi_i^*(\tau) + \text{c.c.}\right]\right\rangle_0 , \qquad (2)$$

with the expectation value taken with respect to

$$S_0 = \frac{1}{2E_c}\sum_i \int d\tau \left(\frac{d\theta_i(\tau)}{d\tau}\right)^2$$

and $J_{ij} = E_J \exp(-iA_{ij})$ for nearest neighbors and zero otherwise. For convenience, we set $\hbar = 1$ and $c=1$.

To study the critical behavior we can consider small fluctuations about mean-field theory which amounts to expanding the second term in Eq. (2) in powers of Ψ. Neglecting the time dependence of the quartic term gives

$$S = \tfrac{1}{2}\int d\tau \sum_{ij} \Psi_i(\tau)^* J_{ij}^{-1}\Psi_j(\tau) - \tfrac{1}{4}\int d\tau \int d\tau' \sum_i g(\tau-\tau')\Psi_i^*(\tau)\Psi_i(\tau') + u\int d\tau \sum_i |\Psi_i(\tau)|^4 , \qquad (3)$$

with $g(\tau-\tau') = \langle e^{i\theta(\tau) - i\theta(\tau')}\rangle_0$.

Now, we concentrate in the $f=\frac{1}{2}$ case. Since $\sum A_{ij} = \pi(\text{mod } 2\pi)$, we set $A_{ij}=\pi$ for the triangular lattice and $A_{ij}=0$ (horizontal), $A_{ij}=\pi x_i$ (vertical bonds) for the square lattice, where x_i is the position of the ith column. This leads to an isotropic antiferromagnetic coupling for the triangular case and alternating ferromagnetic and antiferromagnetic couplings for the square lattice giving rise to frustration. A Ginzburg-Landau-Wilson free energy can now be constructed by expanding about the most fluctuating modes. For the triangular lattice, $J(q)$ has two minima at $\pm Q$ with $Q=(4\pi/3,0)$ and for the square lattice, after diagonalizing the Fourier-transformed coupling matrix $J(q,q')$ one finds two minima at $(0,0)$ and $(0,\pi)$.[10] So, for each case, one needs two order parameters Ψ_1 and Ψ_2. Retaining these modes only, the quartic term in Eq. (3) generates coupling terms of the form

$$u(|\Psi_1|^2 + |\Psi_2|^2)^2 - v_1|\Psi_1|^2|\Psi_2|^2 + v_2 \text{Re}(\Psi_1^*\Psi_2)^2 . \qquad (4)$$

In the square lattice $u, v_1 = v_2 > 0$ but for the triangular lattice $u>0$, $v_1<0$, and $v_2=0$. This latter case may be transformed into the former by the change of variables $\Psi_1 \to (\Psi_1 + i\Psi_2)/\sqrt{2}$ and $\Psi_2 \to (i\Psi_1 + \Psi_2)/\sqrt{2}$.

The final step is to expand the Fourier-transformed quadratic terms in Eq. (3) about these minima and rescale the fields and time to obtain the action

$$S = \int d\tau \int d^2r \left\{ \sum_{a=1,2}\left(\frac{1}{2}r_0|\Psi_a|^2 + \frac{1}{2}\left[(\nabla\Psi_a)^2 + \left(\frac{d\Psi_a}{d\tau}\right)^2\right]\right) + u(|\Psi_1|^2+|\Psi_2|^2)^2 + v[\text{Re}(\Psi_1^*\Psi_2)^2 - |\Psi_1|^2|\Psi_2|^2]\right\} , \qquad (5)$$

with $v>0$ and $r_0=1/J_0-2/E_c$, $J_0=2\sqrt{2}E_J$ and $J_0=3E_J$ for the square and triangular lattices, respectively. A simple mean-field analysis gives a transition at $r_0=0$ which implies $\alpha_c=E_c/E_J=4\sqrt{2}$ and 6 for the square and triangular lattices, respectively, which are significantly smaller than the corresponding $f=0$ values.

To obtain the critical behavior and exponents of the system described by Eq. (5) is a rather more difficult problem. However, Kawamura has studied, by a two-loop renormalization-group and a Monte Carlo analysis, precisely this free energy in another context[15,16] with E_c/E_J playing the role of temperature. The renormalization-group analysis yields a variety of stable fixed points depending on the number of components of the fields Ψ when $v>0$ and runaway when $v<0$. Monte Carlo analysis[16] of a layered triangular antiferromagnet indicates a continuous transition with $\nu\approx 0.53$. Using this result one can infer the behavior of measurable quantities such as $\rho_s\approx(\alpha_c-\alpha)^{(d-1)\nu}$ and, since both q and w scale in the same way, one expects that the crossover temperature to classical behavior and the Mott gap vanish as $(\alpha-\alpha_c)^{\nu}$.

A question now arises concerning the relative magnitude of the universal resistance for $f=\frac{1}{2}$ and $f=0$. To calculate the conductivity we follow the usual procedure of making the gauge-invariant replacement $\nabla\to\nabla-i(2e)\mathbf{a}$ in Eq. (5) and expressing the conductivity in terms of the analytic continuation $w\to -iw$ of the current-current correlation function

$$C_{\mu\nu}(q,w)=\int d\mathbf{r}\,d\tau\, e^{-i\mathbf{q}\cdot\mathbf{r}+iw\tau}\times\frac{\partial}{\partial a_\mu(r,\tau)}\frac{\partial}{\partial a_\nu(0,0)}\ln Z|_{a=0}$$

and $\sigma(w)=\lim_{w\to 0}\mathrm{Im}C_{xx}(0,w)/w$.

The gauge field $\mathbf{a}$ is an infinitesimal $\mathbf{a}$ added to the gauge field due to the external field. One may check that during the transformations described earlier, $\mathbf{a}$ simply appears in the expected gauge-invariant form. We have not been able to evaluate σ for the system described by Eq. (5) since the conductivity will be a universal constant in $d=2+1$ only and an analysis in this dimension is a hopeless task. However, following Ref. 8, we can obtain an approximate value by replacing Ψ_1 and Ψ_2 by n-component complex fields and letting $n\to\infty$. This is equivalent to a Hartree approximation of Eq. (5) which involves a self-consistent decoupling of the quartic terms giving

$$S_H=\int\tau\int d^2r\sum_{\beta=1}^{n}\left\{\sum_{\alpha=1,2}\left(\frac{1}{2}r|\Psi_\alpha^\beta|^2+\frac{1}{2}\left[(\nabla\Psi_\alpha^\beta)^2+\left(\frac{d\Psi_\alpha^\beta}{d\tau}\right)^2\right]\right)+\rho\,\mathrm{Re}[(\Psi_1^\beta)^*\Psi_2^\beta]\right\}, \tag{6}$$

$$r=r_0+4(2u-v)\langle|\Psi|^2\rangle,\qquad \rho=4v\langle\mathrm{Re}(\Psi_1^*\Psi_2)\rangle. \tag{7}$$

For $v>0$, the case relevant to the arrays, there is a continuous transition at $r=0$ with $\rho=0$ for $d+1>2$. If $v<0$, however, a first-order transition[14] is found for $2<d+1<4$. Just at the continuous transition in $d=2$, $r=\rho=0$ and the fields Ψ_1,Ψ_2 decouple and we see that $\sigma_{f=1/2}=2\sigma_{f=0}$, where $\sigma_{f=0}$ is the value obtained from a single field $(\pi/8)(2e)^2/h$ as first calculated in Ref. 8. Since, in $d=2$, the resistance per square is just $R=1/\sigma$, we then obtain

$$R^*_{f=1/2}=\tfrac{1}{2}R^*_{f=0}=\frac{4}{\pi}\frac{h}{(2e)^2},$$

which seems consistent with the experimental results.[2]

One may be tempted to generalize this result to all rational values of $f=p/q$ which would yield $R^*(p/q)=(1/q)R^*(0)$. Many different properties of an array in a field[9,19] depend on q. We do not expect this to be true, in general, but, for low-order rationals like $f=\frac{1}{3}$, this may be a good approximation. Also, the $f=\frac{1}{2}$ case is very special since the Hall conductivity is zero unlike other values of f.

Although our model contains no explicit dissipation mechanism to provide a resistance, all we need is some dissipative effects since the value of R^* at α_c is determined by general scaling arguments for the superfluid density. Vortex tunneling by quantum fluctuations is such a possible collective mechanism.[20] In one-dimensional chains of junctions the analogous mechanism is phase-slip induced by quantum fluctuations[21] for which the resistance at α_c vanishes linearly with temperature. The general scaling arguments used here lead immediately to this result which leads us to believe that the universal resistance at the transition does not depend on the particular dissipation mechanism.

The validity of these results in a real array with $f=\frac{1}{2}$ is open to some question. There are two issues which are unresolved by theory and experiment. The first is the assumption of a continuous superconducting-to-insulating transition which is a prerequisite for a universal resistance and the second is the value of the resistance. Theory only says that a continuous transition is possible but there are many effects which can drive it to first order. For example, the inclusion of a $1/r$ Coulomb interaction between the grains reduces the domain of attraction of the isotropic fixed point,[5] although a slight generalization of the treatment of Ref. 5 to the present case indicates that the Coulomb interaction will flow to zero for some range of parameters. The experiments are also inconclusive on this point. In fact, even in the absence of an applied field, an array which seems to be close to the transition at $T=0$ exhibits noisy voltage spikes[2] at low temperatures which may indicate a first-order transition. However, we believe that the prediction that the universal resistance ratio $R_{1/2}/R_0 \approx 1/2$ will remain approximately true in a real array provided the transitions are continuous. More theoretical and experimental work is needed to clarify these issues and to explore the effects of dissipation and disorder.

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