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ABSTRACT

In this paper we modeled the thrust vector deviations (magnitude and direction) in continuous orbital maneuvers, through one correlation function of their values. We studied one parameter that measures the correlation intensity between the deviations in practical interest range for the usual space missions. This model for the superposed deviations is nearest of the realist phenomena, because admits the correlation between the systematic (random bias) and operational (white noise) deviations in space missions.

INTRODUCTION

The study of the deviations sources in the space vehicle operational systems is very important to the space missions development. These sources produces perturbations through the orbital maneuvers that demands the use of the propulsion systems. During the thrusters burns these non-ideal systems introduces deviations on the keplerian elements and, therefore, dislocates the space vehicle out of the nominal trajectories. These deviations are due the engineering operations (manufacturing kind of thrust operations, etc) and the mass center line dislocation of the vehicle. Recently, many authors have realized studies about these kind of perturbations inside the space trajectories. Porcelli and Vogel (1980) studied the orbit insertion deviations propagation in two-impulsive noncoplanar orbital transfers through the final keplerian elements and velocities components. Adams and Melton (1986) developed algorithm to calculate the propagation of guidance and navigation deviations along a impulsive trajectory involving finite-duration perigee burns in the semi-major axis, in the position and velocity initials, flight-path angle, and burn-on ignition time. Others authors used the propulsion system as control system to reach many purposes. See e.g., Kluever (1997), Javorsek II and Longuski (1999), Vassar and Sherwood (1985), Ulybyshev (1998), etc. Rodrigues (1991) and Santos-Paulo (1998) used one deterministic analysis of thrust deviations effects to the impulsive assumption for the non-punctual satellite. Jesus (1999) developed one extensive numerical and analytical study about orbital transfer under thrust deviations, gaussian and uniform distribution probabilistic deviations. The deviations did not superposed and or correlated, but only individual and non-correlated direction deviations. Numerical and analytical non-linear (near parabolic) cause/effect relation were found between the deviations direction (causes) and the keplerian elements deviations (effect), e. g., for the final mean semi-major axis. The space maneuvers too are affected by the coupling effects of the orbital and rotational motions. Many authors studied these effect: Duboshin, 1958; Barkin, 1985; Beletskii, 1990; Wang, 1991; Wang, 1992 and Maciejewski, 1995. In this paper we extended the Jesus (1999) results for the same problem but under correlated and superposed thrust vector deviations. We present the results of the direction misalignments effects through the keplerian elements for the superposition and correlation thrust deviations. We developed one Monte-Carlo exact numerical analysis and found the cause/effect relation between the final keplerian elements (final mean semi-major axis) and the pitch and yaw direction angle deviations to optimal continuous and non-coplanar transfers maneuvers. These angles are the control variables and provides the optimal (fuel consumption minimum) direction to thruster burn.

MATHEMATICAL FORMULATION

The basic mathematical formulation (equations motion, equations associated optimal control, coordinates systems, etc.) to this orbital problem can be found in Jesus (2002). In this paper we present only the thrust vector general characteristics. In the Figure 1 we present the coordinate system localized in the satellite (TRN system) and the thrust vector applied to this vehicle.

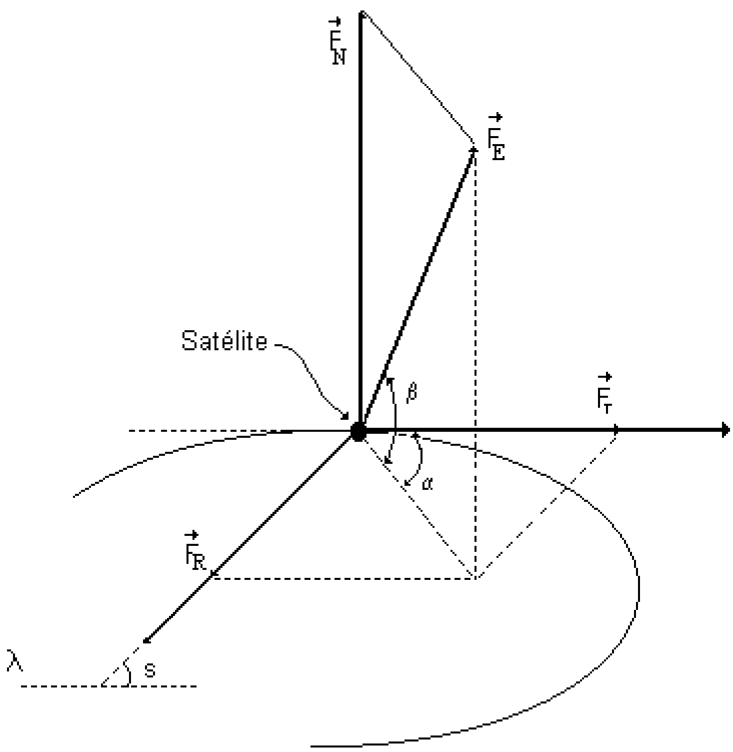


Fig. 1 – The thrust vector applied to the satellite

The thrust components are affected by "pitch" and "yaw" during the burns. The thrust vector is given,

$$\vec{F}_E = \vec{F} + \Delta\vec{F} \tag{1}$$

$$\vec{F}_E = \vec{F}_R + \vec{F}_T + \vec{F}_N \tag{2}$$

$$\left|\vec{F}_E\right| = F_E, \quad \left|\vec{F}\right| = F \tag{3}$$

and their components are,

$$F_R = (F + \Delta F)\cos(\beta + \Delta\beta)\mathrm{sen}(\alpha + \Delta\alpha) \tag{4}$$

$$F_T = (F + \Delta F)\cos(\beta + \Delta\beta)\cos(\alpha + \Delta\alpha) \tag{5}$$

$$F_N = (F + \Delta F)\mathrm{sen}(\beta + \Delta\beta) \tag{6}$$

with,

F, F_T and ΔF (DES1) are the vector without deviation modulus, the vector with deviations, and the vector thrust deviation, respectively; Δα (DES2) and Δβ (DES3) are the "pitch" and "yaw" direction deviations, respectively; F_R, F_T and F_N are the thrust vector components with deviations in the transversal, radial and normal directions, respectively. The DES1, DES2, DES3 are maximum deviations, that is, $\Delta\alpha_{max}$ = DES2 = $\sqrt{3}\,.\sigma_{\Delta\alpha}$ and DES3= $\sqrt{3}\,.\sigma_{\Delta\beta}$ = $\Delta\beta_{max}$.

The equations (4) to (6) present the thrust vector components with superposed deviations. We suggest that during the non-ideal propulsion system operations to realize orbital maneuvers, the thrust vector will be affected through the burns at the deviations ΔF, Δα and Δβ due several operational uncertainness sources. Besides this, these operations causes too propulsion system consuming, therefore, it occurs decay of the propulsion system power. We expect that after the first burn, this system loses energy and power. So, the successive burns will introduce accumulative deviations, affecting more the orbit-target (nominal trajectory). To model these consuming effects we suggest one correlation function thrust deviations, given,

$$J_n = \sum_{k=1}^{L/2}\left[k^{-z}\left(\frac{2\pi}{L}\right)^{(1-z)}\right]^{1/2}\cos\left(\frac{2\pi nk}{L} + \phi_k\right) \tag{7}$$

with,

J_n – the correlated deviations

L – interactions number during the arcs burn

ϕ_k – random phases uniformly distributed in the interval [0,2π] z – correlation parameter

This function provides the individual and correlated thrust deviations ($\Delta\alpha$, $\Delta\beta$, ΔF). The approach based on the use of discrete Fourier transforms (Feder, 1992; Osborne et all, 1989; Greis et all, 1991), a power-law spectral density imposed by construction whenever the deviations are given by Eq. (7). The parameter z defines as much as is strong the correlation between two consecutive deviations values. The non-correlated deviations case is obtained at z=0, inside the practical interest range. So, if we use this correlation function inside the nonlinear keplerian orbital dynamics to model the effects of the non-ideal propulsion system, then the orbital maneuvers of space vehicle will be more realist than the orbital maneuvers under thrust vector non-correlated deviations.

CORRELATED DEVIATIONS EVOLUTION

The correlation parameter, z, determines the robustness and their behavior of the correlation between two consecutive deviations values. The Figures 2 to 7 show the evolution of the J_n behavior with z.

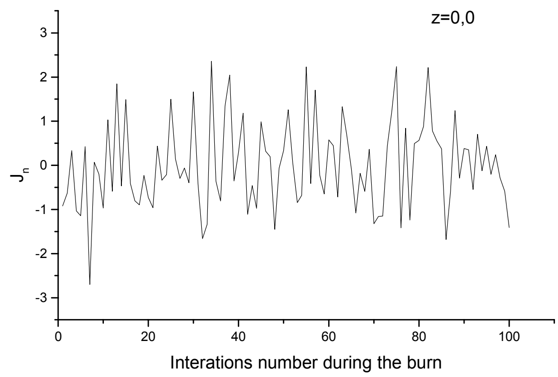


Fig.2 – Correlated deviations vs. L, z=0,0

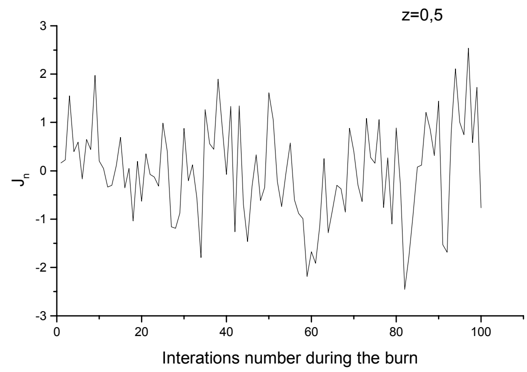


Fig.3 – Correlated deviations vs. L, z=0,5

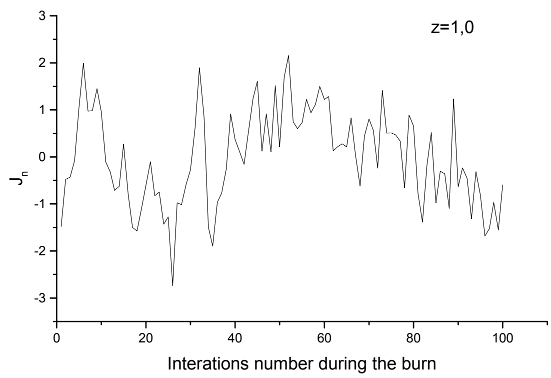


Fig.4 – Correlated deviations vs. L, z=1,0

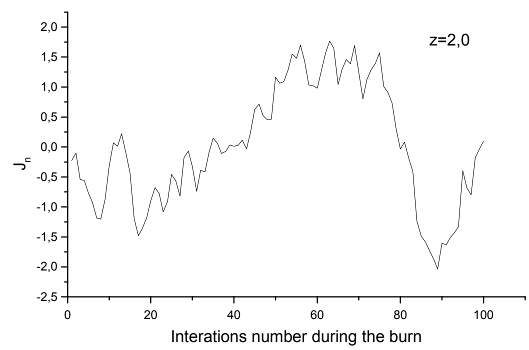
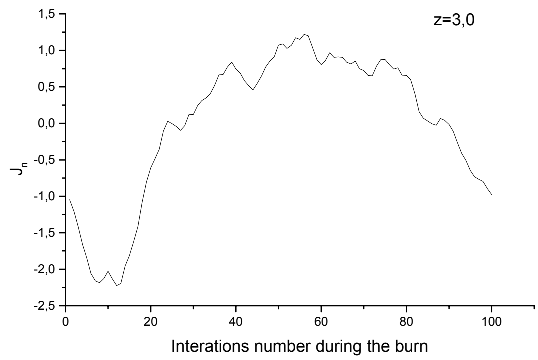


Fig.5 – Correlated deviations vs. L, z=2,0



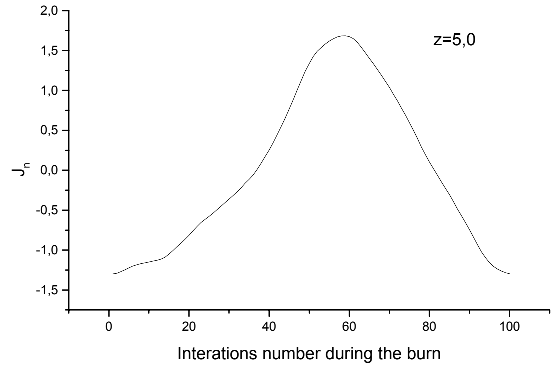


Fig.6 – Correlated deviations vs. L, z=3,0

Fig.7 – Correlated deviations vs. L, z=5,0

We can observe that the correlation is more defined with the increasing of the parameter z. Figure 2 shows the result for the z=0, that is, non-correlated deviations (J_n) case. But, when the parameter z increases the correlation between the J_n turn more evident and precise. Figure 7 shows the behavior correlation for the z=5,0. This value characterizes a strong correlation and can model the demands of the maneuvers under propulsion system so consuming effects.

NUMERICAL SIMULATIONS RESULTS – SEMI-MAJOR AXIS

In this section we present the Monte-Carlo simulations for orbital transfers maneuvers under direction correlated gaussian deviations in the thrust vector. We analyzed two kind of maneuvers. The first is 1) a high orbit low thrust coplanar theoretical transfer (T) used by Biggs (1978 and 1979) and Prado (1989) to test the optimization method; 2) a middle orbit high thrust non-coplanar practical transfer (P) (one of the transfers during the injection of the EUTELSAT II-F2 satellite) from Kuga et alli (1991). These were confirmed and improved with respect to the satisfaction of the initial and final keplerian elements. The deviations are modeled as random-bias (systematic – S) or white noise (operational - O) stochastic processes. We analyzed the correlated deviations effects in the final mean semi-major axis, $E\{a(t_f)\}$, for the both transfers trajectories. Initially, we verified in what range of the parameter z the Equation (7) is valid for model the orbital dynamic treated in our simulations. It is necessary that space missions attend to the operational specifications to be feasible. For example, it is not desirable thrust vector direction deviations more than $2{,}5^0$, because this range deviations is considered not operational (introduce very strong perturbations through the trajectories, requiring correction maneuvers so expensive). The correlated deviations model in (7) reproduces the results of the without correlated case to z=0 inside direction deviations range twice major that practical interest range ($\sim 5{,}5^0$). It is shows the precision of this model. These results can be showed in the Figures (8) to (10) for the theoretical orbit, under pitch, yaw and thrust modulus deviations, respectively. The results from DES1, Figure 10, are too reproduced, that is, there is not cause/effect relation between the final mean semi-major axis and thrust vector modulus deviations. It occurs only mathematical fluctuations as seen in Jesus (1999).

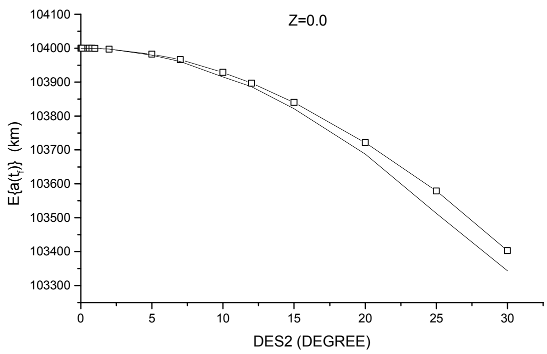


Fig. 8 – **E{a(t$_f$)} vs. DES2, TS, z=0,0**

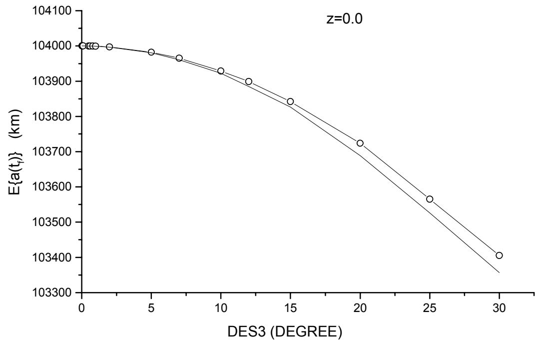


Fig.9 – E{a(t$_f$)} vs. DES3, TS, z=0,0

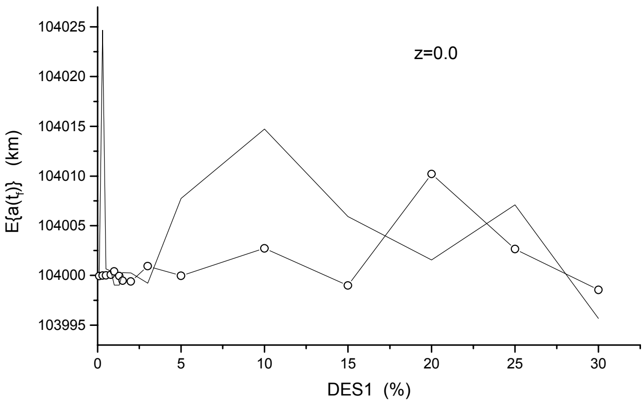
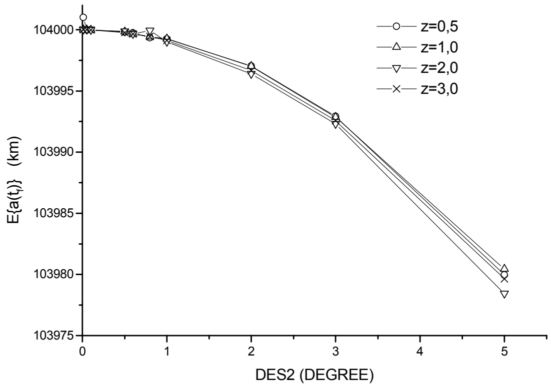


Fig.10 – E{a(t$_f$)} vs. DES1, TS, z=0,0

In the follow we present the semi-major axis values evolution for the z≠0 for the both maneuvers, initially, to individual direction deviations, after, to superposed direction deviations and, finally, superposed and correlated direction deviation. The Figures (11) to (14) show the semi-major axis behavior as function of the increasing z for the direction individual deviations. In these figures we can observe that the correlation effects are perceptible for deviations range more than DES2 = $2,0^0$ and DES3= $4,0^0$ (theoretical orbit). This effect is more strong in this orbit through the pitch angle, because this orbit is in-plane. But, the practical maneuvers is out-plane and, therefore, we expect that the yaw angle has more effect inside the orbit than the pitch angle. The increasing of the correlation effect in this orbit is to produce smaller decay in the E{a(t$_f$)} values than the without correlated deviations case, but in practical interest out-range. This can be seen in the Figure (14). The yaw individual deviations interferes more in this orbit than the pitch individual deviations.



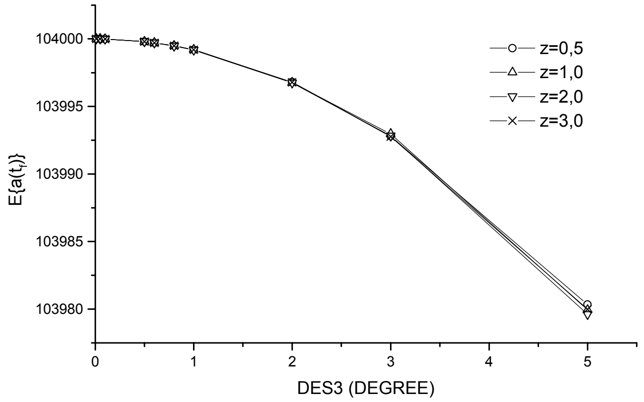
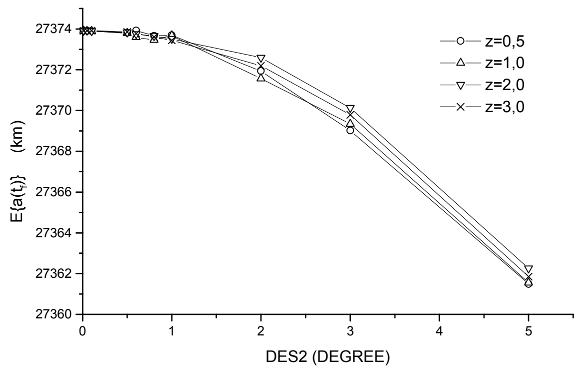


Fig. 11 – E{a(t_f)} vs. DES2, TS, z≠0,0

Fig.12 – E{a(t_f)} vs. DES3, TS, z≠0,0



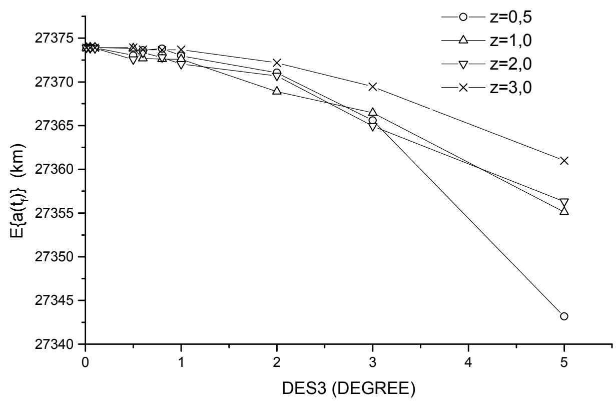
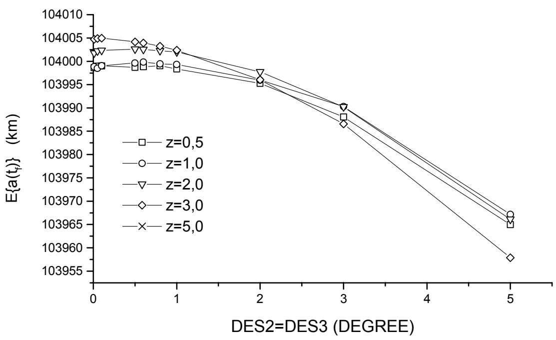


Fig. 13 – E{a(t_f)} vs. DES2, PS, z≠0,0

Fig.14 – E{a(t_f)} vs. DES3, PS, z≠0,0

The Figures (15) and (16) show the results for the both orbits under superposed correlated equals deviations. The superposition and correlation effects can be analyzed for the equals deviations as function of the increasing z.



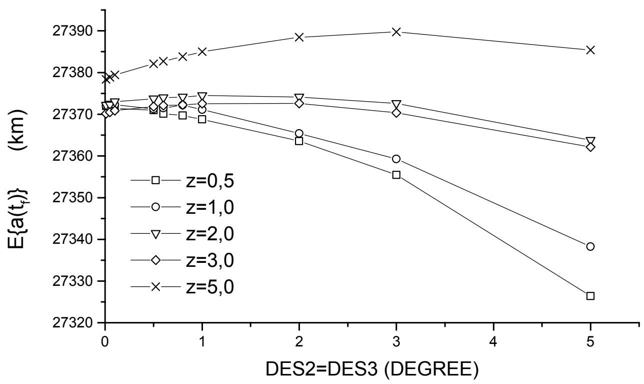


Fig. 15 – E{a(t_f)} vs. DES2=DES3, TS, z≠0,0

Fig.16 – E{a(t_f)} vs. DES2=DES3, PS, z≠0,0

In the Figures (15) and (16) we can observe that the behavior verified for the individual deviations case w.r.t. the correlation is maintained, but it is amplified under superposition direction deviations in the both orbits. Orbits under only superposed deviations were more damaged than those under individual direction deviations (Jesus, 2003). Apparently, the correlation effects inverts this behavior, but interest practical out-range. So, we can say that the model in Equation (7) is very good and secure inside the interest practical range. In the follow we present the results for the more general case, that is, orbits under thrust vector superposed and correlated different deviations. The Figures (17) to (20) show the results to the practical orbit as function z.

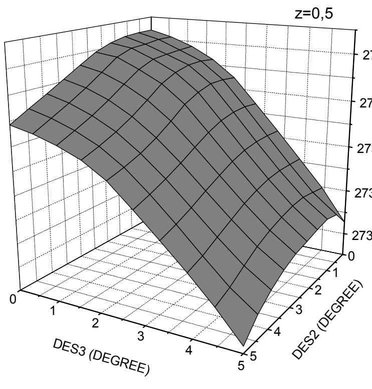


Fig. 17 – E{a(t_f)} vs. DES2≠DES3, PS, z=0,5

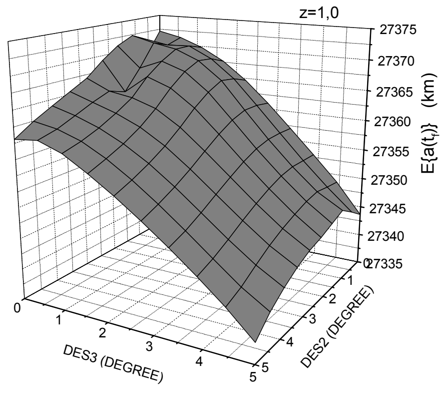


Fig.18 – E{a(t_f)} vs. DES2≠DES3, PS, z=1,0

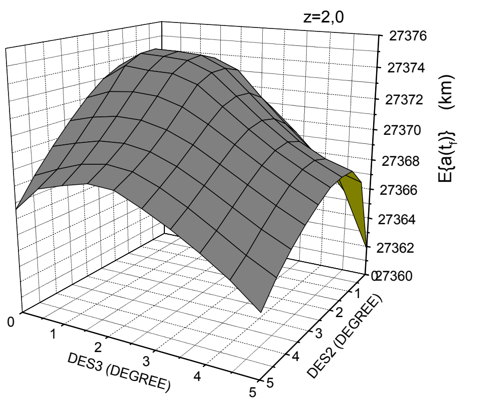


Fig. 19 – E{a(t_f)} vs. DES2≠DES3, PS, z=2,0

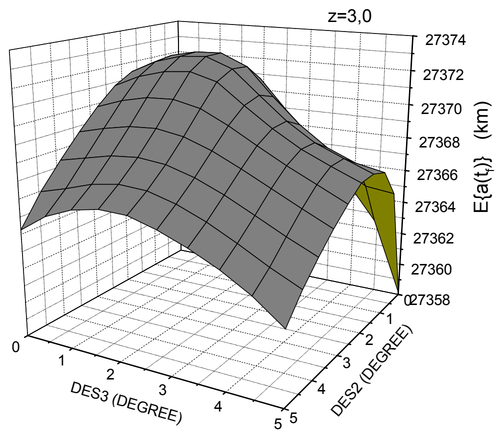


Fig.20 – E{a(t_f)} vs. DES2≠DES3, PS, z=3,0

The decay of the E{a(t_f)} values occurs under superposition different direction deviations. The correlation effect is deform this decay (the surface is deformed through the increasing of the z). This deformation reduces the distance between the E{a(t_f)} values, inside practical interest deviations range. So, to the general case, the correlation plus the superposition effects damage the orbits, and this fact can be compared with the propulsion system consuming, during the transfers maneuvers studied.

This results to the theoretical orbit is too verified, but it not so strong. The Figures (21) to (24) show these results. The theoretical orbit surface is, relatively, little damaged with the correlation effects.

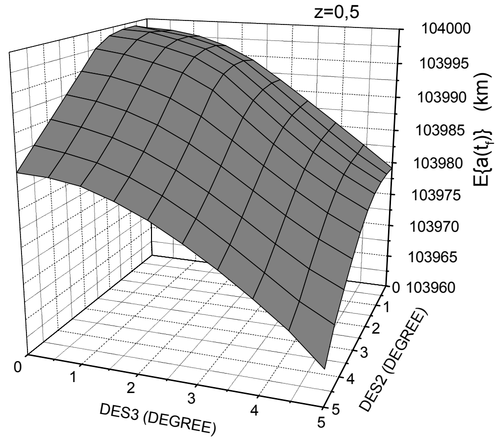


Fig. 21 – E{a(t$_f$)} vs. DES2≠DES3, TS, z=0,5

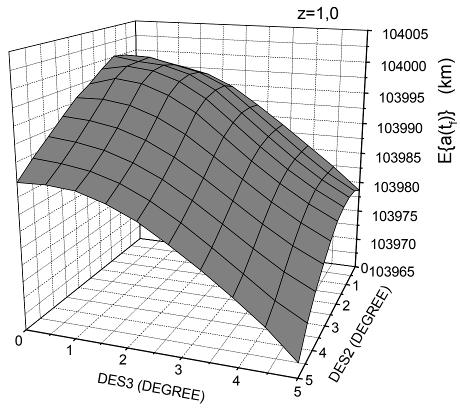


Fig.22 – E{a(t$_f$)} vs. DES2≠DES3, TS, z=1,0

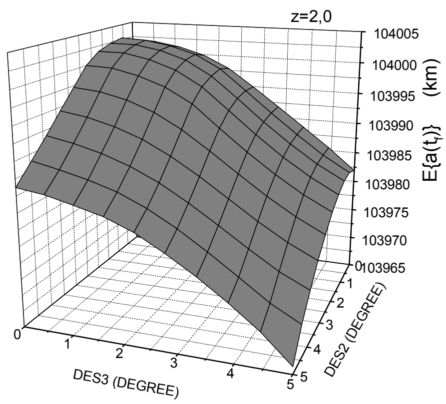


Fig. 23 – E{a(t$_f$)} vs. DES2≠DES3, TS, z=2,0

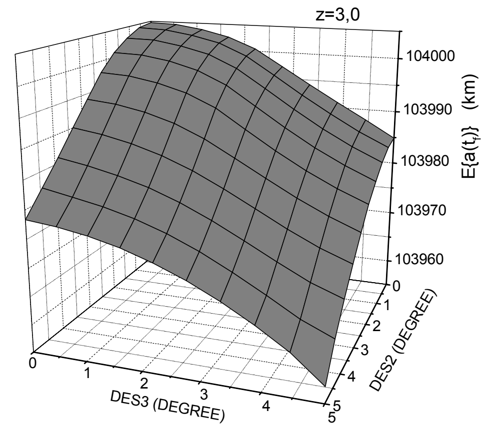


Fig.24 – E{a(t$_f$)} vs. DES2≠DES3, TS, z=3,0.

CONCLUSIONS

We studied the orbital trajectories under thrust vector correlated direction deviations effects. We presented one function of the correlated deviations to model the propulsion system consuming, during the arcs burns in the transfers maneuvers of satellite to in-plane and out-plane orbits. The numerical results (Monte-Carlo simulations) showed that the final mean semi-major axis of the orbits are affected strongly through correlation effect w.r.t. the orbits under only superposed direction deviations. These results are valid mainly inside the practical interest deviations range of the space missions. The dynamic will require more correction maneuvers because the consuming of the thrusters, therefore, it will demand more fuel consumption. To successive maneuvers more loss of the optimality and energy space vehicle will be necessary. This situation can be modeled through increasing correlation parameter z.

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