**Summary/Notes**

This is the first chapter, in a series of 22, written as an introduction to the fundamentals of plasma physics. This introductory chapter is intended to give the reader an overall view of the subject, at a descriptive level. A description is given of the general properties of plasmas, the basic criteria that need to be satisfied for an ionized medium to behave as a plasma, the several types of plasmas which occur in nature, and of some of the most important applications of plasma physics. A brief discussion of the various theoretical approaches that are used in the analysis of plasma phenomena is also presented.

**Remarks**

Revised in October, 1984
PREFACE

This text is intended as a general introduction to the fundamentals of plasma physics. It should be useful primarily for advanced undergraduate and first year graduate students meeting the subject of plasma physics for the first time, and presupposes only a basic elementary knowledge of vector analysis, differential equations and complex variables, as well as courses on classical mechanics and electromagnetic theory beyond the sophomore level. Some effort has been made to make the book self-contained by including in the text developments of the fluid mechanics and kinetic theory that is needed.

Throughout the text the emphasis is on clarity rather than formality. The various derivations are explained in detail and the physical interpretations are emphasized. The equations are presented in such a way that they connect together without requiring the reader to do extensive algebra to bridge the gap. The features of clarity and completeness make the book suitable for self-learning and for self-paced courses.

The structure of this book is as follows. The first chapter consists of a basic introduction to the subject of plasma physics, at a descriptive level, intended to give the reader an overall view of the subject. The motion of charged particles under the
influence of specified electric and magnetic fields is treated in
detail in Chapters 2, 3 and 4. In the next five chapters the basic
equations necessary for an elementary description of plasma phenomena
are developed. Chapter 5 introduces the concepts of phase space and
distribution function, and derives the basic differential kinetic
equation which governs the evolution of the distribution function in
phase space. The definitions of the macroscopic variables in terms of
the phase space distribution function are presented in Chapter 6. The
Maxwell-Boltzmann distribution function is introduced in Chapter 7 as
the equilibrium solution of the Boltzmann equation. In Chapter 8 the
macroscopic transport equations for a plasma considered as a mixture
of various interpenetrating fluids are derived, whereas the
macroscopic transport equations for the whole plasma as a single
conducting fluid are developed in Chapter 9. The remaining of the
book is devoted to applications of these basic equations in the
description of a variety of important phenomena in plasmas. The
problems of electrical conductivity and diffusion in plasmas are
analyzed in Chapter 10, and other basic plasma phenomena, such as
electron plasma oscillations and Debye shielding, are treated in
Chapter 11. Simple applications of the magnetohydrodynamic equations,
such as in plasma confinement by magnetic fields and the pinch
effect, are presented in Chapters 12 and 13. The subject of wave
phenomena in plasmas is organized in the next six chapters. A review
of the basic concepts related to the propagation of electromagnetic
waves in free space is given in Chapter 14. The propagation of very
low frequency waves in a highly conducting fluid is analyzed in
Chapter 15, under the title of magnetohydrodynamic waves. The various modes of wave propagation in cold and warm plasmas are considered in Chapters 16 and 17, respectively. In Chapters 18 and 19 the problems of wave propagation in hot nonmagnetized plasmas and in hot magnetized plasmas, respectively, are analyzed. Collision phenomena in plasmas and the derivation of the Boltzmann collision integral and of the Fokker-Planck collision term, are presented in Chapters 20 and 21. Finally, in Chapter 22 some applications of the Boltzmann equation to the analysis of transport phenomena in plasmas are presented.

A number of problems is provided at the end of each chapter, which illustrate additional applications of the theory and supplement the textual material. The answers to most of the problems are included in their statements.

The numbering of the equations starts over again at each section. When reference is made to an equation, using three numbers, the first number indicates the chapter and the last two numbers indicate the section and the equation, respectively. Within the same chapter the first number is omitted.

The book contains more material than can normally be covered in one semester. This permits some freedom in the selection of topics depending on the level and desired emphasis of the course, and on the interests of the students.
In this, as in any introductory book, the topics included clearly do not cover all areas of plasma physics. No attempt was made to present the experimental aspects of the subject. Moreover, there are some important theoretical topics which are covered only very briefly and some which have been left for more advanced courses on plasma physics, such as plasma instabilities, plasma radiation, nonlinear plasma theory and plasma turbulence.

The system of units used in this text is the rationalized MKSA.
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CHAPTER 1

INTRODUCTION

1. GENERAL PROPERTIES OF PLASMAS

1.1 - Definition of a plasma

The term plasma is used to describe a wide variety of macroscopically neutral substances containing many interacting free electrons and ionized atoms or molecules, which exhibit collective behavior due to the long-range Coulomb forces. Not all media containing charged particles, however, can be classified as plasmas. For a collection of interacting charged and neutral particles to exhibit plasma behavior it must satisfy certain conditions, or criteria, for plasma existence. These criteria will be discussed in the next section. The word plasma comes from the Greek and means "something molded". It was applied for the first time by Tonks and Langmuir, in 1929, to describe the inner region, remote from the boundaries, of a glowing ionized gas produced by electric discharge in a tube, the ionized gas as a whole remaining electrically neutral.

1.2 - Plasma as the fourth state of matter

From a scientific point of view, matter in the known universe is often classified in terms of four states: solid, liquid, gaseous and plasma. The basic distinction between solids, liquids and
gases lies in the difference between the strength of the bonds that hold their constituent particles together. These binding forces are relatively strong in a solid, weak in a liquid, and essentially almost absent in the gaseous state. Whether a given substance is found in one of these states depends on the random kinetic energy (thermal energy) of its atoms or molecules, that is, on its temperature. The equilibrium between this particle thermal energy and the interparticle binding forces determines the state.

By heating a solid or liquid substance the atoms or molecules acquire more thermal kinetic energy until they are able to overcome the binding potential energy. This leads to phase transitions, which occur at a constant temperature for a given pressure. The amount of energy required for the phase transition is called the latent heat.

If sufficient energy is provided, a molecular gas will gradually dissociate into an atomic gas as a result of collisions between those particles whose thermal kinetic energy exceeds the molecular binding energy. At sufficiently elevated temperatures an increasing fraction of the atoms will possess enough kinetic energy to overcome, by collisions, the binding energy of the outermost orbital electrons, and an ionized gas or plasma results. However, this transition from a gas to a plasma is not a phase transition in the thermodynamic sense, since it occurs gradually with increasing temperature.
1.3 - Plasma production

A plasma can be produced by raising the temperature of a substance until a reasonably high fractional ionization is obtained. Under thermodynamic equilibrium conditions the degree of ionization and the electron temperature are closely related. This relation is given by the Saha equation (Chapter 7). Although plasmas in local thermodynamic equilibrium are found in many places in nature, as is the case for many astrophysical plasmas, they are not very common in the laboratory.

Plasmas can also be generated by ionization processes that raise the degree of ionization much above its thermal equilibrium value. There are many different methods of creating plasmas in the laboratory and, depending on the method, the plasma may have a high or low density, high or low temperature, it may be steady state or transient, stable or unstable, and so on. In what follows, a brief description is presented of the most commonly known processes of photoionization and electric discharge in gases.

In the photoionization process, ionization occurs by absorption of incident photons whose energy is equal to or greater than the ionization potential of the absorbing atom. The excess energy of the photon is transformed into kinetic energy of the electron-ion pair formed. For example, the ionization potential energy for the outermost electron of atomic oxygen is 13.6 eV, which can be supplied by radiation of wavelength smaller than about
910 Å i.e. in the far ultraviolet. Ionization can also be produced by X-rays or gamma-rays, which have much smaller wavelengths. The Earth's ionosphere, for example, is a natural photoionized plasma (see Section 3).

In a gas discharge, an electric field is applied across the ionized gas, which accelerates the free electrons to energies sufficiently high to ionize other atoms by collisions. One characteristic of this process is that the applied electric field transfers energy much more efficiently to the light electrons than to the relatively heavy ions. The electron temperature in gas discharges is therefore usually higher than the ion temperature, since the transfer of thermal energy from the electrons to the heavier particles is very slow.

When the ionizing source is turned off, the ionization decreases gradually because of recombination until it reaches an equilibrium value consistent with the temperature of the medium. In the laboratory the recombination usually occurs so fast that the plasma completely disappears in a small fraction of a second.

1.4 - Particle interactions and collective effects

The properties of a plasma are markedly dependent upon the particle interactions. One of the basic features which distinguish the behavior of plasmas from that of ordinary fluids and solids is the existence of collective effects. Due to the long-range of
electromagnetic forces, each charged particle in the plasma interacts simultaneously with a considerable number of other charged particles, resulting in important collective effects which are responsible for the wealth of physical phenomena that take place in a plasma.

The particle dynamics in a plasma is governed by the internal fields due to the nature and motion of the particles themselves, and by externally applied fields. The basic particle interactions are electromagnetic in character. Quantum effects are negligible, except for some cases of close collisions. In a plasma we must distinguish between charge-charge and charge-neutral interactions. A charged particle is surrounded by an electric field and interacts with the other charged particles according to the Coulomb force law, with its dependence on the inverse of the square of the separation distance. Furthermore, a magnetic field is associated with a moving charged particle, which also produces a force on other moving charges. The charged and neutral particles interact through electric polarization fields produced by distortion of the neutral particle's electronic cloud during a close passage of the charged particle. The field associated with neutral particles involves short-range forces, such that their interaction is effective only for interatomic distances sufficiently small to perturb the orbital electrons. It is appreciable when the distance between the centers of the interacting particles is of the order of their diameter, but nearly zero when they are farther apart. Its characteristics can be adequately described only by quantum-mechanical considerations. In many cases this interaction involves permanent or induced electric dipole moments.
A distinction can be made between weakly ionized and strongly ionized plasmas in terms of the nature of the particle interactions. In a weakly ionized plasma the charge-neutral interactions dominate over the multiple Coulomb interactions. When the degree of ionization is such that the multiple Coulomb interactions become dominant, the plasma is considered strongly ionized. As the degree of ionization increases, the Coulomb interactions become increasingly important so that in a fully ionized plasma all particles are subjected to the multiple Coulomb interactions.

1.5 - Some basic plasma phenomena

The fact that some or all of the particles in a plasma are electrically charged and therefore capable of interacting with electromagnetic fields, as well as creating them, gives rise to many novel phenomena that are not present in ordinary fluids and solids. The presence of the magnetic field used, for example, in the heating and confinement of plasmas in controlled thermonuclear research greatly accentuates the novelty of plasma phenomena. To explore all features of plasma phenomena, the plasma behavior is usually studied in the presence of both electric and magnetic fields.

Because of the high electron mobility, plasmas are generally very good electrical conductors, as well as good thermal conductors. As a consequence of their high electrical conductivity they do not support electrostatic fields except, to a certain extent, in a direction normal to any magnetic field present, which inhibits the flow of charged particles in this direction.
The presence of density gradients in a plasma causes the particles to diffuse from dense regions to regions of lower density. Although the diffusion problem in nonmagnetized plasmas is somewhat similar to that which occurs in ordinary fluids, there is nevertheless a fundamental difference. Because of their lower mass, the electrons tend to diffuse faster than the ions, generating a polarization electric field as a result of charge separation. This field enhances the diffusion of the ions and decreases that of the electrons, in such a way as to make ions and electrons diffuse at approximately the same rate. This type of diffusion is called ambipolar diffusion. When there is an externally applied magnetic field, the diffusion of charged particles across the field lines is reduced, which indicates that strong magnetic fields are helpful in plasma confinement. The diffusion of charged particles across magnetic field lines when the diffusion coefficient is proportional to $1/B^2$, where $B$ denotes the magnitude of the magnetic induction, is called classical diffusion, in contrast to Bohm diffusion in which the diffusion coefficient is proportional to $1/B$.

An important characteristic of plasmas is their ability to sustain a great variety of wave phenomena. Examples include high-frequency transverse electromagnetic waves and longitudinal electrostatic plasma waves. In the low frequency regime important waves modes in a magnetized plasma are the so called Alfvén waves and magnetosonic waves. Each of the various possible modes of wave propagation can be characterized by a dispersion relation, which is a functional relation between the wave frequency $\omega$ and the wave number
k, and by its polarization. The study of waves in plasmas provides significant information on plasma properties and is very useful in plasma diagnostics.

Dissipative processes, such as collisions, produce damping of the wave amplitude. This means that energy is transferred from the wave field to the plasma particles. An essentially non-collisional mechanism of wave attenuation also exists in a plasma, the so-called Landau damping. The mechanism responsible for Landau damping is the trapping of some plasma particles (the ones which are moving with velocities close to the wave phase velocity) in the energy potential well of the wave, the net result being the transfer of energy from the wave to the particles. It is also possible to have modes with growing amplitudes, as a result of instabilities, which transfer energy from the plasma particles to the wave field. Instability phenomena are important in a wide variety of physical situations involving dynamical processes in plasmas. The existence of many different types of instabilities in a plasma greatly complicates the confinement of a hot plasma in the laboratory. The study of these instabilities is of essential importance for controlled thermonuclear fusion research.

Another important aspect of plasma behavior is the emission of radiation. The main interest in plasma radiation lies in the fact that it can be used to measure plasma properties. The mechanisms that cause plasmas to emit or absorb radiation can be grouped into two categories: radiation from emitting atoms or
molecules, and radiation from accelerated charges. At the same time that ionization is produced in a plasma, the opposite process, recombination of the ions and electrons to form neutral particles, is normally also occurring. As a result of the recombination process, radiation is often emitted as the excited particles formed during recombination decay to the ground state. This radiation constitutes the line spectra of plasmas. On the other hand, any accelerated charged particle emits radiation. The radiation emitted whenever a charged particle is decelerated by making some kind of collisional interaction is called bremsstrahlung. If the charged particle remains unbound, both before and after the encounter, the process is called free-free bremsstrahlung. Radiation of any wavelength can be emitted or absorbed in bremsstrahlung. If the originally unbound charged particle is captured by another particle, as it emits the radiation, the process is called free-bound radiation. Cyclotron radiation, which occurs in magnetized plasmas, is due to the magnetic centripetal acceleration of the charged particles as they spiral about the magnetic field lines. Blackbody radiation from plasmas in thermodynamic equilibrium is important only in astrophysical plasmas, in view of the large size needed for a plasma to radiate as a blackbody.

2. **CRITERIA FOR THE DEFINITION OF A PLASMA**

2.1 - **Macroscopic neutrality**

In the absence of external disturbances a plasma is macroscopically neutral. This means that under equilibrium with no
external forces present, in a volume of the plasma sufficiently large to contain a large number of particles and yet sufficiently small compared with the characteristic lengths for variation of macroscopic parameters such as density and temperature, the net resulting electric charge is zero. In the interior of the plasma the microscopic space charge fields cancel each other and no net space charge exists over a macroscopic region.

If this macroscopic neutrality was not maintained, the potential energy associated with the resulting Coulomb forces could be enormous compared to the thermal particle kinetic energy. Consider, for example, a plasma with a charged particle number density of $10^{20} \text{m}^{-3}$ and suppose that the electron density ($n_e$) in a spherical volume of $10^{-3} \text{m}$ radius ($r$) were to differ by one per cent from the density of the positive ions ($n_i$). Denoting the ion charge by $e$ and the electron charge by $-e$, the total net charge inside the sphere would be

$$q = \frac{4}{3} \pi r^3 (n_i - n_e)e$$  \hspace{1cm} (2.1)

and the electric potential at the surface of the sphere would be

$$\phi = \frac{1}{4\pi \epsilon_0} \frac{q}{r} = \frac{r^2(n_i - n_e)e}{3\epsilon_0}$$  \hspace{1cm} (2.2)

where $\epsilon_0$ is the permittivity of free space. Plugging numerical values into (2.2) yields $\phi = 6 \times 10^3$ Volts. Recalling that $1 \text{eV} = 1.602 \times 10^{-19}$ joule, we find that $kT = 1 \text{eV}$ when $T = 11\ 600 \text{K}$, where $k$ is Boltzmann's constant ($1.380 \times 10^{-23} \text{ joule/K}$). Therefore, a plasma temperature of several millions of degrees kelvin would be required to balance the electric potential energy with the average thermal particle energy.
Departures from macroscopic electrical neutrality can naturally occur only over distances in which a balance is obtained between the thermal particle energy, which tends to disturb the electrical neutrality, and the electrostatic potential energy resulting from any charge separation, which tends to restore the electrical neutrality. This distance is of the order of a characteristic length parameter of the plasma, called the Debye length. In the absence of external forces, the plasma cannot support departures from macroscopic neutrality over larger distances than this, since the charged particles are able to move freely to neutralize any regions of excess space charge in response to the large Coulomb forces that appear.

2.2 - Debye shielding

The Debye length is an important physical parameter for the description of a plasma. It provides a measure of the distance over which the influence of the electric field of an individual charged particle (or of a surface at some non-zero potential) is felt by the other charged particles inside the plasma. The charged particles arrange themselves in such a way as to effectively shield any electrostatic fields within a distance of the order of the Debye length. This shielding of electrostatic fields is a consequence of the collective effects of the plasma particles. A calculation of the shielding distance was first performed by Debye, for an electrolyte. In Chapter 11 it will be shown that the Debye length, \( \lambda_D \), is directly proportional to the square root of the temperature \( T \) and inversely proportional to the square root of the electron number density, \( n_e \), according to
\[ \lambda_D = \left( \frac{e_0 k T}{n_e e^2} \right)^{1/2} \]  

(2.3)

As mentioned before, the Debye length can also be regarded as a measure of the distance over which fluctuating electric potentials may appear in a plasma, corresponding to a conversion of the thermal particle kinetic energy into electrostatic potential energy.

When a boundary surface is introduced in a plasma the perturbation produced extends only up to a distance of the order of \( \lambda_D \) from the surface. In the neighborhood of any surface inside the plasma there is a layer of width of the order of \( \lambda_D \), known as the plasma sheath, inside which the condition of macroscopic electrical neutrality need not be satisfied. Beyond the plasma sheath region there is the plasma region where macroscopic neutrality is maintained.

Generally \( \lambda_D \) is very small. For example, in a gas discharge where typical values for \( T \) and \( n_e \) are around \( 10^4 \)K and \( 10^{16} \) electrons/m\(^3\), respectively, we have \( \lambda_D = 10^{-4} \)m. For the Earth's ionosphere typical values can be taken as \( n_e = 10^{12} \) electrons/m\(^3\) and \( T = 1000 \)K, which give \( \lambda_D = 10^{-3} \)m. In the interstellar plasma, on the other hand, the Debye length can be several meters long.

It is convenient to define a Debye sphere as a sphere inside the plasma of radius equal to \( \lambda_D \). Any electrostatic fields originated outside a Debye sphere are effectively screened by the charged particles and do not contribute significantly to the electric
field existing at its center. Consequently, each charge in the plasma interacts collectively only with the charges that lie inside its Debye sphere, its effect on the other charges being effectively negligible. The number of electrons \( N_D \), inside a Debye sphere, is given by

\[
N_D = \frac{4}{3} \pi \lambda_D^3 n_e = \frac{4\pi}{3} \left( \frac{e_0 kT}{n_e^{1/3} e^2} \right)^{3/2}
\]

(2.4)

The Debye shielding effect is a characteristic of all plasmas, although it does not occur in every medium that contains charged particles. A necessary and obvious requirement for the existence of a plasma is that the physical dimensions of the system be large compared to \( \lambda_D \). Otherwise there is just not sufficient space for the collective shielding effect to take place and the collection of charged particles will not exhibit plasma behavior. If \( L \) is a characteristic dimension of the plasma, the first criterion for the definition of a plasma is therefore

\[
L \gg \lambda_D
\]

(2.5)

Since the shielding effect is the result of the collective particle behavior inside a Debye sphere, it is also necessary that the number of electrons inside a Debye sphere be very large. A second criterion for the definition of a plasma is therefore

\[
n_e^{1/3} \lambda_D^3 \gg 1
\]

(2.6)
This means that the average distance between electrons, which is roughly given by $n_e^{-2/3}$, must be very small compared to $\lambda_D$. The quantity defined by

$$g = \frac{1}{n_e\lambda_D^2}$$  \hspace{1cm} (2.7)

is known as the plasma parameter and the condition $g \ll 1$ is called the plasma approximation. This parameter is also a measure of the ratio of the mean interparticle potential energy to the mean plasma kinetic energy.

Note that the requirement (2.5) already implies in macroscopic charge neutrality if it is realized that deviations from neutrality can naturally occur only over distances of the order of $\lambda_D$. Nevertheless, macroscopic neutrality is sometimes considered as a third criterion for the existence of a plasma, although it is not an independent one, and can be expressed as

$$n_e = \sum_i n_i$$  \hspace{1cm} (2.8)

2.3 - The plasma frequency

An important plasma property is the stability of its macroscopic space charge neutrality. When a plasma is instantaneously disturbed from the equilibrium condition, the resulting internal space charge fields give rise to collective particle motions which tend to restore the original charge neutrality. These collective motions are characterized by a natural frequency of oscillation known
as the plasma frequency. Since these collective oscillations are high frequency oscillations, the ions, because of their heavy mass, are to a certain extent unable to follow the motion of the electrons. The electrons oscillate collectively about the heavy ions, the necessary collective restoring force being provided by the ion-electron Coulomb attraction. The period of this natural oscillation constitutes a meaningful time scale against which the dissipative mechanisms, tending to destroy the collective electron motions, can be compared.

Consider a plasma initially uniform and at rest, and suppose that by some external means a small charge separation is produced inside it (Fig. 1). When the external disturbing force is removed instantaneously, the internal electric field resulting from charge separation collectively accelerates the electrons in an attempt to restore the charge neutrality. However, because of their inertia, the electrons keep moving beyond the equilibrium position, and an electric field is produced in the opposite direction. This sequence of movements repeats itself periodically, with a continuous transformation of kinetic energy into potential energy, and vice versa, resulting in fast collective oscillations of the electrons about the more massive ions. On the average the plasma maintains its macroscopic charge neutrality.

It will be shown in Chapter 11 that the angular frequency of this collective electron oscillation, called the (electron) plasma frequency, is given by

\[ \omega_{pe} = \left( \frac{n_e e^2}{m_e \epsilon_0} \right)^{1/2} \]  

(2.9)
Fig. 1 - The electric field resulting from charge separation provides the force which generates the electron plasma oscillations.

Collisions between electrons and neutral particles tend to damp these collective oscillations and gradually diminish their amplitude. If the oscillations are to be only slightly damped, it is necessary that the electron-neutral collision frequency, \( v_{en} \), be smaller than the electron plasma frequency,

\[
\nu_{pe} > \nu_{en}
\]

where \( \nu_{pe} = \omega_{pe}/2\pi \). Otherwise, the electrons will not be able to behave in an independent way, but will be forced by collisions to be in complete equilibrium with the neutrals, and the medium can be treated as a neutral gas. Eq. (2.10) constitutes therefore the fourth criterion for the existence of a plasma. This criterion can be alternatively written as
\[ \omega t > 1 \]  

where \( T = 1/\nu_{en} \) represents the average time an electron travels between collisions with neutrals, and \( \omega \) stands for the angular frequency of typical plasma oscillations. It implies that the average time between electron-neutral collisions must be large compared to the characteristic time during which the plasma physical parameters are changing.

Consider for example a gas containing, say, \( 10^{16} \)
electrons/m\(^3\) at a temperature of 1000K, which satisfies both criteria \( L >> \lambda_D \) and \( n_e \lambda_D^3 >> 1 \). If the density of the neutral particles \( n_n \) is relatively small, as in the interstellar gas for example, \( \tau \) is relatively large and the electrons will behave independently, so that the medium can then be treated as a plasma. On the other hand, if \( n_n \) is many orders of magnitude greater than \( n_e \), then the motion of the electrons will be coupled to that of the neutrals and their effect will be negligible.

The basic characteristics of various laboratory and cosmic plasmas are given in Fig. 2 in terms of their temperature \( T \) and electron density \( n_e \), as well as of parameters which depend upon \( T \) and \( n_e \), such as the Debye shielding distance \( \lambda_D \), the electron plasma frequency \( \omega_{pe} \), and the number of electrons \( N_D \) inside a Debye sphere.

3. THE OCCURRENCE OF PLASMAS IN NATURE

With the progress made in astrophysics and in theoretical physics during this century it was realized that most of the matter in
Fig. 2 - Ranges of temperature and electron density for several laboratory and cosmic plasmas and their characteristic physical parameters: Debye length \( \lambda_D \), plasma frequency \( \omega_{pe} \) and number of electrons \( N_D \) in a Debye sphere.
the known universe, with a few exceptions such as the surfaces of cold planets (the Earth, for example) exists as a plasma.

3.1 - The Sun and its atmosphere

The Sun, which is our nearest star and upon which the existence of life on Earth fundamentally depends, is a plasma phenomenon. Its energy output is derived from thermonuclear fusion reactions of protons forming helium ions deep in its interior, where temperatures exceed 12 millions K. The high temperature of its interior and the consequent thermonuclear reactions keep the entire Sun gaseous. Due to its large mass ($2 \times 10^{30}$Kg), the force of gravity is sufficient to prevent the escape of all but the most energetic particles, and of course radiation, from the hot solar plasma.

There is no sharp boundary surface to the Sun. Its visible part is known as the solar atmosphere, which is divided into three general regions or layers. The photosphere, with a temperature of about 6000K, comprises the visible disk, the layer in which the gases become opaque, and is a few hundred kilometers thick. Surrounding the photosphere there is a redish ring called the chromosphere, approximately ten thousand kilometers thick, above which flame-like prominences rise with temperatures of the order of 100,000K. Surrounding the chromosphere there is a tenuous hot plasma, extending millions of kilometers into space, known as the corona. A steep temperature gradient extends from the chromosphere to the hotter corona, where the temperature exceeds 1 million K.
The Sun possesses a magnetic field which at its surface is of the order of $10^{-4}$ Tesla, but in the regions of sunspots (regions of cooler gases) the magnetic field rises to about 0.1 Tesla.

3.2 - The solar wind

A highly conducting tenuous plasma called the solar wind, composed mainly of protons and electrons, is continuously emitted by the Sun at very high speeds into interplanetary space, as a result of the supersonic expansion of the hot solar corona. The solar magnetic field tends to remain frozen in the streaming plasma and, because of solar rotation, the field lines are carried into Archimedean spirals by the radial motion of the solar wind (Fig. 3). Typical values of the parameters in the solar wind are $n_e = 5 \times 10^6 m^{-3}$, $T_i = 10^4 K$, $T_e = 5 \times 10^5 K$, $B = 5 \times 10^{-8}$ Tesla and drift velocity $3 \times 10^5 m/s$.

3.3 - The magnetosphere and the Van Allen radiation belts

As the highly conducting solar wind impinges on the Earth's magnetic field it compresses the field on the sunward side and flows around it at supersonic speeds. This creates a boundary, called the magnetopause, which is roughly spherical on the sunward side and roughly cylindrical in the anti-sun direction (Fig. 4). The inner region, from which the solar wind is excluded and which contains the compressed Earth's magnetic field, is called the magnetosphere.
Fig. 3 - Schematic representation of the Archimedes spiral structure of the interplanetary magnetic field in the ecliptic plane.

Inside the magnetosphere we find the Van Allen radiation belts, in which energetic charged particles (mainly electrons and protons) are trapped in regions where they execute complicated trajectories which spiral along the geomagnetic field lines and at the same time drift slowly around the Earth. The origin of the inner belt is ascribed to cosmic rays, which penetrate into the atmosphere and form proton-electron pairs which are then trapped by the Earth's magnetic field. The outer belt is considered to be due to and maintained by streams of plasma consisting mainly of protons and electrons which are ejected from time to time by the Sun. Depending on solar activity, particularly violent solar eruptions may occur with the projection of hot streams of plasma material into space. The separation into inner and outer belts reflects only an altitude-dependent energy spectrum, rather than two separate trapping regions.
Fig. 4 - Schematic configuration of the magnetosphere in the noon-midnight plane. The dark crescents represent the regions of trapped energetic particles (Van Allen radiation belts). The turbulent region between the shock wave (bow shock) and the magnetopause is known as the magnetosheath. Geocentric distances are indicated in units of earth radii.
3.4 - The ionosphere

The large natural blanket of plasma in the atmosphere, which envelopes the Earth from an altitude of approximately 60 km to several thousands of kilometers, is called the ionosphere. The ionized particles in the ionosphere are produced during the daytime through absorption of solar extreme ultraviolet and X-ray radiation by the atmospheric species. As the ionizing radiation from the Sun penetrates deeper and deeper into the Earth's atmosphere it encounters a larger and larger density of gas particles, producing more and more electrons per unit volume. However, since radiation is absorbed in this process, there is a height where the rate of electron production reaches a maximum. Below this height the rate of electron production decreases, in spite of the increase in atmospheric density, since most of the ionizing radiation was already absorbed at the higher altitudes.

Fig. 5 provides some idea of the relative concentration and altitude distribution of the electrons and the principal positive ions, typical of the daytime ionosphere during solar minimum conditions. The Earth's magnetic field exerts a great influence on the dynamic behavior of the ionospheric plasma. An interesting phenomenon which occurs in the ionospheric polar regions is the aurora. It consists of electromagnetic radiation emitted by the atmospheric species and induced by energetic particles of solar and cosmic origin which penetrate into the atmosphere along the geomagnetic field lines near the poles.
Fig. 5 - Height distribution of the electrons and the principal positive ions, typical of the daytime ionosphere during solar minimum conditions.

3.5 - Plasmas beyond the solar system

Beyond the solar system we find a great variety of natural plasmas in stars, interstellar space, galaxies, intergalactic space, and far beyond to systems quite unknown before the start of astronomy from space vehicles. There we find a variety of phenomena of great cosmological and astrophysical significance: interstellar shock waves from remote supernova explosions, rapid variations of X-ray fluxes from neutron stars with densities like that of atomic nuclei, pulsing radio stars or pulsar (which are theoretically pictured as rapidly rotating neutron stars with plasmas emitting synchrotron radiation from the surface) and the plasma phenomena around the remarkable "black holes" (which are considered to be singular regions
of space into which matter has collapsed, and possessing such a powerful gravitational field that nothing, whether material objects or even light itself, can escape from them).

The behavior of plasmas in the universe involves the interaction between plasmas and magnetic fields. The Crab nebula is a rich source of plasma phenomena because it contains a magnetic field. The widespread existence of magnetic fields in the universe has been demonstrated by independent measurements, and a wide range of field magnitudes has been found, varying from $10^{-9}$ Tesla in interstellar space to 1 Tesla on the surface of magnetic variable stars.

4. APPLICATIONS OF PLASMA PHYSICS

A wide variety of plasma experiments have been performed in the laboratory to aid in the understanding of plasmas, as well as to test and help expand plasma theory. The progress in plasma research has lead to a wide range of plasma applications. A brief exposition of some important practical applications of plasma physics is presented in this section.

4.1 - Controlled thermonuclear fusion

The most important application of man-made plasmas is in the control of thermonuclear fusion reactions, which holds a vast potential for the generation of power. Nuclear fusion is the process whereby two light nuclei combine to form a heavier one, the total final mass being slightly less than the total initial mass. The mass difference
(\Delta m) appears as energy (E) according to Einstein's famous law
\[ E = (\Delta m)c^2, \]
where \( c \) is the speed of light. The nuclear fusion reaction is the source of energy in the stars, including the Sun. The confinement of the hot plasma in this case is provided by the self-gravity of the stars.

In the nuclear fusion of hydrogen the principal reactions involve the deuterium (\(^2\text{H}\)) and tritium (\(^3\text{H}\)) isotopes of hydrogen, as follows:

\[ ^2\text{H} + ^2\text{H} \rightarrow \begin{cases} \ \ \ ^3\text{He} + ^1\text{n} + 3.27 \text{ Mev} \\ \ \ \ ^3\text{H} + ^1\text{H} + 4.03 \text{ Mev} \end{cases} \]  
(4.1a)

\[ ^2\text{H} + ^3\text{H} \rightarrow ^4\text{He} + ^1\text{n} + 17.58 \text{ Mev} \]  
(4.1b)

\[ ^2\text{H} + ^3\text{He} \rightarrow ^4\text{He} + ^1\text{H} + 18.34 \text{ Mev} \]  
(4.1c)

where \(^1\text{n}\) represents a neutron. The basic problem in achieving controlled fusion is to generate a plasma at very high temperatures (with thermal energies at least in the 10 keV range) and hold its particles together long enough for a substantial number of fusion reactions to take place. The need for high temperatures comes from the fact that, in order to undergo fusion, the positively charged nuclei must come very close together (within a distance of order \( 10^{-14} \text{m} \)), which requires sufficient kinetic energy to overcome the electrostatic Coulomb repulsion.
Fig. 6 presents the cross sections, as a function of the incident particle energy, for the nuclear fusion reactions of hydrogen given in (4.1). They are appreciable only for incident particles with energies above at least 10 keV. This means that the plasma must have temperatures of the order of 100 millions K. Other fusion reactions involving nuclei with larger values of the atomic number Z require even higher energies to overcome the Coulomb repulsion.

Fig. 6 - Fusion cross sections, in barns (1 barn = 10^{-28} m^2), as a function of energy, in keV, for the hydrogen reactions given in (4.1).
Many confinement schemes have been suggested and built which use some type of magnetic field configuration. The main experimental efforts for achieving plasma conditions for fusion can be grouped into four approaches: (1) open systems (magnetic mirrors); (2) closed systems (toruses); (3) theta pinch devices; and (4) laser-pellet fusion.

The mirror machines are linear devices with an axial magnetic field to keep the particles from the wall, and with magnetic mirrors (regions of converging magnetic field lines) at the ends to reduce the number of particles escaping at each end (Fig. 7). The four principal toroidal systems differ in the way they twist the magnetic field lines. They are the stellarators (in which the twisting of the field lines is produced by external helical conductors), the tokamaks (in which a poloidal field produced by an internal plasma current is superposed on the toroidal field), the multipoles (which have their magnetic field lines primarily in the poloidal direction and produced by internal conductors), and the Astron (in which internal relativistic particle beams modify a mirror field into a form having stable confinement regions with closed lines of force). In the theta pinch devices a plasma current in the azimuthal ($\theta$) direction and a longitudinal magnetic field produce a force which compresses the cross-sectional area of the plasma. Finally, the scheme to ignite a fusion reaction using pulsed lasers consists in focusing converging laser beams on a small pellet of solid deuterium-tritium material producing a rapid symmetrical heating of the plasma, followed by an expansion of the heated surrounding shell and compression of the pellet core by the recoil (Fig. 8).
Fig. 7 - Schematic illustration showing the magnetic field configurations of some basic schemes for plasma confinement. (a) Magnetic mirror system. (b) Tokamak. (c) Linear $\theta$-pinch.
In addition to the heating and confinement problems attention must be given to the energy loss by radiation (predominantly electron-ion bremsstrahlung and electron cyclotron radiation). These radiation losses constitute a serious factor in maintaining a self-sustaining fusion device. To generate more energy by fusion than is required to heat and confine the plasma, and to supply the radiation losses, a condition is imposed on the plasma density, $n$, and the confinement time, $\tau$, as well as on the temperature. It turns out that the product $n \tau$ must be higher than a minimum value which, for example, is estimated to be about $10^{20} \text{m}^{-3} \text{s}$ for deuterium-tritium (with $T > 10^7\text{K}$) and about $10^{22} \text{m}^{-3} \text{s}$ for deuterium-deuterium (with $T > 10^8\text{K}$).
This condition is known as the **Lawson criterion**. Consequently, controlled fusion can be achieved either by having a large number density of hot plasma particles confined for a short period of time, or by having a smaller number density of particles confined for a longer period of time. For this reason some fusion experiments operate in the regime of high density and short confinement time utilizing a pulsed mode of operation.

Since controlled nuclear fusion can provide an almost limitless source of energy, it is certainly one of the most important scientific challenges man faces today, and its achievement will cause an enormous impact on our civilization.

4.2 - The magnetohydrodynamic generator

The magnetohydrodynamic (MHD) energy generator converts the kinetic energy of a dense plasma flowing across a magnetic field into electrical energy. While a rigorous discussion of this device becomes quite involved, its basic principle is quite simple.

Suppose (Fig. 9) that a plasma flows with velocity \( v \) (along the \( x \) direction) across an applied magnetic field \( B \) (in the \( y \) direction). The Lorentz force \( q(v \times B) \) causes the ions to drift upward (in the \( z \) direction) and the electrons downward so that if electrodes are placed in the walls of the channel and connected to an external circuit then a current density \( J = \sigma E_{\text{ind}} = \sigma v \times B \) (where \( \sigma \) denotes the plasma conductivity and \( E_{\text{ind}} \) is the induced electric field) flows across the plasma stream in the \( z \) direction.
Fig. 9 - Schematic diagram illustrating the basic principle of the MHD energy generator.

This current density, in turn, produces a force $\mathbf{J} \times \mathbf{B}$ (in the x direction) which decelerates the flowing plasma. The net result is the conversion of some of the plasma kinetic energy entering the generator into electrical energy that can be applied to an external load. This process has the advantage that it operates without the inefficiency of a heat cycle.

4.3 - Plasma propulsion

Plasma propulsion systems for rocket engines are based on a process which converts electrical energy into plasma kinetic energy, that is, the reverse of the MHD generator process.

The plasma rocket engine is accomplished by having both electric and magnetic fields applied perpendicular to each other,
across a plasma (Fig. 10). The resulting current density \( \mathbf{J} \) flowing in the direction of the applied \( \mathbf{E} \) field gives rise to a \( \mathbf{J} \times \mathbf{B} \) force which accelerates the plasma out of the rocket. The associated reaction force, due to conservation of momentum, accelerates the rocket in the direction opposite to the plasma flow. The ejected plasma must always be neutral, otherwise the rocket will become charged to a large potential.

An important characteristic of plasma propulsion systems is that they are capable of generating a certain amount of thrust (although small) over a very long time period, contrarily to chemical propulsion systems. Since the force the plasma rocket engine provides is too modest to overcome the Earth's gravitational field, chemical rockets must still be used as the first stage of any plasma propulsion system in order to produce the extremely high values of thrust required to leave the Earth's gravity. The plasma rocket engine is appropriate for long interplanetary and interstellar space travel.

Fig. 10 - Schematic diagram illustrating the basic principle of the plasma rocket engine.
4.4 - Other plasma devices

A number of other practical applications of plasma physics should be mentioned in addition to controlled fusion, MHD energy conversion and plasma propulsion.

The thermionic energy converter is a device which utilizes a cesium plasma between two electrodes to convert thermal energy into electrical energy. The cathode is heated so that electrons are emitted from the surface, and the anode is cooled. Due to the presence of the cesium plasma, very large electrical currents can be produced at the expense of a significant fraction of the thermal energy applied to the cathode.

Examples of applications involving gas discharges include the ordinary fluorescent tubes and neon lights used for illumination and for signs, mercury rectifiers, spark gaps, a number of specialized tubes like the hydrogen thyratrons and the ignitrons, which are used for switching, and the arc discharges or plasma jets, which are the source of temperatures two or more times as high as the hottest gas flames and which are used in metallurgy for cutting, melting and welding metals.

Two major applications in the area of communications are the long-distance radio wave propagation by reflection in the ionospheric plasma and the communication with a space vehicle through the plasma layer which forms around it during the reentry period into the Earth's atmosphere.
Finally, there is the realm of solid state plasmas. If the usual lattice temperature is considered, it can be easily verified that solids do not satisfy the plasma shielding criterion \( N_D \gg 1 \). Nevertheless, quantum mechanical effects associated with the uncertainty principle give some solids an effective electron temperature high enough to make \( N_D \) sufficiently large, so that plasma behavior can be observed. It has been demonstrated that the free electrons and holes in appropriate solid materials, particularly semiconductors, exhibit the same sort of oscillations and instabilities as gaseous plasmas. The most likely application of solid state plasmas is in electronic circuitry.

5. THEORETICAL DESCRIPTION OF PLASMA PHENOMENA

The dynamic behavior of a plasma is governed by the interactions of the plasma particles with the internal fields produced by the particle themselves and with externally applied fields. As the charged particles in a plasma move around, they can generate local concentrations of positive or negative charge, which give rise to electric fields. Their motion can also generate electric currents and therefore magnetic fields. The dynamics of the particles in a plasma is adequately described by the laws of classical (non-quantum) mechanics. Generally, the momentum of the plasma particles is high and the density low enough to keep their De Broglie wavelengths much smaller than the inter-particle distance. Quantum effects turn out to be important only at very high densities and very low temperatures.
5.1 - General considerations on a self-consistent formulation

The interaction of charged particles with electromagnetic fields is governed by the Lorentz force. For a typical particle of charge $q$ and mass $m$, moving with velocity $v$ in the presence of electric ($E$) and magnetic induction ($B$) fields, the equation of motion is

$$\frac{dp}{dt} = q(E + v \times B)$$  \hspace{1cm} (5.1)

where $p$ denotes the momentum ($p = mv$). It is conceivable, at least in principle, to describe the dynamics of a plasma by solving the equations of motion for each particle in the plasma under the combined influence of externally applied force fields and internal fields generated by all the other plasma particles. If the total number of particles is $N$, we will have $N$ nonlinear coupled differential equations of motion to solve simultaneously. A self-consistent formulation must be used since the fields and the trajectories of the particles are intrinsically coupled, that is, the internal fields associated with the presence and motion of the plasma particles influence their motions which, in turn, modify the internal fields.

The electromagnetic fields obey Maxwell equations

$$\nabla \times E = -\frac{\partial B}{\partial t}$$  \hspace{1cm} (5.2)

$$\nabla \times B = \mu_0 (J + \epsilon_0 \frac{\partial E}{\partial t})$$  \hspace{1cm} (5.3)
\[ \nabla \cdot \mathbf{E} = \frac{\rho_c}{\varepsilon_0} \]  
(5.4)

\[ \nabla \cdot \mathbf{B} = 0 \]  
(5.5)

where \( \rho_c, \mathbf{J}, \varepsilon_0 \) and \( \mu_0 \) denote, respectively, the total charge density, the total electric current density, the electric permittivity and the magnetic permeability of free space. The plasma charge density and current density can be expressed, respectively, as

\[ \rho_{cp} = \frac{1}{\Delta V} \sum_i q_i \]  
(5.6)

\[ \mathbf{J}_p = \frac{1}{\Delta V} \sum_i q_i \mathbf{v}_i \]  
(5.7)

where the summation is over all charged particles contained inside a suitably chosen small volume element \( \Delta V \). Note that since we are dealing with a discrete distribution of charges and therefore also of current densities, \( \rho_{cp} \) and \( \mathbf{J}_p \) should actually be expressed in terms of Dirac delta functions. If point charges are considered, the problem gets even more complicated because the fields become singular at the particle positions. However, if \( \Delta V \) is chosen big enough to contain a fairly large number of particles, then (5.6) and (5.7) will give smooth functions for \( \rho_{cp} \) and \( \mathbf{J}_p \) which are suitable for analytical calculations.

Although this self-consistent approach is conceivable in principle, it cannot be carried out in practice. According to the laws
of classical mechanics, in order to determine the position and velocity of each particle in the plasma as a function of time under the action of known forces, it is necessary to know the initial position and velocity of each particle. For a system consisting of a very large number of interacting particles these initial conditions are obviously unknown. Furthermore, in order to explain and predict the macroscopic phenomena observed in nature and in the laboratory, it is not of interest to know the detailed individual motion of each particle, since the observable macroscopic properties of a plasma are due to the average collective behavior of a large number of particles. To try to interpret the complex individual particle motions in terms of a few macroscopic observables would be a very inefficient way of making predictions. We reject, therefore, the possibility of solving simultaneously the equations of motion for a large number of interacting particles, even because the results would be of no practical interest.

5.2 - Theoretical approaches

For the theoretical description of plasma phenomena there are basically four principal approaches with several different choices of approximations, each of which applies to different circumstances.

One useful approximation, known as particle orbit theory, consists in studying the motion of each charged particle in the presence of specified electric and magnetic fields. This approach is not really plasma theory, but rather the dynamics of a charged
particle in given fields. Nevertheless it is important, since it provides some physical insight for a better understanding of the dynamical processes in plasmas. It has proven to be useful for predicting the behavior of very low density plasmas, which is determined primarily by the interaction of the particles with external fields. This is the case, for example, of the highly rarefied plasmas of the Van Allen radiation belts and the solar corona, as well as for cosmic rays, high energy accelerators and cathode ray tubes.

Since a plasma consists of a very large number of interacting particles, in order to provide a macroscopic description of plasma phenomena it is appropriate to adopt a statistical approach. This implies in a great reduction in the amount of information to be handled. In the kinetic theory statistical description it is necessary to know only the distribution function for the system of particles under consideration. The problem consists in solving the appropriate kinetic equations which govern the evolution of the distribution function in phase space. One example of a differential kinetic equation is the Vlasov equation, in which the interaction between the charged particles is described by smeared out internal electromagnetic fields consistent with the distributions of electric charge density and current density inside the plasma, and the effects of short-range correlations (close collisions) are neglected.

When collisions between the plasma particles are very frequent so that each species is able to maintain a local equilibrium distribution function, then each species can be treated as a fluid
described by a local density, local macroscopic velocity and local temperature. In this case the plasma is treated as a mixture of two or more interpenetrating fluids. This theory is called *two-fluid* or *many-fluid theory*. In addition to the usual electrodynamic equations, there is a set of hydrodynamic equations expressing conservation of mass, momentum and energy for each particle species in the plasma.

Another approach consists in treating the whole plasma as a *single conducting fluid* using lumped macroscopic variables and their corresponding hydrodynamic conservation equations. This theory is usually referred to as the *one-fluid theory*. An appropriately simplified form of this theory, applicable to the study of very low frequency phenomena in highly conducting fluids immersed in magnetic fields, is the *magnetohydrodynamic (MHD)* theory.
1.1 - The interatomic or intermolecular forces are usually represented in terms of a potential energy function \( V(r) \) such that

\[
F = -\frac{dV(r)}{dr}
\]

For neutral particles, at large internuclear distances, there is a slight attractive potential between the particles called the van der Waals potential (which is the long-range part of the Lennard-Jones potential). For like atoms or molecules in like states the van der Waals interaction potential can be represented by

\[
V(r) = -C \text{ Ry} \left(\frac{a_0}{r}\right)^6
\]

where \( C \) is a constant (which depends on the type of particle), \( a_0 \) is the Bohr radius (0.529 \( \text{Å} \)) and \( \text{Ry} \) denotes the Rydberg energy unit (13.605 eV). Calculate the van der Waals force of attraction between two hydrogen molecules (for which \( C = 24.0 \)), and compare with the Coulomb force between a proton and an electron at a distance \( r = N a_0 \), where \( N >> 1 \).

1.2 - Consider an initially uniform plasma in which the electron and ion number densities are each equal to \( n \). By some external means, let a one-dimensional perturbation occur such that the electrons in an infinite plane (y-z plane) are displaced by a small amount \( x \) (see Fig. P1.1).
(a) Using Gauss' law show that the electric field which appears across the perturbed plane is given by

\[ E = \left( \frac{ne}{c_0} \right)_x \]

(b) Show that the equation of motion (Newton's second law) for each electron, under the action of this electric field, is

\[ \frac{d^2x}{dt^2} + \left( \frac{ne^2}{m c_0} \right)_x = 0 \]

Verify that this is the equation of a harmonic oscillator of frequency

\[ \omega_{pe} = \left( \frac{ne^2}{m c_0} \right)^{1/2} \]

Fig. P1.1
1.3 - (a) Calculate the amount of energy released by the fusion of 1g of deuterium according to the nuclear reactions indicated in (4.1), considering as end products \(^6\text{He}, \:^1\text{H}\) and \(^1\text{n}\). Assume that the two possible results shown in (4.1), for the reaction \(^2\text{H} + \:^2\text{H}\), occur with equal probability.

(b) How much energy can be released from the fusion of all the deuterium that exists in one liter of ordinary water? Compare this much energy with the energy obtained from the combustion of one liter of gasoline.

1.4 - Calculate the Coulomb repulsion force and the associated electric potential energy due to the Coulomb interaction of two deuterium nuclei when brought together at a distance of \(10^{-2.4}\) m. What temperature must have the nuclei in a deuterium plasma, if their average thermal kinetic energy is to be equal to this electric potential energy?

1.5 - In a MHD generator a plasma of conductivity \(\sigma\) is driven with velocity \(u\) (in the x direction) across a magnetic field \(B\) (in the y direction). Two electrode plates, each of area \(A\) and separated by a distance \(d\), are placed oriented parallel to the x-y plane, as shown in Fig. 9 of Chapter 1.

(a) Show that the open-circuit electric potential difference between the two electrode plates is given by

\[ \phi = Bud \]
(b) If an external load of resistance \( R_L \) is connected between the electrodes, show that the current that flows is given by

\[
I = \frac{Bud}{R_L + R_p}
\]

where \( R_p \) denotes the internal plasma resistance.

(c) Show that the power delivered to the load is

\[
P_L = \frac{B^2 u^2 d^2 R_L}{(R_L + R_p)^2}
\]

Verify that this has a maximum \( (dP_L/dR_L = 0) \) when \( R_L = R_p \) and show that the maximum power that can be delivered to the load is given by

\[
P_{L,\text{max}} = \frac{1}{4} B^2 u^2 d \sigma A
\]

(d) Determine numerical results for items (a), (b) and (c) when \( B = 1 \) Tesla, \( u = 100 \) m/s, \( \sigma = 100 \) mho/m, \( d = 0.1 \) m and \( A = 1 \) m².

1.6 - Consider a rocket once it is beyond the Earth's gravitational field. Let:

\( v \) = constant velocity of the exhaust gas \textit{relative to the rocket}.

\( u(t) \) = instantaneous velocity of the rocket.

\( M(t) \) = instantaneous total mass of the rocket.

\[- \frac{dM}{dt} \] = constant time rate of decrease of \( M(t) \), that is, the mass expelled per unit time.
(a) Verify that the equation of motion (Newton's second law) for the rocket is
\[
\frac{d}{dt} [M(t) u(t)] = - \frac{dM}{dt} [v - u(t)]
\]
and show that the instantaneous acceleration of the rocket is
\[
\frac{du}{dt} = - \frac{v}{M(t)} \frac{dM}{dt}
\]

(b) Integrate the equation of motion to show that
\[
u(t) = u(t_0) + \int v \frac{M(t_0)}{M(t)} \, dt
\]

(c) If the rocket burns for a time interval \( t - t_0 = T_0 \) and if \( M(t) \ll M(t_0) \), show that the initial acceleration of the rocket is
\[
\left. \frac{du}{dt} \right|_{t_0} = \frac{v}{M(t_0)} \frac{[M(t_0) - M(t)]}{T_0} = \frac{v}{T_0}
\]

(d) Calculate numerically \( du/dt \big|_{t_0} \) and \( u(t) \) for a chemical rocket with \( v = 10^3 \text{ m/s} \) and \( T_0 = 10 \text{ s} \); and for a plasma propulsion system with \( v = 10^4 \text{ m/s} \) and \( T_0 = 100 \text{ days} \). For the calculation of \( u(t) \) consider \( u(t_0) = 0 \) and \( M(t_0) = 10 M(t) \).

1.7 - Using Maxwell equations (5.3) and (5.4) derive the charge conservation equation
\[
\frac{\partial \rho}{\partial t} + \nabla \cdot J = 0
\]
This result shows that conservation of electric charge is implied by Maxwell equations.

1.8 - From Maxwell curl equation (5.2) derive the equation

\[ \nabla \cdot \mathbf{B} = \text{constant} \]

Therefore, (5.5) can be considered as an initial condition for (5.2) since, if \( \nabla \cdot \mathbf{B} = 0 \) at the initial time, then (5.2) implies that this condition will remain satisfied for all subsequent times.

1.9 - Using Maxwell equations derive the following energy conservation law for electromagnetic fields, known as Poynting's theorem,

\[ \frac{\partial}{\partial t} \int \left( \frac{1}{2} \varepsilon \mathbf{E}^2 + \frac{1}{2} \mu \mathbf{H}^2 \right) d^3r + \oint (\mathbf{E} \cdot \mathbf{H}) \cdot d\mathbf{S} = -\oint \mathbf{J} \cdot \mathbf{E} \, d^3r \]

for a linear isotropic medium, for which \( \mathbf{D} = \varepsilon \mathbf{E} \) and \( \mathbf{B} = \mu \mathbf{H} \). Give the physical interpretation for each term in this equation. What are the physical dimensions of these terms?

1.10 - Consider the following Maxwell equations:

\[ \nabla \times \mathbf{B} = \mu_0 \left( \mathbf{J}_t + \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right) \]

\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \]
\[ \nabla \cdot \vec{E} = \rho_{ct}/\varepsilon_0 \]

\[ \nabla \cdot \vec{D} = \rho_c \]

For a general medium for which

\[ \vec{D} = \varepsilon_0 \vec{E} + \vec{P} \]

\[ \vec{B} = \mu_0 (\vec{H} + \vec{M}) \]

where \( \vec{P} \) is the polarization vector and \( \vec{M} \) is the magnetization vector, show that the total electric charge density (\( \rho_{ct} \)) and current density (\( \vec{J}_t \)) are given by

\[ \rho_{ct} = \rho_c - \nabla \cdot \vec{P} \]

\[ \vec{J}_t = \vec{J} + \partial \vec{P}/\partial t + \nabla \times \vec{M} \]

Explain why \( \vec{E} \) and \( \vec{B} \) are usually considered as fundamental fields, whereas \( \vec{D} \) and \( \vec{H} \) are partial fields.