DINCON 2006
Brazilian Conference on Dynamics, Control and Their Applications
May 22 – 26, 2006, Guaratinguetá, SP, Brazil

COMPARING RECURSIVE LEAST SQUARES ALGORITHMS APPLIED TO ORBIT DETERMINATION

Hélio Koiti Kuga [1], *Paula Cristiane Pinto Raimundo* [2], *Roberto Vieira da Fonseca Lopes* [3]

[1] DMC / INPE, São José dos Campos, Brazil, hkk@dem.inpe.br
[2] DMC / INPE, São José dos Campos, Brazil, paula-cristiane@uol.com.br
[3] DSS / INPE, São José dos Campos, Brazil, roberto@dss.inpe.br

Abstract: The Global Positioning System is a powerful and low cost means to allow computation of orbits for artificial Earth satellites. Normally the task is carried out off-line using Batch Least Squares methods for orbit determination of satellites with an onboard GPS receiver. Usually, pseudo-range measurements taken from the GPS receiver are available to the orbit estimation algorithm. In this work, recursive least squares methods are studied, considering the conventional Kalman form, UD Kalman form, and sequential Givens rotations. The main aim here is to implement these routines and perform comparisons between the algorithms. The study case is the satellite Topex/Poseidon which carries onboard a GPS receiver which measurements are freely available through Internet. The work pinpoints the differences of implementation between the Kalman form, UD Kalman form, and the less conventional orthogonal transformation via Givens rotations. Simulations were carried out processing real data from the satellite Topex/Poseidon and the algorithms were compared to each other in terms of speed of convergence, accuracy and computer burden.

Keywords: orbit determination, recursive least squares, Kalman filter.

1. INTRODUCTION

The problem of orbit determination consists of estimating parameters values that completely specifies the body trajectory in the space, processing a set of information (measurements) from this body.

The Global Positioning System is a powerful and low cost means to allow computation of orbits for artificial Earth satellites. The Topex/Poseidon (T/P) satellite is an example of using this system for space positioning.

Usually, the iterative improvement of the position parameters of a satellite is carried out using the least squares methods. On a simple way, the least squares estimation algorithms are based on the data equations that describe the linear relation between the residual measurements and the estimation parameters.

The standard Kalman form is a recursive algorithm which allows the state and the state error covariance matrix estimates, through a sequential form. The factorization methods are based on the errors covariance matrix factorization. Such methods involve factorizations with no square roots and have numerical properties very superior than the normal methods.

2. LEAST SQUARES METHODS

Parameters estimation aims at estimating things that are constant along the estimation process. It is necessary a set of measurements to shape the relation between these measurements and the parameters to be estimated.

One of the most used parameter estimator is the least squares algorithm. Basically, the algorithm minimizes the cost function of the residuals squared [1]. The recursive least squares algorithms, when applied to parameters estimation, presents two advantages: avoids matrix inversion in the presence of uncorrelated measurement errors; and needs smaller matrices, which means less need of memory storage.

2.1 *Kalman Form*

In the Kalman form, the equations to implement recursive least squares are given by:

- Kalman Gain:

$$K_i = P_{i-1} H_i^T \left[H_i P_{i-1} H_i^T + R_i \right]^{-1}$$

- Estimated state:

$$\Delta\hat{x}_i = \Delta\hat{x}_{i-1} + K_i (\Delta y_i - H_i \Delta\hat{x}_{i-1})$$

- Error covariance matrix on the state:

$$\hat{P}_i = (I - K_i H_i) P_{i-1}$$

where H is the matrix relating measurements to parameters to be estimated, R is the measurement error covariance matrix, and y is the measurement vector.

Although the normal equations produce a simple and straight implementation of the least squares estimation methods, these can carry out a deficient numerical stability, in the event of bad conditioned estimation problems [2]. For solving this deficiency, alternative formulations were developed, based on QR factorization methods. The Givens rotation [3] is a method for solving least squares problems [4] through orthogonal transformations.

2.2 *Recursive Least Squares Using Sequential Givens Rotations*

The Givens rotations are used when it is fundamental to cancel specific elements of a matrix. The full transformation generically can be given by:

- $$\begin{pmatrix} R \\ 0 \end{pmatrix} = (U_m \quad U_{m-1} \quad \ldots \quad U_3 \quad U_2) H = Q^T H$$

- $$\begin{pmatrix} d \\ r \end{pmatrix} = (U_m \quad U_{m-1} \quad \ldots \quad U_3 \quad U_2) y = Q^T y$$

where R is triangular. At each step the orthogonalization of the H matrix is performed (producing a transformed measurement vector d and r) and the results are stored to the next set of measurements. At the end the final solution is computed. See details in [2, 4].

3. KALMAN FILTER

3.1 *Standard Kalman Filter*

There are two differences between the least squares solution and the standard Kalman filter solution: this can account for dynamic noise on the dynamic state model; and this is an estimator with real time characteristics. The standard Kalman filter has two stages: time-update and measurement-update [1, 5].

The time-update formal equations are:

- $\overline{x}_k = \varphi_{k,k-1} \hat{x}_{k-1}$
- $\overline{P}_k = \varphi_{k,k-1} \hat{P}_{k-1} \varphi_{k,k-1}^t + \Gamma_k Q_k \Gamma_k^t$

where $\overline{x}_k$ and $\overline{P}_k$ represent the state and the covariance updated for the instant k.

And the measurement-update formal equations are:

- $K_k = \overline{P}_k H_k^t \left[H_k \overline{P}_k H_k^t + R_k \right]^{-1}$
- $\hat{P}_k = (I - K_k H_k) \overline{P}_k$
- $\hat{x}_k = \overline{x}_k + K_k (y_k - Hk\overline{x}_k)$

where K_k is the Kalman gain and $\hat{x}_k$ and $\hat{P}_k$ are the state and the covariance updated for the instant k.

3.2 *UD Fatorization*

This technique consists of the covariance matrix P factorization such that:

$$P = UDU^T,\ P^{1/2} = UD^{1/2} = U\, diag\left(\sqrt{d_1}, \ldots, \sqrt{d_n}\right)$$

where U is an upper triangular matrix, and D a diagonal matrix.

The characteristics of the UD algorithm are: it is a square root like algorithm; it has precision properties of square root algorithms; and it is almost comparable, considering speedy and number of computations, to the standard Kalman filter [5, 6].

4. RESULTS

The test conditions used real pseudo-range measurement data from T/P satellite, gathered by the onboard GPS receiver, at 1993/11/18, with selective availability (SA) on.

For the recursive least squares estimator via Givens rotations, the period of analysis covered two hours of data (about one orbital period of T/P); and for Kalman filter like estimators, it was considered a period of only 300s, which will be explained later.

Figures 4.1 to 4.6 show residuals of pseudo-range measurements and errors in position behavior for the three estimators. Table 4.1 shows the minimum and maximum values of residual of pseudo-range and errors in position, again for the three algorithms here studied.

The recursive least squares via Givens rotations method manifested very nice performance during a long interval of time (7200s). Meanwhile, for the standard Kalman form and UD form, in the short interval tested, the residuals decreased, which indicates convergence if the measurements were processed for a larger interval of time. Nevertheless, even in such a short interval of 300s, the position error increased tremendously, meaning that the estimators did not converge numerically. Users of the Kalman filter call this phenomenon as divergence, in which the statistical consistency between residuals and state estimates are not achieved.

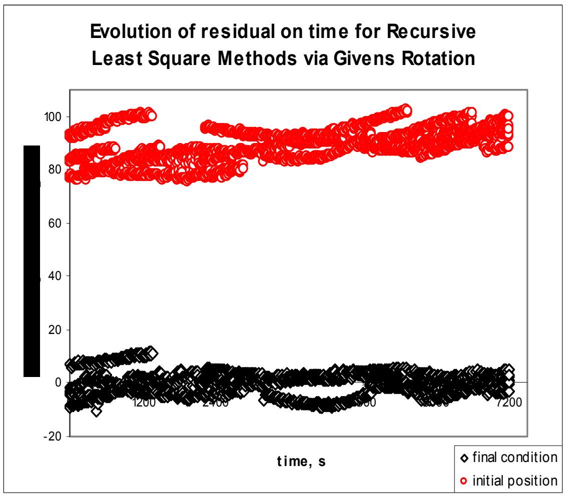

Figure 4.1 – Residual of pseudo-range, through Givens methods.

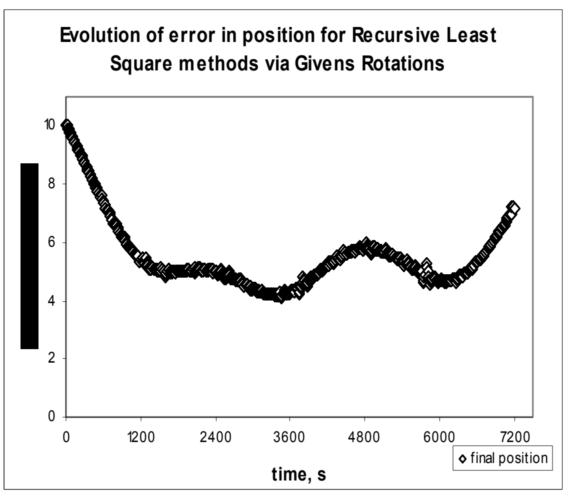

Figure 4.2 – Errors in position, through Givens methods.

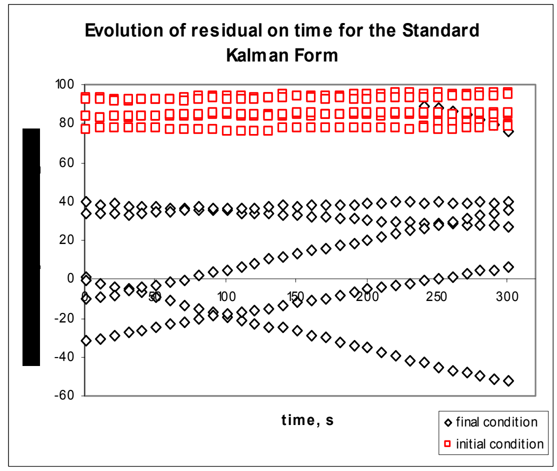

Figure 4.3 – Residual of pseudo-range, through standard Kalman form.

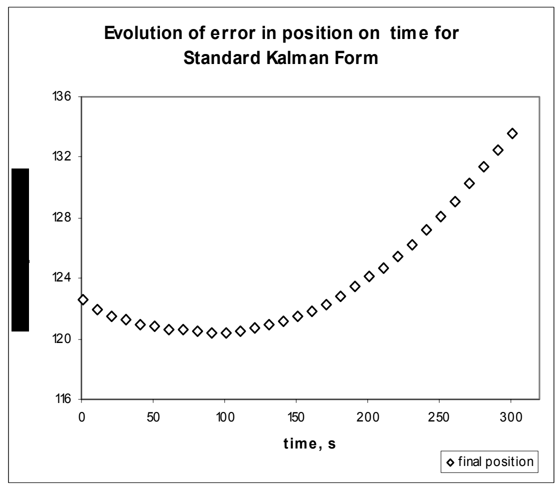

Figure 4.4 – Errors in position, through standard Kalman form.

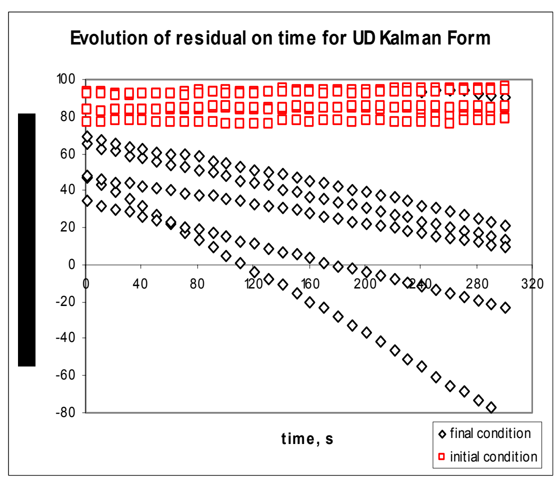

Figure 4.5 – Residual of pseudo-range, through UD Kalman form.

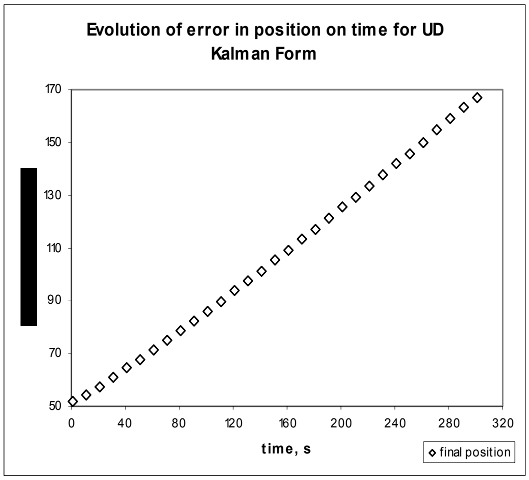

Figure 4.6 – Errors in position, through UD Kalman form.

Estimator	Parameter	Initial Condition	Final Condition	Nº Iterations
Recursive Least Square	Residual (min), m	75,784	-10,683	6
	Residual (max), m	102,282	11,891	
	Error (min), m		4,113	
	Error (max), m		10,047	
Standard Kalman Filter	Residual (min), m	75,962	-52,392	more than 400
	Residual (max), m	95,769	89,041	
	Error (min), m		120,387	
	Error (max), m		133,576	
UD Kalman Form	Residual (min), m	75,962	-81,799	more than 400
	Residual (max), m	95,769	94,072	
	Error (min), m		51,637	
	Error (max), m		167,253	

Table 4.1 – Comparison table between pseudo-range residuals and errors in position for the three estimators.

5. CONCLUSIONS

Although the technological progress have established the state estimation techniques via Kalman filter, in the parameters estimation application, when a lot of measurements are estimated for a long time, the recursive least square methods (here, via orthogonal Givens rotations) appears more powerful and with better numerical precision than the equivalent Kalman form (Standard and UD).

The Kalman form, in the two presented versions, yielded misleading estimates and statistics, primarily because they did not converge. On the other hand, the recursive Givens estimator shows that the dynamics is well modeled and the computational program well formulated, with attained convergence in few (6) iterations.

The most important fact to conclude is: the three estimators are very good and powerful, when applied to the right problems. The recursive least squares method via Givens rotation is very efficient in cases of parameters estimation, when there is the processing of a lot of measurements and for a long period of time; whereas standard Kalman form and UD form are better in cases of state estimation, in real time.

ACKNOWLEDGEMENTS

This work was partially financed by Fapesp master fellowship 04/11635-8, and CNPQ grants 303143/2004-5 and ?????????.

REFERENCES

[1] KUGA, H. K.: "Noções Práticas de Técnicas de Estimação". Class Notes of Optimization on Dynamical Systems II. INPE, São José dos Campos: 2005.

[2] SILVA, A. A.: "Determinação de Órbitas com o GPS através de Mínimos Quadrados Recursivo com Rotações de Givens". Dissertation of Master's Degree. FEG-UNESP, Guaratinguetá: 2001.

[3] GIVENS, J.W. "Computation of plane unitary rotations transforming a general matrix to triangular form." SIAM Journal of Applied Mathematics, Vol. 6, p. 26-50, 1958.

[4] MONTENBRUCK, O.; SUAREZ, M. "A modular Fortran library for sequential least-squares estimation using QR-Factorization." DLR, German Space Operations Center, Oberpfaffenhofen. Germany: 1984 (DLR-GSOC IB 94-05).

[5] BIERMAN, G.J.: "Factorization methods for discrete sequential estimation." Academic, New York: 1977.

[6] KUGA, H. K.: "Análise Numérica Preliminar dos Métodos de Estimação Seqüencial". INPE - 2517 - RPI/072, Internal Publication. INPE, São José dos Campos: 1981.

Helio: Não entendi o que é para eu colocar no lugar das ???????? nos agradecimentos. O restante eu corrigi.