MULTI-OBJECTIVE OPTIMIZATION APPLIED TO SATELLITE CONSTELLATIONS II: INITIAL APPLICATIONS OF THE SMALLEST LOSS CRITERION

Evandro Marconi Rocco
Marcelo Lopes de Oliveira e Souza
Antonio Fernando Bertachini de Almeida Prado
Instituto Nacional de Pesquisas Espaciais – INPE
C.P. 515 CEP 12201-970 – São José dos Campos, SP, Brasil
E-mail: evandro@dem.inpe.br; marcelo@dem.inpe.br; prado@dem.inpe.br

ABSTRACT

In this work the problem of orbital maintenance of symmetrical constellations of satellites, with minimum fuel consumption, is studied using impulsive maneuvers with time constraint. To perform the station keeping of a constellation of n satellites we have the problem of simultaneously optimizing the maneuvers for n satellites. When we consider all the satellites it is not simple to determine the optimal maneuver strategy that minimizes the fuel consumption with time constraint. Therefore, the goal of this work is to formulate and to study maneuver strategies that make possible to obtain solutions with small fuel consumption considering all the satellites in the constellation. The problem can be formulated as a multi-objective problem due to the nature of the station-keeping of a satellite constellation. Thus, the multi-objective problem applied to satellite constellations is defined and a new multi-objective optimization method was applied. This method can consider n conflicting objectives simultaneously without reducing the problem to an optimization of only one objective, as occur with most of the methods found in the literature. This method, called the Smallest Loss Criterion, was presented in Rocco et al. (2003). It was compared with other existent methods and it was verified that it is capable to supply better results to the problem of the station-keeping of satellite constellations. In this work, some applications of the Smallest Loss Criterion is presented, as a complementation of the previous work, presented in Rocco et al. (2003).

THE MULTI-OBJECTIVE PROBLEM

According to Cohon (1978), the static optimization of problems with one objective can be defined in the following way:

$$\begin{aligned} &\text{Maximize} \quad Z(\mathbf{x}) \quad \text{with relation of } \mathbf{x} \in \mathbf{R}^n \\ &\text{Subject to} \quad g_i(\mathbf{x}) \leq 0 \qquad i = 1, 2, \ldots, m \\ &\qquad\qquad\quad \mathbf{x} \geq 0 \end{aligned} \tag{1}$$

Given $Z(\cdot)$, $g_i(\cdot)$ or

$$\text{Maximize } Z(\mathbf{x}) \text{ with relation to } \mathbf{x} \in \mathbf{R}^n \quad (2)$$

Subject to $\mathbf{x} \in \mathbf{F}_d$

Given $Z(.)$, $\mathbf{F}_d$

where $\mathbf{F}_d$ is the feasible area of the decision space, defined by:

$$\mathbf{F}_d = \{\mathbf{x} \in \mathbf{R}^n \mid g_i(\mathbf{x}) \leq 0, i = 1, 2, \ldots, m; \mathbf{x} \geq 0\} \quad (3)$$

The multi-objective problem can be defined by:

$$\text{Maximize } \mathbf{Z}(\mathbf{x}) = [Z_1(\mathbf{x}), Z_2(\mathbf{x}), \ldots, Z_p(\mathbf{x})] \quad (4)$$

Subject to $\mathbf{x} \in \mathbf{F}_d$

Therefore, in this case, the objective function, is a vector with dimension p. Thus, the multi-objective optimization consists of the minimization (or maximization) of a vector of objectives $\mathbf{Z}(\mathbf{x})$ that may be subject of constraints or bounds. This problem type is very common in station keeping of satellite constellations because during the maneuvers many parameters must be optimized at the same time.

In problems of unidimensional optimization (when we have one objective), the possible solutions ($\mathbf{x} \in \mathbf{F}_d$) can be compared by means of the objective function, that is, given two solutions $\mathbf{x}^1$ and $\mathbf{x}^2$ we can compare $Z(\mathbf{x}^1)$ with $Z(\mathbf{x}^2)$ and determine the optimal solution so that $\mathbf{x} \in \mathbf{F}_d$ doesn't exist such that $Z(\mathbf{x}) > Z(\mathbf{x}^*)$. In problems of multi-dimensional optimization (multi-objective problem), in general, it is not possible to compare all the possible solutions because the comparison on the basis of one objective can be contradicted with the comparison based on another objective. Namely, supposing that:

$$\begin{aligned} \mathbf{Z}(\mathbf{x}^1) &= [Z_1(\mathbf{x}^1), Z_2(\mathbf{x}^1)] \\ \mathbf{Z}(\mathbf{x}^2) &= [Z_1(\mathbf{x}^2), Z_2(\mathbf{x}^2)] \end{aligned} \quad (5)$$

$\mathbf{x}^1$ is better than $\mathbf{x}^2$ if and only if:

$$Z_1(\mathbf{x}^1) > Z_1(\mathbf{x}^2) \text{ and } Z_2(\mathbf{x}^1) \geq Z_2(\mathbf{x}^2) \quad (6)$$

or

$$Z_1(\mathbf{x}^1) \geq Z_1(\mathbf{x}^2) \text{ and } Z_2(\mathbf{x}^1) > Z_2(\mathbf{x}^2) \quad (7)$$

If $Z_1(\mathbf{x}^1) > Z_1(\mathbf{x}^2)$ and $Z_2(\mathbf{x}^1) < Z_2(\mathbf{x}^2)$ we cannot conclude anything regarding $\mathbf{x}^1$ and $\mathbf{x}^2$, that is, $\mathbf{x}^1$ e $\mathbf{x}^2$ cannot be compared.

THE SMALLEST LOSS CRITERION

As previously shown in Rocco et al. (2003), there are several multi-objective optimization methods. The methodologies found in the literature generally began the problem with a multi-objective approach but ended reducing the problem to the unidimensional case, by means of simplifications or influence factors. Or, when the approach was really multi-objective, the found result was a group of solutions candidates to the optimal solution, and in this case, for the choice of the optimal solution we should use other approaches external to the problem. In practical applications, it would be convenient to apply a methodology capable to find the solution that assists all the objectives.

The Smallest Loss Criterion, presented in Rocco (2002) and Rocco et al. (2003), can be enunciated in the following way: In a problem with n conflicting objectives, where the intention is to optimize the n objectives simultaneously, privileging none of them, the solution should be the solution which result in the smallest loss for each one of the objectives. Because doesn't exist a solution that optimize simultaneously the n objectives individually.

An attempt to find this solution would be to find de barycenter of a normalized n-dimensional figure, which the vertexes are the optimal solution for each objective. Therefore, for problems with three objectives the solution would be in the center of a normalized triangle, for n objectives the solution would be in the center of a normalized n-dimensional figure. Figure 1 shown an example of this criterion applied to a problem with three conflicting objectives. In this example, $S1$, $S2$ and $S3$ are the optimal solutions for each one of the objectives, considered separately. B is the barycenter of the triangle formed by the segments $\overline{S1S2}$, $\overline{S2S3}$ and $\overline{S3S1}$. By the barycenter definition, the distance from B to the vertexes of the triangle represented by the solutions $S1$, $S2$ and $S3$ is the same. So, if the barycenter B is adopted as a solution for the multi-objective problem, the segment $\overline{S1B}$ represents the loss in relation to the objective 1, and in the same way, the segments $\overline{S2B}$ and $\overline{S3B}$ represent the loss in relation to the objectives 2 and 3 respectively. Thus, from the Figure 1 it can be conclude that if the three objectives are equally considered, the best solution is that one which coincides with the barycenter of the triangle.

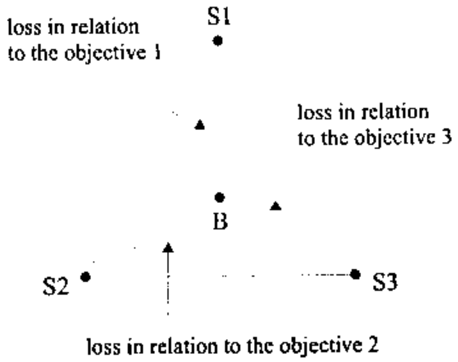

Fig. 1 – The Smallest Loss Criterion.

RESULTS OBTAINED WITH THE SMALLEST LOSS CRITERION

To test this methodology, it was adopted, just as an example, a constellation in low Earth orbit. In this way, the effect of the atmospheric drag was accentuated. Thus, the variation of the semi-major axis was accentuated and the software developed was submitted to a situation of worse case with relation to the orbital maintenance of the satellites. Three satellites with circular and equatorial nominal orbits compose the constellation studied. Therefore, the nominal elements of the satellites are given by (a and l in km and angles in radians):

e	= 0.00000000	a	= 6603.13900000
l	= 6603.13900000	i	= 0.00000000
ω	= 0.00000000	Ω	= 0.00000000

It may be assumed that in the initial instant the satellite 1 is entering in the visibility cone of the ground tracking station and the actual orbital elements of the satellite 1 are given by:

Satellite 1:

a_1 = 6601.93044509	Ω_1 = 0.00000000	u_1 = 0.00000000
e_1 = 0.00000000	ω_1 = 0.00000000	f_1 = 0.00000000
i_1 = 0.00000000	M_1 = 0.00000000	θ_1 = 0.00000000

where M is the mean anomaly, u is the eccentric anomaly, f it is the true anomaly and θ is the true longitude. The actual orbital elements of the others satellites are given by:

Satellite 2:

a_2 = 6601.93044509	Ω_2 = 0.00000000	u_2 = 2.09439522
e_2 = 0.00000000	ω_2 = 0.00000000	f_2 = 2.09439522
i_2 = 0.00000000	M_2 = 2.09439522	θ_2 = 2.09439522

Satellite 3:

a_3 = 6601.93044509	Ω_3 = 0.00000000	u_3 = 4.18879042
e_3 = 0.00000000	ω_3 = 0.00000000	f_3 = 4.18879042
i_3 = 0.00000000	M_3 = 4.18879042	θ_3 = 4.18879042

To assist the specifications of the mission, the satellites should be positioned in such a way that the difference between the true longitudes ($\Delta\theta_1$, $\Delta\theta_2$ and $\Delta\theta_3$) should be 2.09439435 rad (120 degrees). With the actual true longitudes θ_1, θ_2 and θ_3 we can calculate the position constraints $\delta\theta_1$, $\delta\theta_2$, $\delta\theta_3$ and $\delta\theta$, which represent the position error of the satellites, as shown in the Figure 2:

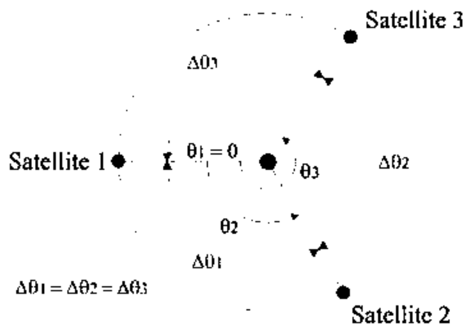

Fig. 2 – Nominal position of the satellites.

$$\theta_1 \le \theta_2: \quad \delta\theta_1 = (\theta_2 - \theta_1) - \frac{2\pi}{3} \tag{8}$$

$$\theta_2 < \theta_1: \quad \delta\theta_1 = (2\pi - \theta_1 + \theta_2) - \frac{2\pi}{3} \tag{9}$$

$$\theta_2 \le \theta_3: \quad \delta\theta_2 = (\theta_3 - \theta_2) - \frac{2\pi}{3} \tag{10}$$

$$\theta_3 < \theta_2: \quad \delta\theta_2 = (2\pi - \theta_2 + \theta_3) - \frac{2\pi}{3} \tag{11}$$

$$\theta_3 \le \theta_1: \quad \delta\theta_3 = (\theta_1 - \theta_3) - \frac{2\pi}{3} \tag{12}$$

$$\theta_1 < \theta_3: \quad \delta\theta_3 = (2\pi - \theta_3 + \theta_1) - \frac{2\pi}{3} \tag{13}$$

$$\delta\theta = \frac{|\delta\theta_1| + |\delta\theta_2| + |\delta\theta_3|}{3} \tag{14}$$

If the difference between the nominal and actual elements, and the position constraint $\delta\theta$ do not satisfy the tolerance previously specified, at least a correction maneuver becomes necessary. The actual position error for the satellites are: $\delta\theta_1 = \delta\theta_2 = \delta\theta_3 = \delta\theta = 0$, but the error in the semi-major axis is -1.20855491 km. Considering that it is necessary to execute the maneuver, several possible maneuvers are calculated, each one of them with different values of the semi-major axis of the final orbit and different values of the time spent for the maneuver. The semi-major axis and the time, vary from predefined values belonging to an operation range for the satellite. Thus we obtain the orbital elements of the transfer orbit, where Δv is the total velocity increment and t is the time spent in the maneuver.

Maneuver 1: $a_{final} = a_{nominal} = 6603.139$ km

a = 3813.50753965 e = 0.84297654 ω = 3.29684376 Δv = 10.43563617 t = 701.00770076

Maneuver 2: $a_{final} = 1.01\, a_{nominal} = 6603.364$ km

a = 3838.94341245 e = 0.82943908 ω = 3.30375052 Δv = 10.16880281 t = 702.00882124

Maneuver 3: $a_{final} = 1.02\, a_{nominal} = 6602.914$ km

a = 3832.39271231 e = 0.82972598 ω = 3.30147425 Δv = 10.15092314 t = 692.00916737

Maneuver 4: $a_{final} = 1.03\, a_{nominal} = 6603.589$ km

a = 3826.99885076 e = 0.83567624 ω = 3.30053543 Δv = 10.28923606 t = 701.10831091

Maneuver 5: $a_{final} = 1.04\, a_{nominal} = 6602.689$ km

a = 3832.00577843 e = 0.83006536 ω = 3.30139725 Δv = 10.15886021 t = 692.50914418

Maneuver 6: $a_{final} = 1.05\, a_{nominal} = 6603.814$ km

a = 3836.77670075 e = 0.83146479 ω = 3.30331350 Δv = 10.21398945 t = 704.45390971

Maneuver 7: $a_{final} = 1.06\, a_{nominal} = 6602.464$ km

a = 3836.67153417 e = 0.82958744 ω = 3.30024999 Δv = 10.16508475 t = 699.00894038

Maneuver 8: $a_{final} = 1.07\, a_{nominal} = 6604.039$ km

a = 3834.28236173 e = 0.83205939 ω = 3.30253352 Δv = 10.21971060 t = 702.00859016

Maneuver 9: $a_{final} = 0.996\, a_{nominal} = 6602.239$ km

a = 3834.83954066 e = 0.82894987 ω = 3.30220747 Δv = 10.14092168 t = 694.00916232

Applying the Smallest Loss Criterion to select the best maneuver, using normalized measures with $\delta\theta_{max} = 0.7\,\text{rad}$, $\Delta v_{max} = 11\,\text{km/s}$ and $T_{max} = 1000\,\text{s}$ to calculate the barycenter, it can be obtained the following table.

TABLE 1 – Choose of the Best Maneuver.

	$\delta\theta$		Δv		*Time*		*Classification*
1	0.34398092	9°	10.43563617	9°	701.00770076	5°	9°
2	0.33559710	5°	10.16880281	5°	702.00882124	8°	5°
3	0.33072576	1°	10.15092314	2°	692.00916737	1°	1°

TABLE 1 – Choose of the Best Maneuver (Cont.).

4	0.33920262	8°	10.28923606	8°	701.10831091	6°	8°
5	0.33118940	2°	10.15886021	3°	692.50914418	2°	2°
6	0.33815711	7°	10.21398945	6°	704.45390971	9°	7°
7	0.33415824	4°	10.16508475	4°	699.00894038	4°	4°
8	0.33730895	6°	10.21971060	7°	702.00859016	7°	6°
9	0.33122408	3°	10.14092168	1°	694.00916232	3°	3°
B	0.33122408		10.14092168		694.00916232		

● Vertexes ● Best Maneuver ● Barycenter

Therefore, the maneuver 3 is selected, because it is the closest maneuver of the barycenter. Now, it is considered that the satellite 3 is in visibility ($\theta_3 = 0$). Therefore, the satellites present the following orbital elements:

Satellite 1:

a_1 = 6602.68166455 Ω_1 = 0.00000000 u_1 = 1.59913823

e_1 = 0.00000000 ω_1 = 0.00000000 f_1 = 1.59913823

i_1 = 0.00000000 M_1 = 1.59913823 θ_1 = 1.59913823

Satellite 2:

a_2 = 6601.36490371 Ω_2 = 0.00000000 u_2 = 4.18996899

e_2 = 0.00000000 ω_2 = 0.00000000 f_2 = 4.18996899

i_2 = 0.00000000 M_2 = 4.18996899 θ_2 = 4.18996899

Satellite 3:

a_3 = 6601.36490371 Ω_3 = 0.00000000 u_3 = 0.00000000

e_3 = 0.00000000 ω_3 = 0.00000000 f_3 = 0.00000000

i_3 = 0.00000000 M_3 = 0.00000000 θ_3 = 0.00000000

Thus, the actual position error for the satellites are: $\delta\theta_1$ = -0.49643566; $\delta\theta_2$ = 0.00117879; $\delta\theta_3 = 0.49525687$; $\delta\theta = 0.33095711$. Calculating the maneuvers for the satellite 3, we obtain:

Maneuver 1: $a_{final} = a_{nominal} = 6603.139$ km

a = 3820.10647821 e = 0.83967098 ω = 3.29880702 Δv = 10.37217642 t = 702.20795943

Maneuver 2: $a_{final} = 1.01\, a_{nominal} = 6603.364$ km

a = 3838.33164219 e = 0.82962949 ω = 3.30368228 Δv = 10.17237866 t = 701.80880615

Maneuver 3: $a_{final} = 1.02\, a_{nominal} = 6602.914$ km

a = 3820.02868970 e = 0.83647339 ω = 3.29834695 Δv = 10.28484948 t = 692.32301320

Maneuver 4: $a_{final} = 1.03\, a_{nominal} = 6603.589$ km

a = 3831.09118064 e = 0.83285319 ω = 3.30165542v Δv = 10.22995622 t = 699.50860888

Maneuver 5: $a_{final} = 1.04\, a_{nominal} = 6602.689$ km

a = 3829.89510389 e = 0.83097526 ω = 3.30092382 Δv = 10.17578723 t = 692.00905341

Maneuver 6: $a_{final} = 1.05\, a_{nominal} = 6603.814$ km

a = 3801.74263575 e = 0.85048278 ω = 3.29372503 Δv = 10.59662285 t = 704.00701406

Maneuver 7: $a_{final} = 1.06\, a_{nominal} = 6602.464$ km

a = 3821.94279967 e = 0.83753216 ω = 3.29916105 Δv = 10.32212647 t = 699.00822263

Maneuver 8: $a_{final} = 1.07\, a_{nominal} = 6604.039$ km

a = 3827.14559359 e = 0.83613392 ω = 3.30076197 Δv = 10.30244922 t = 702.80820995

Maneuver 9: $a_{final} = 0.996\, a_{nominal} = 6602.239$ km

a = 3821.52887093 e = 0.83498276 ω = 3.29863995 Δv = 10.25135796 t = 690.50874504

Applying the Smallest Loss Criterion to select the best maneuver, using normalized measures with $\delta\theta_{max} = 0.7$ rad, $\Delta v_{max} = 11$ km/s and $T_{max} = 1000$ s to calculate the barycenter, it can be obtained the following table.

TABLE 2 – Choose of the Best Maneuver.

	$\delta\theta$		Δv		*Time*		*Classification*
1	0.34323784	8°	10.37217642	8°	702.20795943	7°	8°
2	0.33646255	4°	10.17237866	1°	701.80880615	6°	4°
3	0.33606968	3°	10.28484948	5°	692.32301320	3°	5°
4	0.33739004	5°	10.22995622	3°	699.50860888	5°	3°
5	0.33236134	1°	10.17578723	2°	692.00905341	2°	1°
6	0.35149893	9°	10.59662285	9°	704.00701406	9°	9°
7	0.34016808	6°	10.32212647	7°	699.00822263	4°	6°
8	0.34122538	7°	10.30244922	6°	702.80820995	8°	7°
9	0.33416546	2°	10.25135796	4°	690.50874504	1°	2°
B	0.33432978		10.19984128		694.77553487		

● Vertexes ● Best Maneuver ● Barycenter

Therefore, the maneuver 5 is selected. Now, it is considered that the satellite 2 is in visibility ($\theta_2 = 0$). Therefore, the satellites present the following orbital elements:

Satellite 1:

a_1 = 6602.25313837 Ω_1 = 0.00000000 u_1 = 3.69287951

e_1 = 0.00000000 ω_1 = 0.00000000 f_1 = 3.69287951

i_1 = 0.00000000 M_1 = 3.69287951 θ_1 = 3.69287951

Satellite 2:

a_2 = 6600.67050929 Ω_2 = 0.00000000 u_2 = 0.00000000

e_2 = 0.00000000 ω_2 = 0.00000000 f_2 = 0.00000000

i_2 = 0.00000000 M_2 = 0.00000000 θ_2 = 0.00000000

Satellite 3:

a_3 = 6602.43722227 Ω_3 = 0.00000000 u_3 = 1.59657315

e_3 = 0.00000000 ω_3 = 0.00000000 f_3 = 1.59657315v

i_3 = 0.00000000 M_3 = 1.59657315 θ_3 = 1.59657315

Thus, the actual position error for the satellites are: $\delta\theta_1$ = 0.32893532; $\delta\theta_2$ = 0.32240998; $\delta\theta_3 = 0.00652534$; $\delta\theta = 0.21929022$. Calculating the maneuvers for the satellite 2, we obtain:

Maneuver 1: $a_{final} = a_{nominal} = 6603.139$ km

a = 3772.37771504 e = 0.86553757 ω = 3.28511293 Δv = 10.90875480 t = 700.50599655

Maneuver 2: $a_{final} = 1.01\, a_{nominal} = 6603.364$ km

a = 3823.66473495 e = 0.83557977 ω = 3.29961906 Δv = 10.27556023 t = 695.80850117

Maneuver 3: $a_{final} = 1.02\, a_{nominal} = 6602.914$ km

a = 3798.81533139 e = 0.84792405 ω = 3.29267829 Δv = 10.51590970 t = 692.00758398

Maneuver 4: $a_{final} = 1.03\, a_{nominal} = 6603.589$ km

a = 3813.45483938 e = 0.84224231 ω = 3.29699006 Δv = 10.41646132 t = 699.00782878

Maneuver 5: $a_{final} = 1.04\, a_{nominal} = 6602.689$ km

a = 3815.40470459 e = 0.84007344 ω = 3.29739101 Δv = 10.36610269 t = 696.00810762

Maneuver 6: $a_{final} = 1.05\, a_{nominal} = 6603.814$ km

a = 3822.87235742 e = 0.83877470 ω = 3.29977568 Δv = 10.35876729 t = 704.10794359

Maneuver 7: $a_{final} = 1.06\, a_{nominal} = 6602.464$ km

a = 3834.51473078 e = 0.82946812 ω = 3.30244565 Δv = 10.15517129 t = 695.50905470

Maneuver 8: $a_{final} = 1.07\, a_{nominal} = 6604.039$ km

a = 3832.27494773 e = 0.83282349 ω = 3.30220482 Δv = 10.23427641 t = 701.50853489

Maneuver 9: $a_{final} = 0.996\, a_{nominal} = 6602.239$ km

a = 3831.52178764 e = 0.83010164 ω = 3.30149199 Δv = 10.16054631 t = 692.50911115

Applying the Smallest Loss Criterion to select the best maneuver, using normalized measures with $\delta\theta_{\max} = 0.7$ rad, $\Delta v_{\max} = 11$ km/s and $T_{\max} = 1000$ s to calculate the barycenter, it can be obtained the following table.

TABLE 3 – Choose of the Best Maneuver.

	$\delta\theta$		Δv		*Time*		*Classification*
1	0.02871318	9º	10.90875480	9º	700.50599655	7º	9º
2	0.00575679	3º	10.27556023	4º	695.80850117	4º	1º
3	0.01195674	7º	10.51590970	8º	692.00758398	1º	8º
4	0.01179272	6º	10.41646132	7º	699.00782878	6º	6º
5	0.00879068	5º	10.36610269	6º	696.00810762	5º	3º
6	0.01514568	8º	10.35876729	5º	704.10794359	9º	7º
7	0.00164066	2º	10.15517129	1º	695.50905470	3º	5º
8	0.00687667	4º	10.23427641	3º	701.50853489	8º	2º
9	0.0012963	1º	10.16054631	2º	692.50911115	2º	4º
B	0.00496478		10.27720910		693.34191661		

● Vertexes ● Best Maneuver ● Barycenter

Therefore, the maneuver 2 is selected.

The comparison among the results obtained with the application of the multi-objective methodology developed in this work (Criterion of the Smallest Loss) and the results obtained with the application of nominal maneuvers (each satellite is maneuvered to reach the nominal orbit), can be seen in the Figures 3 to 5. Figure 3 compares the position error, after maneuvering the three satellites of the constellation. Figure 4 compares the sum of the velocity increment necessary to maneuver the satellites of the constellation. Figure 5 compares the sum of the time spend to maneuver all the satellites.

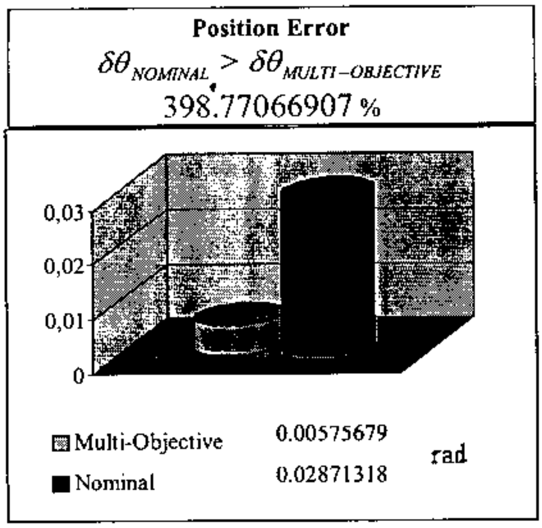

Fig. 3 – $\delta\theta_{MULTI-OBJECTIVE}$ and $\delta\theta_{NOMINAL}$.

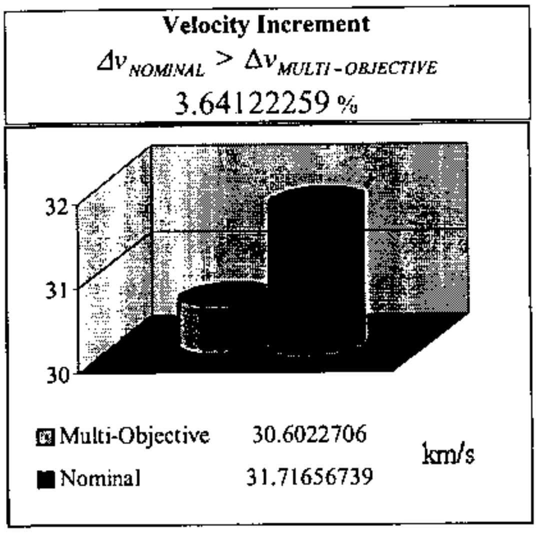

Fig. 4 – $\Delta v_{MULTI-OBJECTIVE}$ and $\Delta v_{NOMINAL}$.

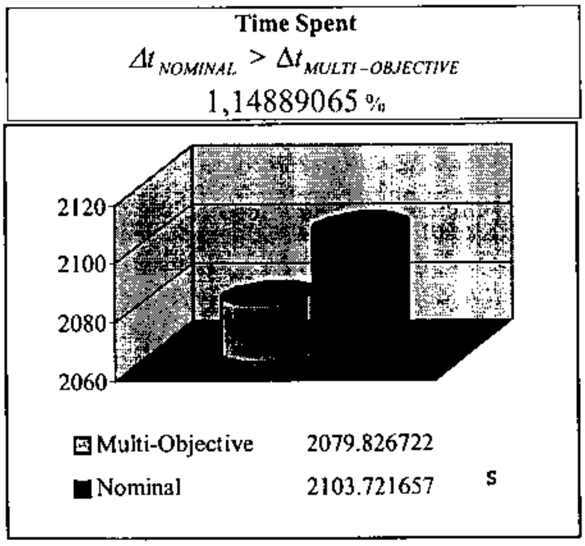

Fig. 5 – $\Delta t_{MULTI-OBJECTIVE}$ e $\Delta t_{NOMINAL}$.

Figures 3 to 5 shown that the multi-objective methodology presented better results. The position error obtained by the multi-objective methodology was considerably smaller than the error obtained by the nominal methodology. The necessary velocity increment and the time spend to maneuver the satellites were also smaller when the multi-objective methodology was applied.

CONCLUSION

In this article, which is part of the work developed by Rocco (2002), the problem of orbital station keeping of satellite constellations was studied as a problem of multi-objective optimization. The multi-objectives methodologies found in the literature, generally began the problem with an multi-objective approach but ended reducing the problem to the mono-objective case, by means of simplifications or influence factors. Or, when the approach was really multi-objective, the found result was a group of solutions candidates to the optimal solution, and in this case, for the choice of the optimal solution we should use other approaches external to the problem. The best methodology found in the literature, that bases on Pareto (1909), present this deficiency, as shown in Rocco et al. (2003). Therefore, it seems that doesn't exist in the revised literature any method really capable to accomplish the multi-objective optimization, considering all the objectives of the problem equally. Thus, it was used a methodology that at least to consider equally all the objectives. This methodology is based on what it was called Smallest Loss Criterion. It was considered in the example presented in this work three objectives, but the Smallest Loss Criterion allow to consider so many objectives as necessary. The constellation considered here was composed by $n = 3$ satellites. However, the multi-objective optimization method and the developed software for the control of the constellation allow easily to consider more than 3 satellites. But the concept for the problem for $n = 3$ is identical to the problem for $n > 3$, but in this case, we would have a larger computer effort. So, it was opted to consider $n = 3$. The developed software was tested and it was verified that it is capable to generate reliable solutions. The software was built in modules, so it can be enlarged considering a larger number of satellites and/or other geometric configurations of the constellation, according to the necessity. The application of the multi-objective methodology generated results clearly superiors, as shown in the Figures 3 to 5. With the application of this methodology it was possible to obtain better results with relationship to the three objectives simultaneously. In a real application, the economy obtained can be extremely important.

REFERENCES

Cohon, L. J. *Multiobjective programming and planning.* New York: Academic Press, 1978. 333 p.

Pareto, V. *Manuale di economia politica com una introduzione alla scienza sociele.* Milão: Società Editrice Libraria, 1909.

Rocco, E. M. *Manutenção orbital de constelações simétricas de satélites utilizando manobras impulsivas ótimas com vínculo de tempo.* São José dos Campos. 178 p. Tese (Doutorado em Engenharia e Tecnologia Espaciais / Mecânica Espacial e Controle) - Instituto Nacional de Pesquisas Espaciais, 2002.

Rocco, E.M.; Souza, M.L.O.; Prado, A.F.B.A. *Multi-Objective Optimization Applied to Satellite Constellations I: Formulation of the Smallest Loss Criterion.* IAC 2003 54st International Astronautical Congress. Bremen, Germany, September 29 - October 3, 2003.