ESTIMATION OF TORQUE IN A REACTION WHEEL USING A BRISTLE MODEL FOR FRICTION

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Abstract. *The objective of this work is to estimate the friction torque in a reaction wheel using a bristle model for nonlinear friction effects. The estimated friction torque is used to improve the response quality of the reaction wheel speed control. For this estimation, measurements from the reaction wheel are simulated through motor's friction dynamic equation integration. The measurements are corrupted by a random noise in order to simulate the uncertainties present in a real equipment. Following, the Kalman filtering theory is applied to obtain the friction torque time behavior. The results are compared with those obtained by a speed controller where the friction torque is neglected.*

Keywords: *estimation, friction, bristle model, control, torque*

1. Introduction

Instituto Nacional de Pesquisas Espaciais (INPE) has, for long time, developed satellite orbit and attitude control activities. Nowadays, there is a struggle to consolidate knowledge in three axis stabilized satellites. New generations of satellites developed at INPE will present chalenges as the complete knowledge of friction force in reaction wheels. Better levels of precision in attitude control could be reached by considering more accurate models for friction force.

The satellite named Multi-Mission Platform (PMM) is being developed at INPE and uses reaction wheels as part of actuators for its attitude control. This work has as goal to contribute for better understanding the friction present in reaction wheels for the PMM project and for the following projects in the future.

Real knowledge of all forces acting on a system is fundamental for precise modeling and, consequently, for a better performance in system control.

In recent work (Colhour and Nair, 1994), it is stated that the effect of friction force in precise control mechanism in low speed can be dominant and hard to model. This situation particulary occur in reaction wheels used for artificial satellite attitude control. The focus of this work is to know all forces acting in such a system and in modeling the friction force.

Many control systems use friction cancellation for better precision. Rigid body dynamics and movement measures are used to estimate unknown friction force in a system, as Ramasubramanian and Ray (2001).

To obtain friction force in a reaction wheel motor axis it is used a bristle model for surface friction and also noisy measurements of motor's angular momentum, which is related to friction force. Friction force is then estimated through Kalman filter for reaching more precise results than the ones obtained in dynamic model integration.

Ray and Remine (1998) says it is improbable that a single model holds for all kind of loads, lubrificant conditions and diferent lubrification regimes experienced by a machine. Real time methods for modeling friction or to determine if a model holds for a particular operation condition are desired for repetitive compensation and friction diagnoses.

Other techniques to obtain friction force are described in Colhour and Nair (1994), Ramasubramanian and Ray (2000), Ramasubramanian and Ray (2001) and Ray and Remine (1998), such as, neural networks and Kalman-Bucy filter.

The study presented in this article verifies that the use of a control law considering real friction force present in a reaction wheel leads to a better system performance.

Although there are reaction wheels that can be commanded in speed, which would avoid the kind of problem treated in this article, it is not a general rule. There are reaction wheels that can only be commanded in torque, which applies to the study herein.

This article is divided in two parts: the first is the simulation phase and the second is estimation. The simulation phase was necessary to generate a set of measurements to be utilized in the Kalman filter. This phase could be neglected if it was available, for example, measurements from a real reaction wheel. The estimation phase is the real objective of this article and Kalman filter is used for that. Control with and without estimation are compared as an ilustrative example of improvement in response using Kalman filter.

2. Equations

Canudas et al. (1995) model interface between two surfaces as sets of bristle contacts. It is shown in Eq. (1):

$$\dot{Z} = v - \frac{|v|}{g(v)}Z \tag{1}$$

Z is mean bristle deflection, v is relative speed between the surfaces and $g(v)$ models changes from static to Coulomb friction states. This change in friction is also known as Stribeck effect.

The $g(v)$ function that describes Stribeck effect is modeled by Eq. (2):

$$g(v) = \frac{1}{\sigma_0}\left(F_c + (F_s - F_c)e^{(v/v_s)^2}\right) \tag{2}$$

σ_0 is bristle stiffness coeficient, F_c is Coulomb force, F_s is Stribeck force and v_s is Stribeck effect velocity, i. e., beyong this velocity the system deperts from rest and starts to move under the effect of a new friction coefficient (dynamic friction coefficient).

According to Hirschorn and Miller (1999), the Stribeck velocity depends on material properties and lubrification. It is normaly found empirically. Its value varies from 0.00001 to $0.1\ m/s$.

For the case where the motor is already in movement ($v \gg v_s$) the velocity will be in only one direction ($|v| = v$), therefore friction force will not present static friction coefficient. In this case $g(v)$ function would be described as $g(v) = F_c/\sigma_0$.

The contact surface used to obtain the results shown in this article is the motor axis in contact with reaction wheel internal circular surface. In this surface, velocity v is described by Eq. (3)

$$v = \frac{r}{J}h \tag{3}$$

r is motor axis radius, J is reaction wheel inertia moment and h is reaction wheel angular momentum.

Relationship between friction torque (N) and bristle deflection (Z) is given by Eq. (4)

$$N = (\sigma_0 Z + \sigma_1 \dot{Z} + \sigma_2 v)r \tag{4}$$

σ_0 is bristle stiffness coefficient, as said before, σ_1 is dumping coefficient during friction transients and σ_2 is viscous friction coefficient.

For simulation phase, where measurements will be generated, Eq. (1), (2), (3) and (4) became Eq. (5)

$$\begin{aligned}
\dot{Z} &= \frac{r}{J}h_{sim} - \frac{\sigma_0}{F_c + (F_s - F_c)e^{(v/v_s)^2}}\left|\frac{r}{J}h_{sim}\right| Z \\
N_{sim} &= (\sigma_0 Z + \sigma_1 \dot{Z} + \sigma_2 \frac{r}{J}h_{sim})r \\
\dot{h}_{sim} &= u - N_{sim}
\end{aligned} \tag{5}$$

Simulated angular momentum and friction torque are denoted by h_{sim} and N_{sim}.

To design Kalman filter it was used a linear system for reaction wheel dynamics described by Eq. (6). Doing that nonlinear equations, as Eq. (5) are avoided in Kalman filtering dynamic model.

$$\dot{X} = AX + Bu + Gn \qquad X = \begin{bmatrix} h \\ N \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad G = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{6}$$

h is reaction wheel angular momentum, already described, N is motor axis torque,u is control function and n is dynamic model noise, with zero mean and covariance Q. Matrix A, B and G are denoted feedback, control and disturbance matrix.

Equation (6) corresponds to:

$$\left\{ \begin{aligned} \dot{h} &= u - N \\ \dot{N} &= n \end{aligned} \right. \tag{7}$$

The desired steady state behavior is the one where the reaction wheel is rotating with angular momentum $h = h_{ref}$. So, necessary commanded torque (u) for cancelling friction torque must be as described in Eq. (8):

$$u = \hat{N} - \gamma(\hat{h} - h_{ref}) \tag{8}$$

γ is the reaction wheel controller gain and notation $\hat{X} = [\hat{h}\ \hat{N}]^T$ refers to estimated state, with dynamical equations denoted by:

$$\begin{cases} \dot{\hat{h}} &= -\gamma(\hat{h} - h_{ref}) \\ \dot{\hat{N}} &= 0 \end{cases} \tag{9}$$

State estimation must be done based on reaction wheel measurements, which is based on wheel angular momentum, as described in Eq. (10):

$$Y = h_{sim} + V \tag{10}$$

V is random noise with mean zero and covariance R.

3. Kalman Filter

Kalman filter can embed dynamic noise in state model and is a estimator with real time characteristics, i. e., it gives estimates for the instant when measurements are processed.

Recursive procedure for state estimation is given by time update and measurement update phases. Time update phase determines state and covariance evolution from time instant t_{k+1} to t_k, it follows the dynamic model described by Eq. (11).

$$\begin{aligned} \bar{h}_k &= [\hat{h}_{k-1} - h_{ref}]e^{-\gamma(t_k - t_{k-1})} + h_{ref} \\ \bar{N}_k &= \hat{N}_{k-1} \\ \bar{P}_k &= \Phi_{k,k-1}\hat{P}_{k-1}\Phi^T_{k,k-1} + \Gamma_k Q_k \Gamma^T_k \end{aligned} \tag{11}$$

Notation $\bar{X} = [\bar{h}_k\ \bar{N}_k]^T$ and $\bar{P}_k$ refers to state and covariance updated to instant k. $\hat{X} = [\hat{h}_k\ \hat{N}_k]^T$ and $\hat{P}_k$ refers to state and covariance updated to instant k, $\Phi_{k,k-1}$ is the covariance transition matrix from instant $k-1$ to k, Γ_k is noise addition matrix and Q_k is dynamics noise covariance matrix.

Covariance transition matrix, responsible for covariance time update is given by Eq. (12).

$$\Phi(t_{k+1} - t_k) = \begin{bmatrix} 1 & -(t_{k+1} - t_k) \\ 0 & 1 \end{bmatrix} \tag{12}$$

Dynamic noise covariance matrix is equal to $Q = \sigma_1^2$ and dynamic noise addition matrix (Γ) presents the relationship described by Eq. (13).

$$\Gamma_k Q \Gamma^T_k = Q \begin{bmatrix} \frac{(t_{k+1}-t_k)^3}{3} & -\frac{(t_{k+1}-t_k)^2}{2} \\ -\frac{(t_{k+1}-t_k)^2}{2} & (t_{k+1} - t_k) \end{bmatrix} \tag{13}$$

Measurement update phase corrects state and covariance at instant t_k considering measurements Y_k. In this phase the equations are described by:

$$\begin{aligned} K_k &= \bar{P}_k H^T_k (H_k \bar{P}_k H^T_k + R)^-1 \\ \hat{P}_k &= (1 - K_k H_k)\bar{P} \\ \hat{X}_k &= \bar{X}_k + K_k(Y_k - H_k X_k) \end{aligned} \tag{14}$$

K_k is Kalman gain and R is observation error covariance matrix.

4. Simulation Algorithm

For friction estimation it is used the following procedure to generate simulated measurements:

1. Previous data

   Radius (r), inertia moment (J) and reference angular momentum (h_{ref}) must be chosen using commercial on the shelf reaction wheels. The radius and inertia moment are specified by the chosen wheel, but the reference angular momentum must be within the acting limits of this wheel. These limits are defined by a maximum and minimum angular velocity, i. e., a limited angular momentum based on Eq. (3).

   Uncertanties in measurements (ρ), initial state (ρ_0) and in the model (ρ_s) must also be provided.

   The following parameters belong to bristle model: F_c, F_s, v_s, σ_0, σ_1 and σ_2.

2. Initialization

$$\begin{aligned} Z(t_0) &= Z_0 \\ \hat{h}(t_0) &= h_0 \end{aligned}$$

With this initialization, it is imposed that bristle deflection is Z_0 and the reaction wheel presents angular momentum h_0, i. e., if both are null the wheel is stopped and without bristle deflection.

3. Simulation equation propagation

There were obtained analytical solutions from Eq. (7) and this solutions were inserted in the propagation equation set to generate measuremets. Eq. (7) presents analytical solutions given by:

$$\begin{aligned} \bar{h}(t) &= (\hat{h}_k - h_{ref})e^{-\gamma(t-t_k)} + h_{ref} \\ \bar{N}(t) &= \hat{N}_k \\ t &\in [t_k, t_{k+1}] \end{aligned} \tag{15}$$

k means that the parameter belongs to time instant k because it has been propagated from time t_k to t_{k+1}.

The solution described by Eq. (15) was substituted in Eq. (8) resulting in:

$$\begin{aligned} \bar{u}(t) &= \bar{N}_k - \gamma(\bar{h}_k - h_{ref})e^{-\gamma(t-t_k)} \\ t &\in [t_k, t_{k+1}] \end{aligned} \tag{16}$$

Equations (5), (15) and (16) describes state behavior of a bristle model for friction between reaction wheel motor axis and its inertial mass.

Equations (5) were integrated from time t_k to t_{k+1} through Runge-Kutta method by using ode45 MATLAB function.

4. Measurements simulation

To obtain a set of measurements for further filtering, they were generated folowing Eq. (10), where the, uncertainties were simulated like a real reaction wheel data acquisition.

5. Estimation Algorithm

In this paper, Kalman filter is used for angular momentum and friction torque estimation in a system with simulated measurements.

1. Initialization

The following matrices were used in the Kalman filter:

$$\hat{X}_0 = [0 \quad 0]^T, \quad \hat{P}_0 = \rho_0^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad R = \rho^2, \quad Q = \rho_1^2 \tag{17}$$

2. Time update phase

In this phase Eq. (11), (12) and (13) are used.

3. Measurement update phase

In this phase Eq. (14) is used.

4. Return to step 2.

The k number, which represents the number of time interactions, is incremented and time update phase is realized again.

6. Simulations

The parameters used in this simulation came from Hirschorn and Miller (1999), they are: $\sigma_0 = 100000\ N/m$, $\sigma_1 = 495\ Ns/m$, $\sigma_2 = 4.6\ Ns/m$, $F_c = 2\ N$, $F_s = 2.5\ N$ and $v_s = 0.01\ m/s$. Specific parameters of this article are: number of interactions = 300, $h_{ref} = 1\ Nms$, $\gamma = 0.02$, $\rho = 0.01$, $\rho_0 = 0.1$, $\rho_1 = 0.1$, $r = 0.006\ m$ and $J = 0.0191\ kgm^2$.

Reference angular momentum (h_{ref}) was choosen to be equal to 1 Nms, because it represents, in a reaction wheel with this characteristics of radius and inertia moment, a rotation of $500\ rpm$. The reaction wheel parameters were chosen from a commercial equipment and presents maximum rotation of $3000\ rpm$.

Figure 1 shows angular momentum behavior which starts at inicial condition $h_0 = 0$ going to the reference value $h_{ref} = 1\ Nms$. The estimated angular momentum is initially 1% close to dynamic equation simulated value, this is because initial covariance has this initial state precision, as can be seen in Fig. 1.

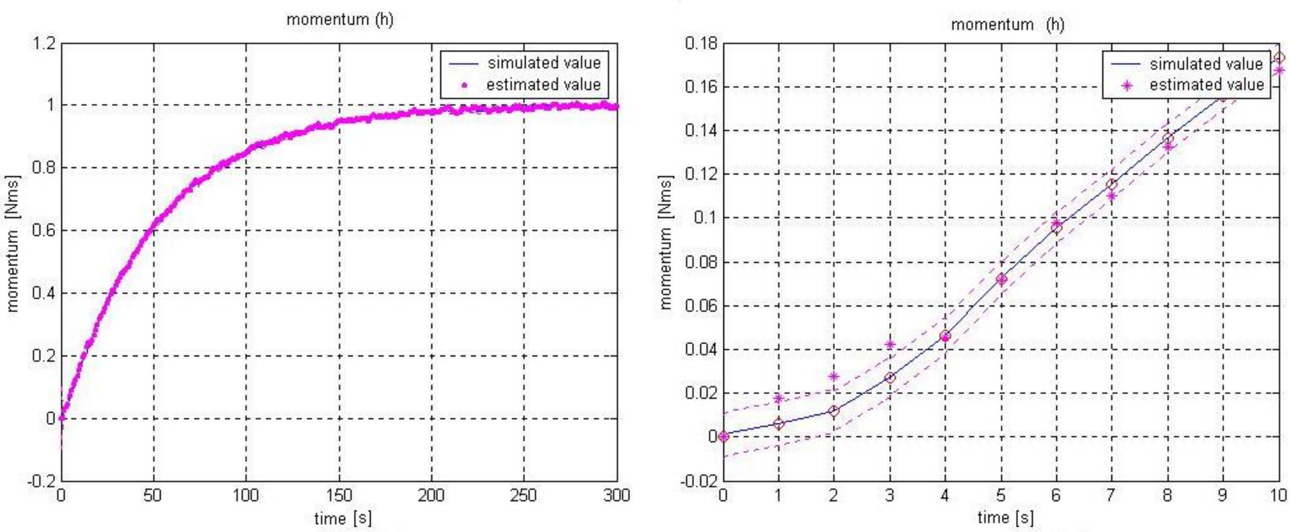

Figure 1. Reaction wheel simulated angular momentum

In Fig. 1, the continuous line represents simulated angular momentum, which is the propagation using step 3 from the simulation algorithm. These equations are (5), (15) and (16).

Measurements instants and consequently the same instants that kalman filter updates the states and covariances are given by integer time instants. The resulting angular momentum from filter update is shown in Fig. 1, as "+" symbol and standard deviation as dashed lines.

Simulation equations evolution occurs during all time interval. The value of angular momentum from simulation equations is shown in Fig. 1 with "o" symbol.

Friction torque, resulted from dynamics integration is shown in Fig. 2. It goes to a constant value equal to $0.02\ Nm$. At time instant of 300 seconds, with a constant angular momentum a constant friction is obtained.

In Fig. 2, the continuous line represents the torque value obtained through equations (5), (15) and (16) simulation, which composes simulation algorithm step 3. Symbol "o" denote the simulation value for measurements generation time and symbol "+" represents estimated torque values from Kalman filter. Initially, the estimated values are separated from the simulated values by 1%, because it wasn't processed a set of sufficient data. With more data being processed, the estimated values approach the simulated ones, representing the convergence of Kalman filter.

Figure 3 describes the control law time behavior. The continuous line is the commanded control, this control represents controller output. The dots are the effective control in the reaction wheel, i. e., the torque in the reaction wheel that is greater than the friction torque. The effective torque is equal to commander torque minus friction torque.

In an amplified view, Fig. 3 shows in more details the behavior of the control. The commanded torque appears in a continuous line and the effective torque in a doted one. The symbol "o" represents the commanded torque in a discrete time, equals to the measurement time. The symbol "+" represents the effective torque value in a discrete time also.

Figure 4 shows that the mean bristle deflection (Z) converges to $2 \times 10^{-5}\ m$, this value was expected because from Eq. (1), when time derivative of Z is null, then $Z = g(v)$. Function $g(v)$ is described by Eq. (2), when $v \gg v_s$ this resuls in $g(v) = F_c/\sigma_0$ and substituing the values used in this simulation results in $2 \times 10^{-5}\ m$.

Residue is the difference between state observation (measurement) and the state itself related by matrix H. Residue is denoted by $Y - HX$, and the closer to zero is the residue, better are measurements precision. For this simulation Fig. 4 has a maximum module of 1.7×10^{-2}.

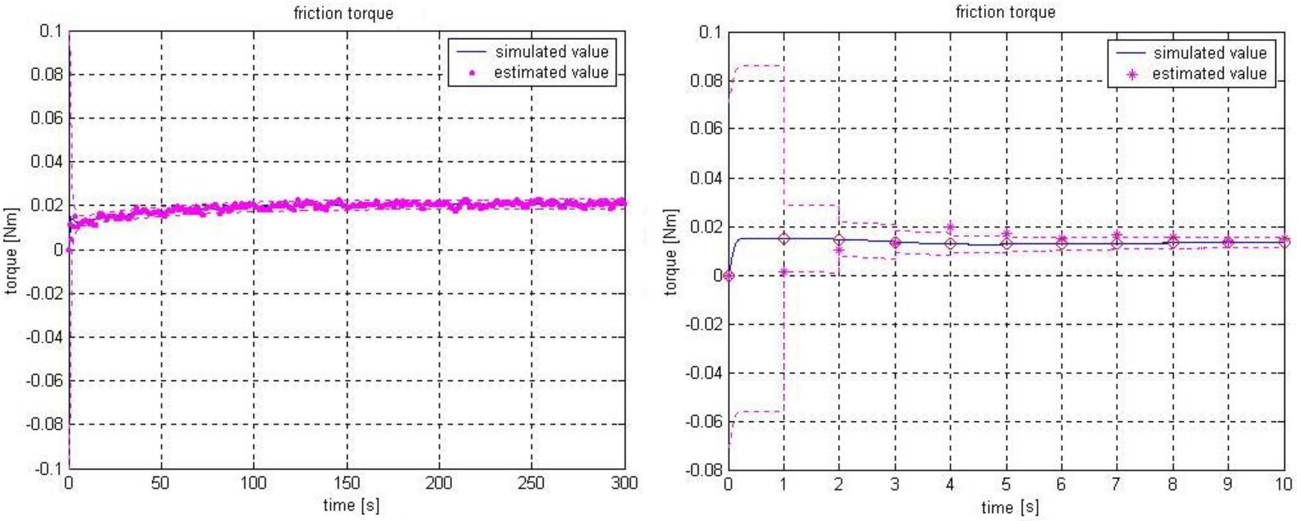

Figure 2. Reaction wheel simulated torque

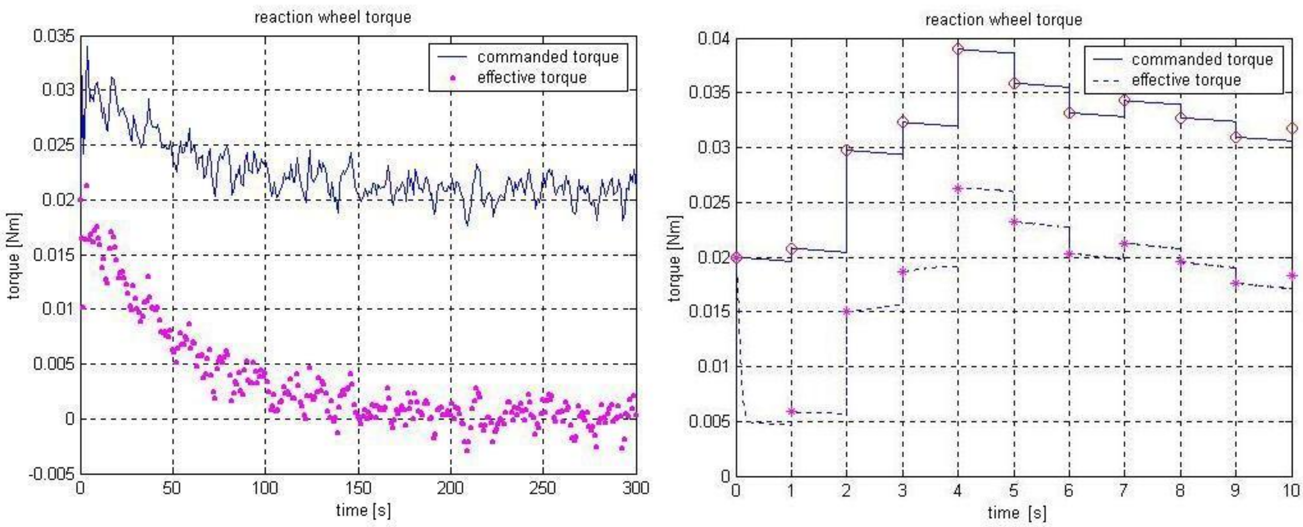

Figure 3. Reaction wheel control torque

The following results correspond to angular momentum and friction torque simulation without state estimation, and consequently without using the estimated state as a feedback for reaction wheel control law.

Not considering the estimation procedure, a new simulation procedure is realized, in it Eq. (5) is integrated again, but the control law is now described by Eq. (18).

$$u = \gamma(Y - h_{ref}) \tag{18}$$

In Fig. 5 the angular momentum, for the case without estimation, goes to value 1 Nms. The steady value is the same for the case with the estimator (dots) and without estimator (continuous line). The control does not stop until the reference value is reached. The difference is how these two cases reach the reference value. With the estimator the evolution is assyntotic and without the estimator it oscillates, overshooting the reference value and descending to almost 0.9 Nms at 280 seconds.

The friction torque, in Fig. 5, compares the estimator case (dots), also described in Fig. 2, with the case without estimator (continuos line). Both go to value 0.02 Nms. To the same angular velocity, the behavior of friction is the same, but friction from estimation is more spread than without estimation case.

Because control law is based in friction torque less the difference between actual angular momentum and reference angular momentum, it's expected that the control reaches the same friction torque value when the angular momentum reaches the reference angular momentum.

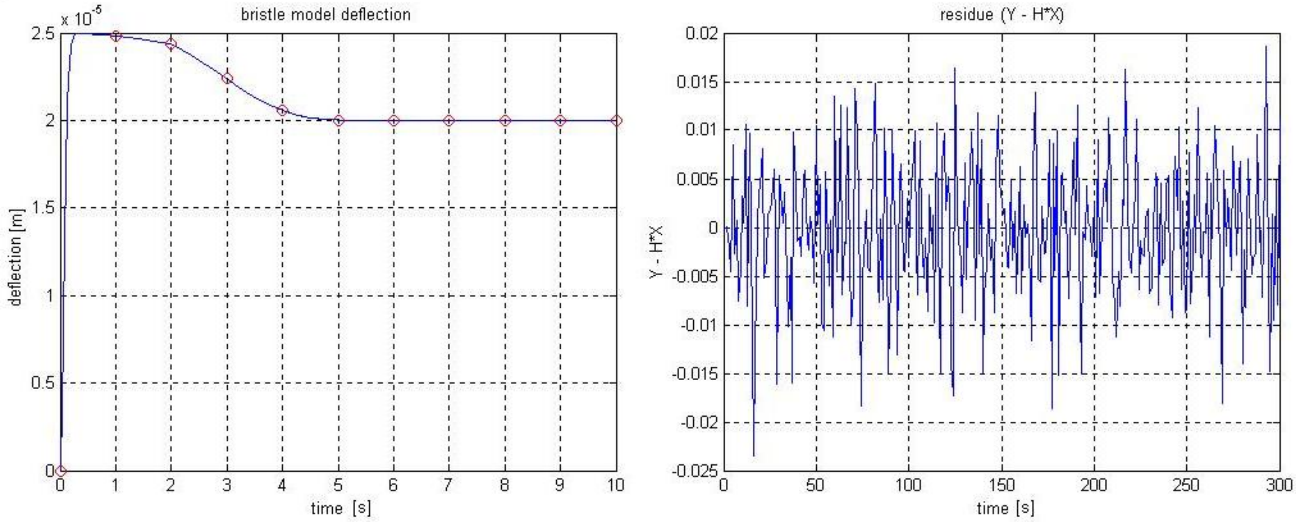

Figure 4. Britle deflection in friction model and reaction wheel state residue

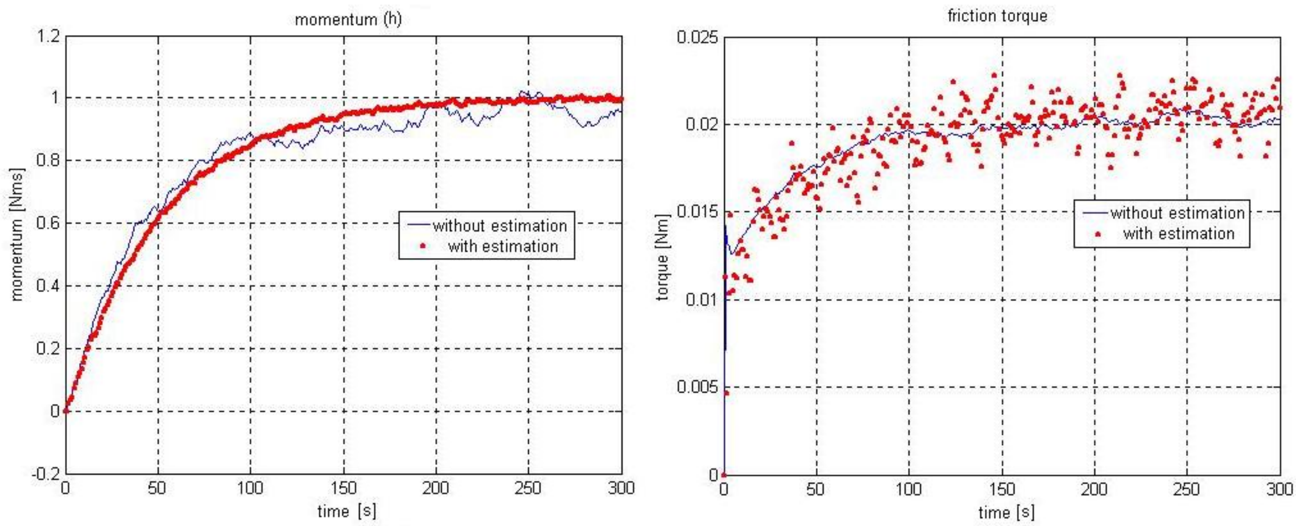

Figure 5. Comparation of angular momentum and friction torque with and without estimation

It can be noted, comparing the effective torque in Fig. 6 with Fig 3, that the bigger effective torque occurs in the simulation considering estimation within the control loop.

The mean bristle deflection has its steady value of $2 \times 10^{-5}\ m$, like Fig. 6 and in the simulation with estimation shown in Fig. 4.

7. Conclusions

The estimator presence in reaction wheel speed control algorithm, for the considerations assumed in this paper, represented better efficiency in the control because the error between the effective state value and its reference was small.

The control laws used in this paper had the objective of friction compensation and no optimization study was realized for their use.

The reaction wheel angular momentum presented asymptotic response in reaching the reference value (h_{ref}), so it does not overpass the desired value. In doing that satellite's energy is not wasted.

Friction torque estimation and its consideration in control applied to a reaction wheel had a better performance in making the state reaching its desired reference value.

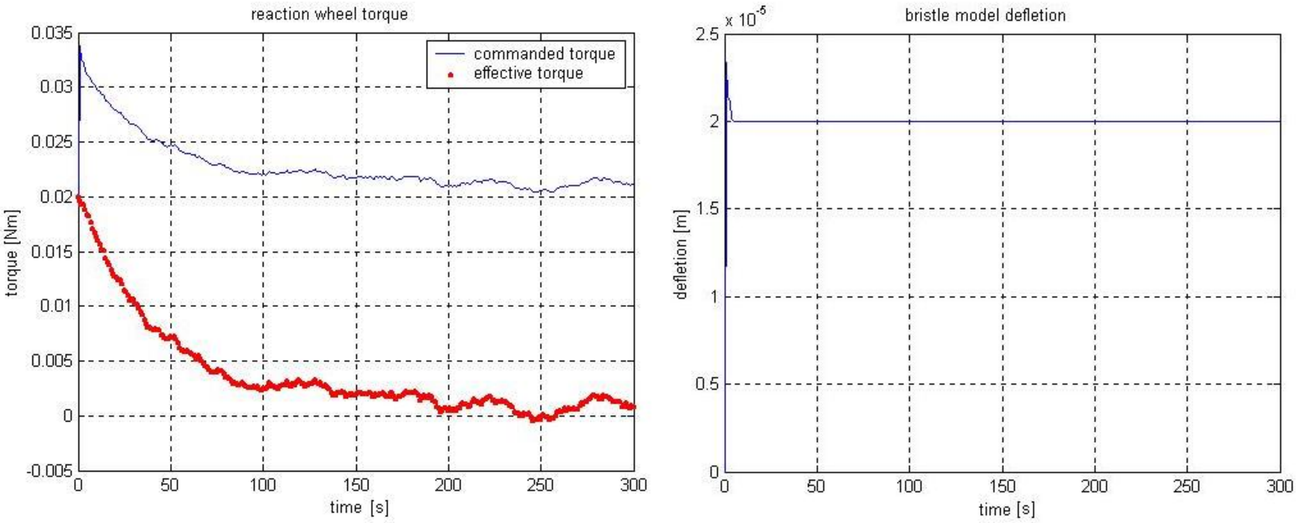

Figure 6. Reaction wheel control torque and deflection without estimation

8. Acknowledgements

Thanks to CAPES, for supporting this study. Thanks also to Adenilson da Silva for suggesting the execution of this work.

9. References

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10. Responsibility notice