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ABSTRACT

This investigation addresses the problem of the deployment of a solar array on a satellite. For this purpose, the solar panel is considered as a rigid multibody system. The mathematical model is developed through the Lagrangian formalism and the resulting governing equations of motion are integrated through a fourth order Runge-Kutta. All nonlinear terms are kept in the analysis and the numerical simulations show the effects of these nonlinearities on the evolution of the system dynamics before, during and after the deployment of the solar array.

INTRODUCTION

The knowledge of the attitude angles is very important to determine the behavior of the satellite (Roberson, 1979) and for the control issues. If the launch vehicle is previously stabilized, as the Long March 4B, used to launch the China Brazil Earth Resources Satellite CBERS-1, the attitude is easily determined but after the deployment phase it is disturbed, consequently that knowledge of the attitude becomes important (Thomson and Reiter, 1960). The determination of these variables through mathematical modeling and computer simulations (Wie, 1986; Porro, 2002), allows the verification of the impact of some critical action, as the draft of solar panels to the sun or as in this work of solar array deployment, under satellite attitude (Meirovitch and Calico, 1972; Wie, 1998).

In this work the orbit described by the satellite is considered circular and every part of the satellite is considered as a rigid body. Three different phases were considered: the instant in which the appendages are closed, the instant in which the appendages are opening, and the instant in which the appendages are completely opened. It was used a prescribed torque profile whose value was adjusted to make the system as stable as possible, to control the opening of the appendages.

All the nonlinear terms associated with the centripetal and Coriolis effects are considered (Smith, Balchandran and Nayfeh, 1992; Porro, 2002), and the control issues are not included. The Lagrangian formulation is utilized for the derivation of the governing equations of motion. A fourth order Runge-Kutta algorithm is utilized to integrate these equations.

DERIVATION OF THE GOVERNING EQUATIONS OF MOTION

Figure (1) shows the system considered in this work.

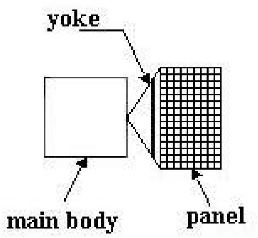

Figure 1. Satellite Model

Table 1. Nomenclature

Symbol	Description
θ_1	angular position of satellite
θ_2	opening angle of yoke
θ_3	opening angle of panel
α	attitude angle of satellite

The governing equations of the motion, obtained applying Lagrange`s equations are given by:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)-\frac{\partial L}{\partial q}=Q \qquad (1)$$

Where:

$$L=T-V \qquad (2)$$

The potential energy is considered equal to zero, because the considered orbit is circular.

The total kinetic energy of the system is given by:

$$T = \sum_{i=1}^{3} T_i \tag{3}$$

The energy given by Eq. (3) is composed of two terms: the first related to the kinect energy associated to the orbital and the attitude motion of all (Eq. (4)) and the second related to the kinect energy associated to the appendage motion (Eq. (5)).

$$T_1 = \frac{1}{2} m_1 r_1^2 \dot{q}_1^2 + \frac{1}{2} \{\dot{a}\}^T [I_1] \{\dot{a}\} \tag{4}$$

$$T_i = \frac{1}{2} m_i r_i^2 \dot{q}_i^2 + \frac{1}{2} \{\dot{q}_i\}^T [I_i] \{\dot{q}_i\} \tag{5}$$

In Eq. (4) and Eq. (5),

$$\{\dot{a}\} = \begin{Bmatrix} \dot{a}_z \\ \dot{a}_y \\ \dot{a}_x \end{Bmatrix} \tag{6}$$

$$\{\dot{q}_i\} = \begin{Bmatrix} 0 \\ 0 \\ \dot{q}_i \end{Bmatrix} \tag{7}$$

$$[I_1] = \begin{bmatrix} I_{x1} & 0 & 0 \\ 0 & I_{y1} & 0 \\ 0 & 0 & I_{z1} \end{bmatrix} \tag{8}$$

$$[I_i] = \begin{bmatrix} I_{xi} & I_{xyi} & I_{xzi} \\ I_{xyi} & I_{yi} & I_{yzi} \\ I_{xzi} & I_{yzi} & I_{z1} \end{bmatrix} \tag{9}$$

Substituting Eq. (2) in Eq. (1), results:

$$M(q) \cdot \ddot{q} + f(q, \dot{q}) \cdot \dot{q} = \boldsymbol{t} \tag{10}$$

The inertia matrix, *M(q)*, is nonlinear in the variables $(a_1, a_2, a_3, q_1, q_2, q_3)$ and the vector *f* is nonlinear in the variables and in the velocities.

The generalized forces Q_i, in the Eq. (1) are the torques $(\boldsymbol{t})$ necessary to move the appendages, supplied by the attitude jets used to correct the satellite attitude.
The influence of the nonlinearities in the whole system dynamics is investigated multiplying all the nonlinear terms by a small parameter, $\in$, and slowly changing its value from 0 to 1. So the Eq. (10) can be written such as:

$$[M_l(q)+\in M_{nl}(q)]\cdot \ddot{q}+\in f(q,\dot{q})\cdot \dot{q}=\boldsymbol{t} \tag{11}$$

where: M_l is the linear part of the inertia matrix and M_{nl} is the nonlinear part of the inertia matrix. When ($\in$=0) the system is linear and when ($\in$=1) the system is nonlinear.
Great values of angular velocities (both during attitude control and the solar array deployment phases) strongly feed these nonlinearities.

NUMERICAL RESULTS

Four values to the small parameter ($\in$) were considered in the numerical results. The solid line represents the system when it is completely linear ($\in$=0); the dash line represents the system when it is completely nonlinear ($\in$=1).

Figure (2) refers to the applied torques on the appendages rotation axes to their deployment.

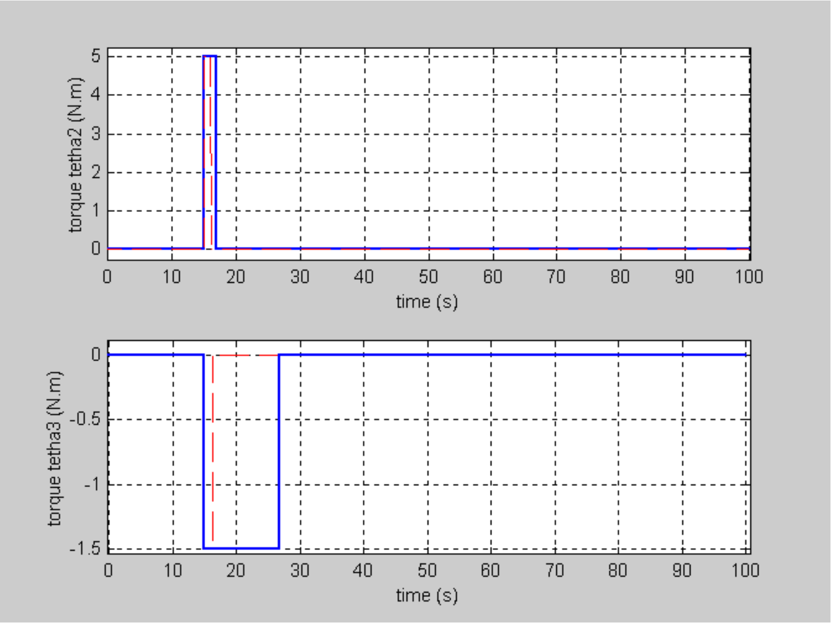

Figure 2. Profiles of torque – appendages angles

Figure (3) shows the behavior of the orbital angle. The small pictures show the three different phases considered. When the nonlinearities were included they did not influence the orbital angle behavior.

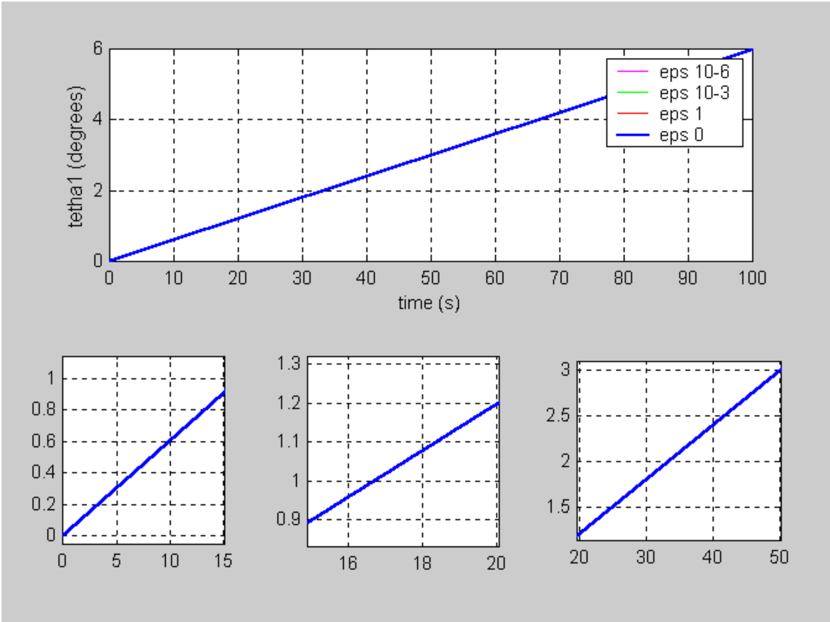

Figure 3. Orbital angle

Figure (4) shows the behavior of the deployment angle of the yoke. The instant in which the appendages are opening, the nonlinearities make this opening faster than if the system were completely linear.

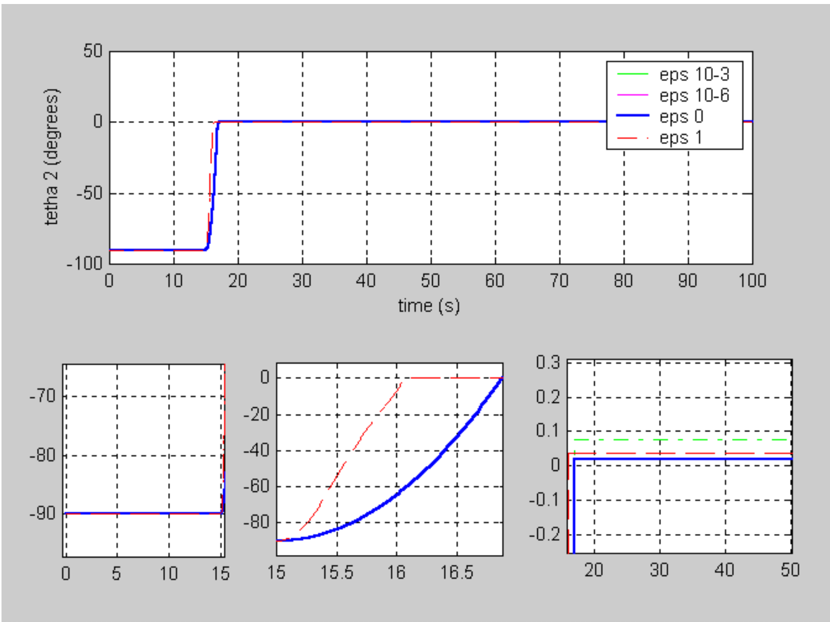

Figure 4. Deployment angle of the yoke

Figure (5) shows the behavior of the deployment angle of the panel.

The instant in which the appendages are closed and the instant in which the appendages are completely opened, the nonlinearities do not influence the system.

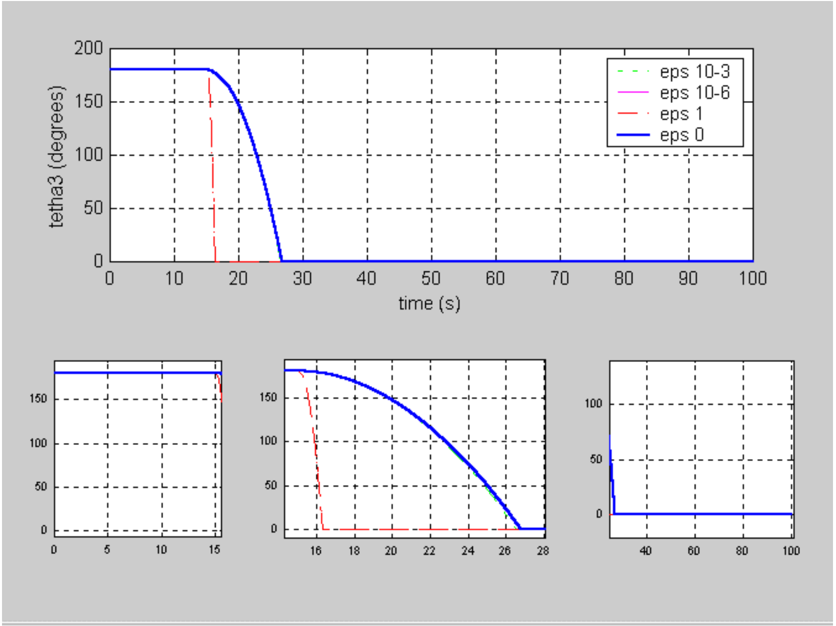

Figure 5. Deployment of the panel

Figure (6) shows the behavior of the attitude angle alpha1 (pitch angle). When the nonlinearities have their value increased, this attitude angle shows oscillatory behavior.
When the appendages are closed, and when the system is nonlinear, it shows a pulse, because at this moment a torque was applied in the satellite.

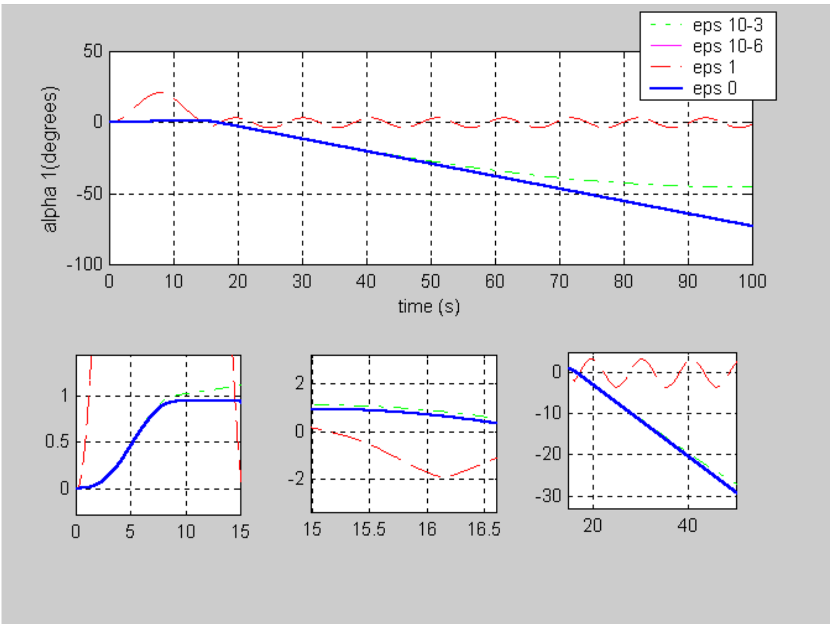

Figure 6. pitch angle

Figure (7) shows the behavior of the attitude angle alpha2 (Roll angle). When the nonlinearities have their value increased, this attitude angle shows oscillatory behavior. When the system is completely linear, after the deployment phase, in this axis, the satellite becomes stabilized.

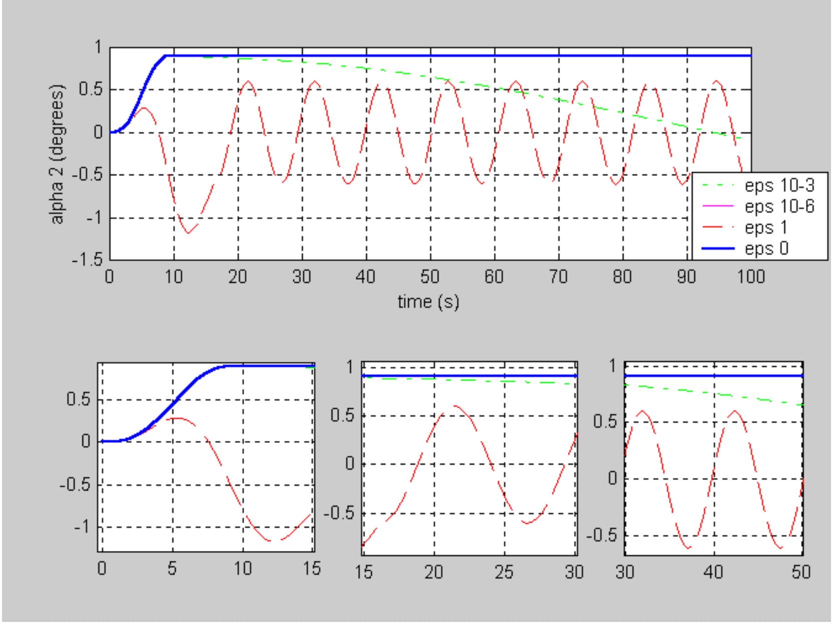

Figure 7. Roll angle

Figure (8) shows the behavior of the attitude angle alpha3 (Yaw angle). When the system is completely linear, in this axis, the satellite becomes stabilized, after the deployment phase.

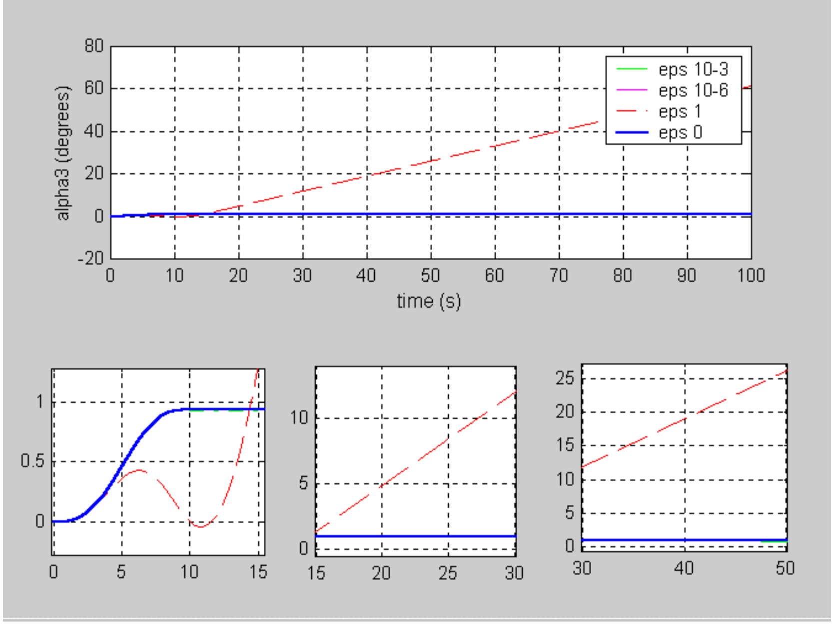

Figure 8. Yaw angle

CONCLUSIONS

The nonlinearities show strong influence in the behavior of the attitude and opening of the appendage angles, causing instability in the same ones. In some cases these terms are more representative than the prescribed torques used, determining the dynamic behavior of the system. This influence was studied in this work, because a small parameter, $\in$, was introduced in the governing equations of motion multiplying all the nonlinear terms. When this parameter is considered equal to 0, the system is linear and when it is considered equal to 1, the system becomes strongly nonlinear. Any intermediate value between 0 and 1 will indicate a bigger or minor influence of the nonlinearities in the dynamic behavior of the system. This influence will depend on the deployment velocities and on the involved panel masses and inertias.

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