BI-IMPULSIVE ORBITAL TRANSFERS BETWEEN NON-COPLANAR ORBITS WITH TIME LIMIT

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ABSTRACT

In this work, we consider the problem of two-impulse orbital transfers between non-coplanar elliptical orbits with minimum fuel consumption but with a time limit for this transfer. This time limit imposes a new characteristic to the problem that rules out the majority of transfer methods. Then, we used the equations presented by Eckel and Vinh, add some new equations to consider cases with different geometries, and solved those equations to develop a software for orbital maneuvers. This software can be used in the next missions developed by INPE. The original method developed by Eckel and Vinh was presented without numerical results. Thus, the modifications considering cases with different geometries, the implementation and the solutions using this method are contributions of the present work. The software was tested with success.

1- INTRODUCTION

The majority of the spacecraft that have been placed in orbit around the Earth utilize the basic concepts of orbital transfers. During the launch, the spacecraft is placed in a parking orbit distinct from the final orbit, which the spacecraft was designed. Therefore, to reach the desired final orbit the spacecraft must perform orbital transfer maneuvers. Beyond this, the spacecraft orbit must be corrected periodically because there are perturbations acting on the spacecraft. In Brazil, we have important applications with the launch of the Remote Sensing Satellites RSS1 and RSS2 that belongs to the Complete Brazilian Space Mission and with the launch of the China Brazil Earth Resources Satellites CBERS1 and CBERS2.

In this work, we consider the problem of two-impulse orbital transfers between non-coplanar elliptical orbits with minimum fuel consumption but with a time limit for this transfer. This time limit imposes a new characteristic to the problem that rules out the majority of the transfer methods available in the literature: Hohmann (1925), Hoelker and Silber (1959), McCue and Bender (1965), Gobetz and Doll (1969), Prado (1989), etc. Therefore, the transfer methods must be adapted to this new constraint: Wang (1963), Lion and Handelsman (1968), Gross and Prussing (1974), Prussing (1969, 1970), Prussing and Chiu (1986), Ivashkin and Skorokhodov (1981), Eckel (1982), Eckel and Vinh (1984) Lawden (1993), Taur et al. (1995). Then, we used the equations presented by Eckel and Vinh (1984), add some new equations to extend the method to cases with different geometry, and solved those equations to develop a software for orbital maneuvers.

2- DEFINITION OF THE PROBLEM

The orbital transfer of a spacecraft from an initial orbit to a desired final orbit consists in a change of state (position, velocity and mass) of the spacecraft, from initial conditions $\vec{r}_0$, $\vec{v}_0$ and m_0 at time t_0 to the final conditions $\vec{r}_f$, $\vec{v}_f$ and m_f at time t_f ($t_f \geq t_0$) as shown in Figure 1. The maneuver can be classified as: maneuvers partially free, when one or more parameter is free (for example, the time spent with the maneuver); or maneuvers completely constrained, when all parameters are constrained. In this case the spacecraft performs an orbital transfer maneuver from a specific point in the initial orbit to another specific point in the final orbit (for example, rendezvous maneuvers). In this work, based in Rocco (1997) we consider the orbital transfer maneuvers partially free, and that the spacecraft propulsion system is able to apply an impulsive thrust. Therefore, we have the instantaneous variation of the spacecraft velocity.

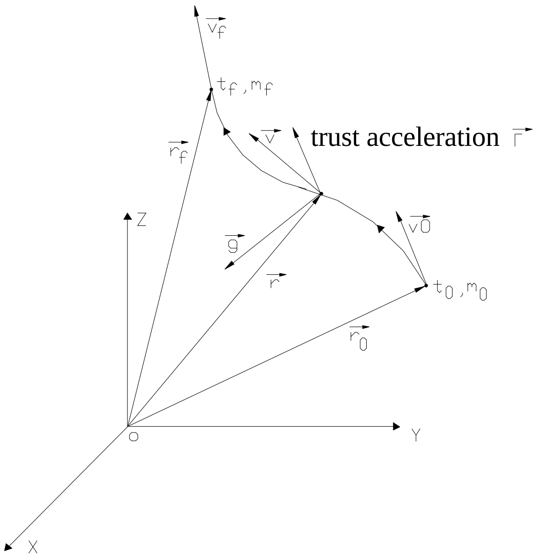

Fig. 1 – Orbital Transfer.

3- PRESENTATION OF THE METHOD

The bases for this method are the equations presented by Eckel and Vinh (1984), that provides: the transfer orbit between non-coplanar elliptical orbits with minimum fuel and fixed time transfer; or minimum time transfer for a prescribed fuel consumption; but in this work we consider only the problem with fixed time transfer. The equations were presented in the literature but the method was not implemented neither tested by Eckel and Vinh, and it is only valid for a specific geometry. They used the plane of the transfer orbit as the reference plane but we decide to use the equatorial plane as the reference plane because in this way it is easy to obtain and to apply the results in real applications. Using the transfer orbit as the reference plane almost all the results obtained belongs to the same specific geometry, so we change the reference system, adding the equations 1 to 6 to consider cases with more complex geometry. Thus, the modification, the implementation and the solutions using this method are contributions of this work. Therefore, the method was implemented to develop a software for orbital maneuvers. By varying the time spent in the maneuver the software developed provides a set of results that are the solution of the problem of bi-impulsive optimal orbital transfer with time limit.

Given two non-coplanar terminal orbits we desire to obtain a transfer orbit with minimum fuel consumption and fixed time transfer. The orbits are specified by their orbital elements, as shown in Table 1 (subscript 1: initial orbit; subscript 2: final orbit; no subscript: transfer orbit):

Table 1 – Orbital Elements.

a	Semi-major axis
e	Eccentricity
p	Semi-latus rectum
ω	Longitude of the periapsis
i	Inclination
Ω	Longitude of the ascending node
M	Mean anomaly
E	Eccentric anomaly
λ	Angle between the planes of the initial and final orbits
β_1	True anomaly of the ascending node N obtained in the plane of the initial orbit
β_2	True anomaly of the ascending node N obtained in the plane of the final orbit
I_1	Location of the first impulse
I_2	Location of the second impulse
Δ	Transfer angle obtained in the plane of the transfer orbit
γ_1	Plane change angle result of the first impulse
γ_2	Plane change angle result of the second impulse
V_1	Velocity increment generated by the first impulse
V_2	Velocity increment generated by the second impulse
V	Total velocity increment
T	Time spent in the maneuver
α_1	True anomaly of the point I_1 obtained in the plane of initial orbit
α_2	True anomaly of the point I_2 obtained in the plane of final orbit
r_1	Distance from point I_1
r_2	Distance from point I_2
f_1	True anomaly of the point I_1 obtained in the plane of the transfer orbit
f_2	True anomaly of the point I_2 obtained in the plane of the transfer orbit
x_1	Radial component of the first impulse
x_2	Radial component of the second impulse
y_1	Transverse component of the first impulse in the plane of the initial orbit
y_2	Transverse component of the second impulse in the plane of the transfer orbit
z_1	Component of the first impulse orthogonal of the initial orbit
z_2	Component of the second impulse orthogonal of the transfer orbit
h_i	Horizontal component of V_i

Depending on the intersection of the orbital planes, we have four possible geometries for the maneuver:

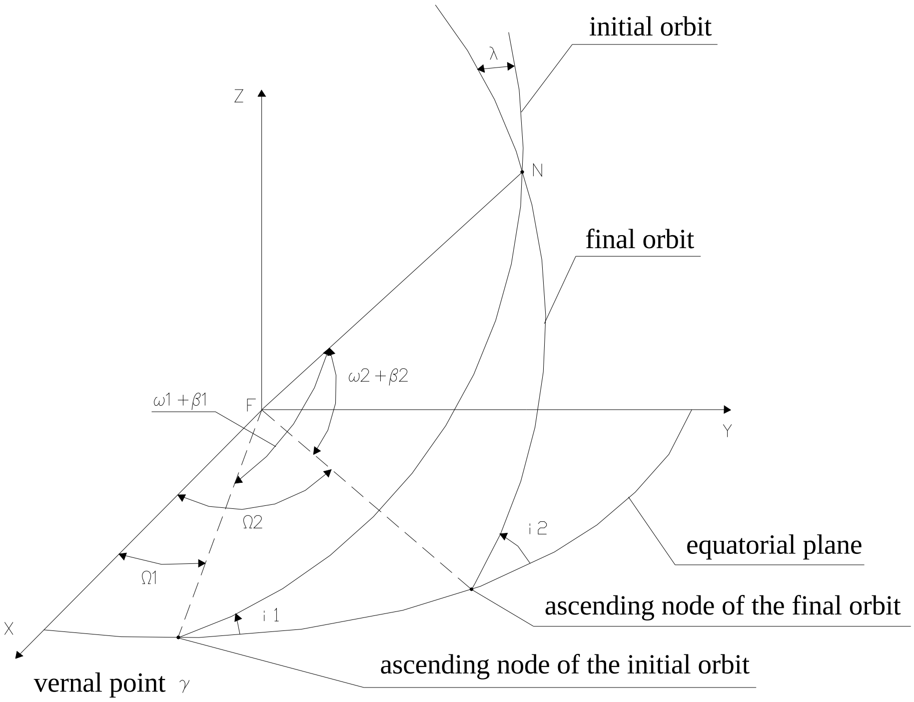

Fig. 2 – Geometry of the Maneuver When $\Omega_2 > \Omega_1$ and $i_2 > i_1$.

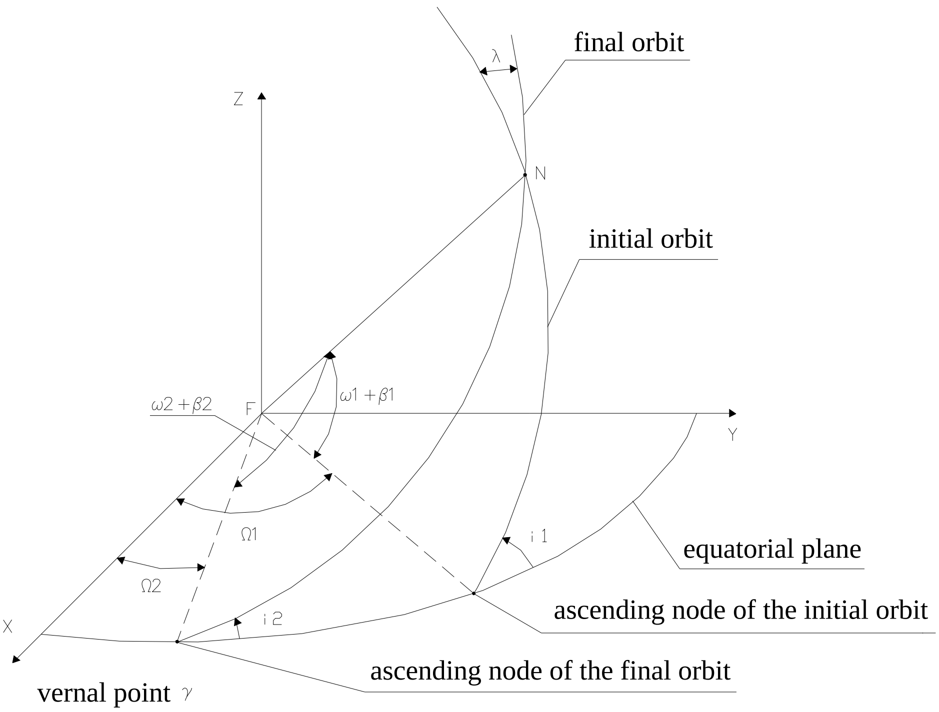

Fig. 3 – Geometry of the Maneuver When $\Omega_1 > \Omega_2$ and $i_1 > i_2$.

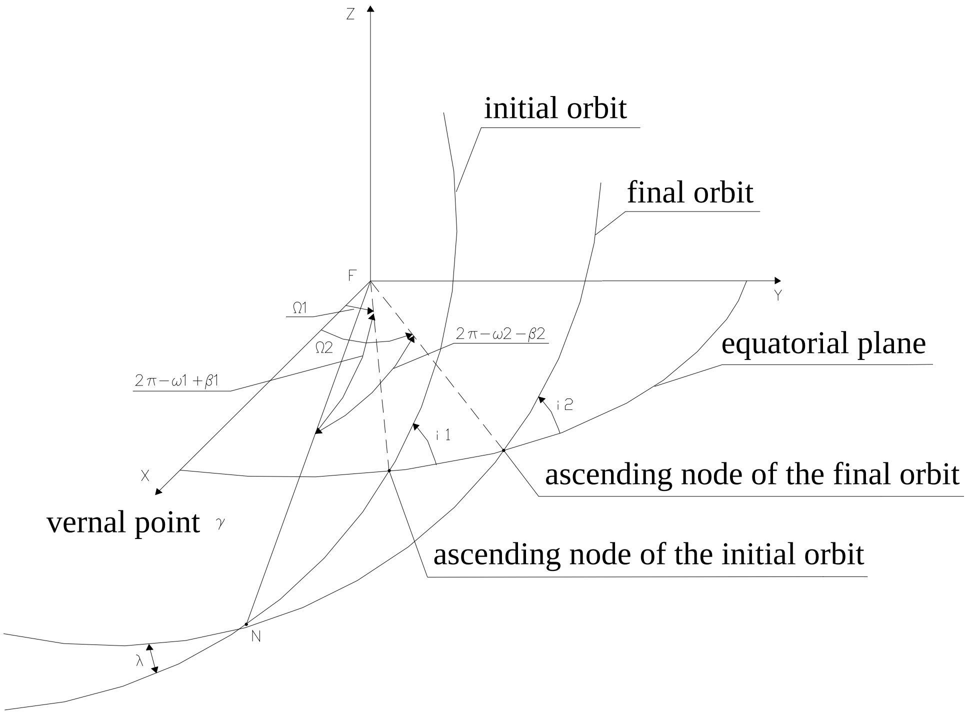

Fig. 4 – Geometry of the Maneuver When $\Omega_2 > \Omega_1$ and $i_1 > i_2$.

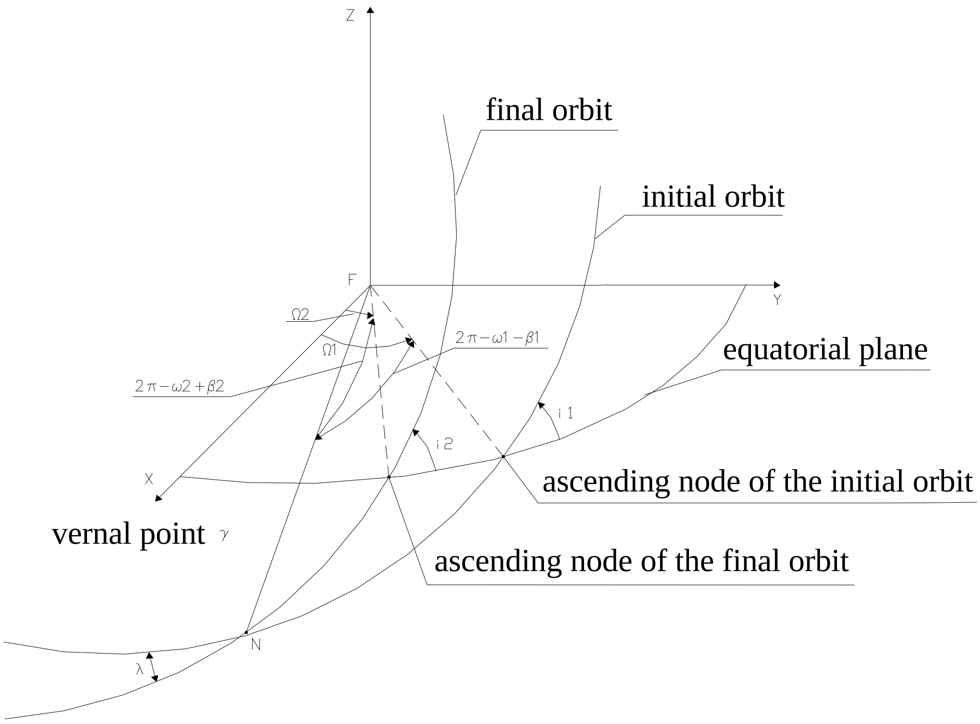

Fig. 5 – Geometry of the Maneuver When $\Omega_1 > \Omega_2$ and $i_2 > i_1$.

Depending on the locations of the impulses, we have three possible cases:

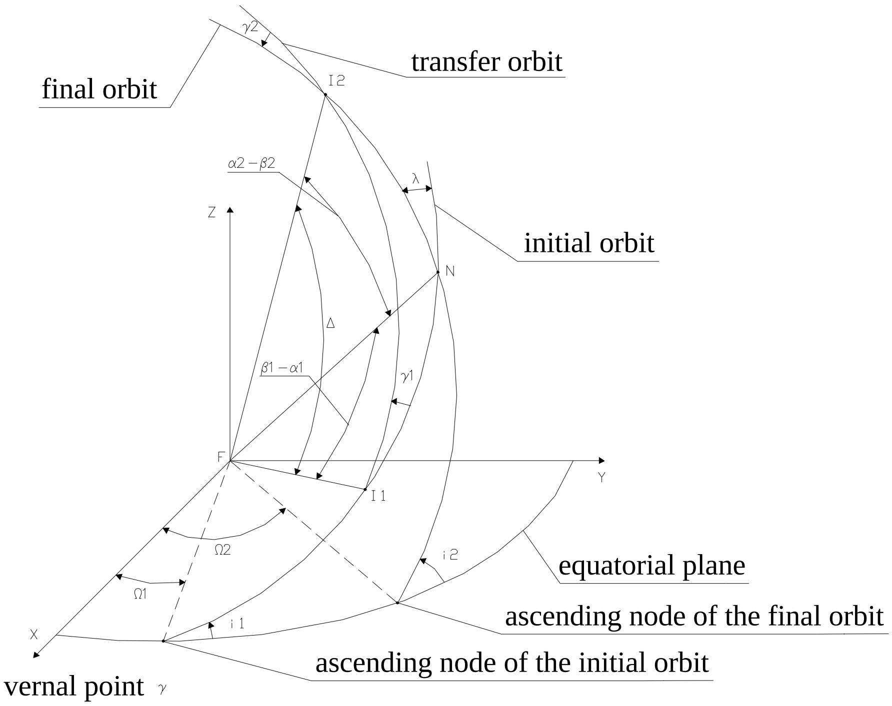

Fig. 6 – Case 1: I_1 before N and I_2 after N.

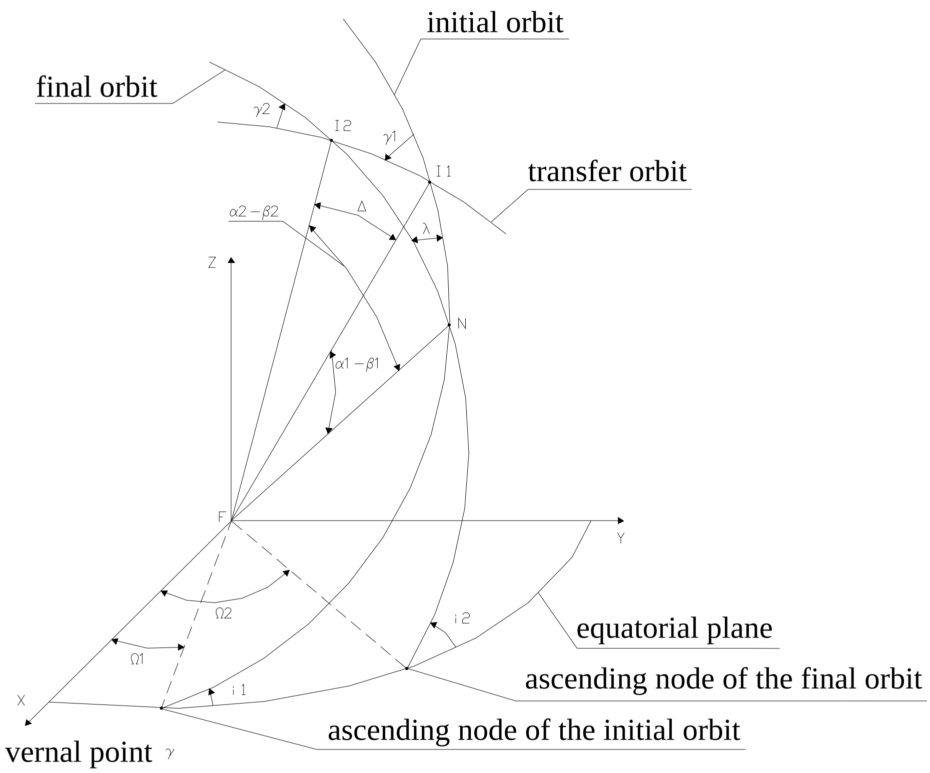

Fig. 7 – Case 2: I_1 and I_2 after N.

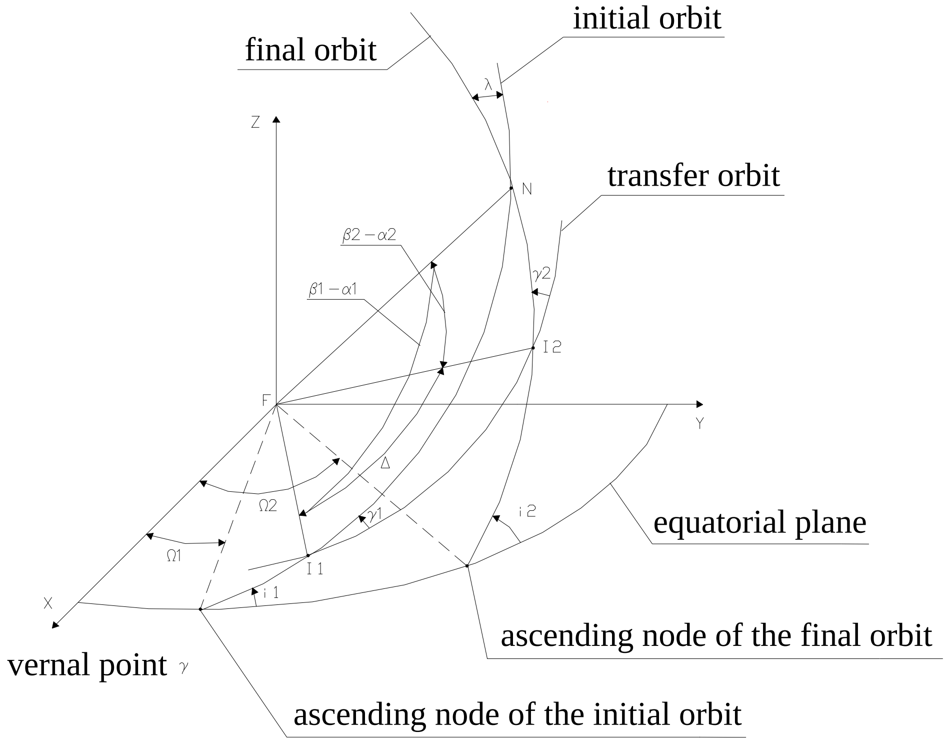

Fig. 8 – Case 3: I_1 and I_2 before N.

Combining these three cases with the four possible geometries for the transfer maneuver we have a set of twelve cases that can be solved by the software developed. Thus, from the geometry of the maneuver we obtain β_1, β_2, λ and the transfer angle Δ:

$$\beta_1 = \arctan\left[\frac{\sin(\Omega_2 - \Omega_1)\tan(180^\circ - i_2)}{\sin i_1 + \tan(180^\circ - i_2)\cos i_1 \cos(\Omega_2 - \Omega_1)}\right] - \omega_1 \tag{1}$$

$$\beta_2 = \arctan\left[\frac{\sin(\Omega_2 - \Omega_1)\tan i_1}{\sin i_2 + \tan i_1 \cos(180^\circ - i_2)\cos(\Omega_2 - \Omega_1)}\right] - \omega_2 \tag{2}$$

$$\lambda = \arcsin\left[\frac{\sin(\Omega_2 - \Omega_1)\sin i_1}{\sin(\omega_2 + \beta 2)}\right] = \arcsin\left[\frac{\sin(\Omega_2 - \Omega_1)\sin i_2}{\sin(\omega_1 + \beta_1)}\right] \tag{3}$$

$$\cos\Delta = \cos(\beta_1 - \alpha_1)\cos(\alpha_2 - \beta_2) + \sin(\beta_1 - \alpha_1)\sin(\alpha_2 - \beta_2)\cos(180^\circ - \lambda) \tag{4}$$

$$\sin\Delta = \frac{\sin(\alpha_2 - \beta_2)\sin(180^\circ - \lambda)}{\sin B} \tag{5}$$

$$B = \arctan\left[\frac{\sin(180^\circ - \lambda)}{\sin(\beta_1 - \alpha_1)\cot(\alpha_2 - \beta_2) - \cos(\beta_1 - \alpha_1)\cos(180^\circ - \lambda)}\right] \tag{6}$$

Considering that the spacecraft propulsion system is able to apply an impulsive thrust and that maneuver is bi-impulsive, the total velocity increment is:

$$V = V_1 + V_2 = F(X) \tag{7}$$

where X is an arbitrary variable for the transfer.

The time of the transfer is:

$$T = G(X) \tag{8}$$

Therefore, the problem is the minimization of V for a prescribed T. If the transfer time is prescribed being equal to a value T_0, we have the constraint relation:

$$T - T_0 = 0 \tag{9}$$

Thus, we have the performance index:

$$J = V + k(T - T_0) \tag{10}$$

From Eckel and Vinh (1984) we have that the solution of the problem depend on three variables: the semi-latus rectum p of the transfer orbit and the true anomaly α_1 and α_2 that define the position of impulses in the initial and final orbits. Therefore, we have the necessary conditions:

$$\frac{\partial V}{\partial p} + k\frac{\partial T}{\partial p} = 0 \qquad \frac{\partial V}{\partial \alpha_1} + k\frac{\partial T}{\partial \alpha_1} = 0 \qquad \frac{\partial V}{\partial \alpha_2} + k\frac{\partial T}{\partial \alpha_2} = 0 \tag{11}$$

By eliminating the Lagrange’s multiplier k from equations 11 we have the set of two equations:

$$\frac{\partial V}{\partial p}\frac{\partial T}{\partial \alpha_1} - \frac{\partial V}{\partial \alpha_1}\frac{\partial T}{\partial p} = 0 \quad ; \quad \frac{\partial V}{\partial p}\frac{\partial T}{\partial \alpha_2} - \frac{\partial V}{\partial \alpha_2}\frac{\partial T}{\partial p} = 0 \tag{12}$$

Evaluating the partial derivatives in these equations and doing some simplifications we have the final optimal conditions:

$$(X_1 + YZe\sin f_2)(S_1 q_1 - T_1 e \sin f_1) + S_1 T_1 + W_1\left(\frac{W_1 - W_2}{\sin \Delta} q_2 - W_1 \tan\frac{\Delta}{2}\right) - \frac{W_1 Z\ e\ r_1 e_1 \sin\alpha_1}{q_1 p_1 \sin f_1 \sin\gamma_1} = 0 \tag{13}$$

$$(X_2 + YZe\sin f_1)(S_2 q_2 - T_2 e \sin f_2) + S_2 T_2 - W_2\left(\frac{W_2 - W_1}{\sin \Delta} q_1 - W_2 \tan\frac{\Delta}{2}\right) + \frac{W_2 Z\ e\ r_2 e_2 \sin a_2}{q_2 p_2 \sin f_2 \sin\gamma_2} = 0 \tag{14}$$

which utilize the following relations:

$$r_i = \frac{p_i}{1 + e_i \cos\alpha_i} \tag{15}$$

$$f_1 = \arctan\left[\cot\Delta - \frac{r_1(p-r_2)}{r_2(p-r_1)\sin\Delta}\right]$$

$$f_2 = \arctan\left[\frac{r_2(p-r_1)}{r_1(p-r_2)\sin\Delta} - \operatorname{cotg}\Delta\right] \tag{16}$$

$$p = \frac{r_1 r_2(\cos f_1 - \cos f_2)}{r_1\cos f_1 - r_2\cos f_2} \tag{17}$$

$$e = \frac{r_2 - r_1}{r_1\cos f_1 - r_2\cos f_2} \tag{18}$$

$$a = \frac{p}{1-e^2} \tag{19}$$

$$\gamma_1 = \arcsin\left[-\frac{\sin(\beta_2-\alpha_2)}{\sin\Delta}\sin\phi\right]$$

$$\gamma_2 = \arcsin\left[\frac{\sin(\beta_1-\alpha_1)}{\sin\Delta}\sin\phi\right] \tag{20}$$

$$x_1 = \sqrt{\mu}\left(\frac{e}{\sqrt{p}}\sin f_1 - \frac{e_1}{\sqrt{p_1}}\sin\alpha_1\right)$$

$$x_2 = \sqrt{\mu}\left(\frac{e_2}{\sqrt{p_2}}\sin\alpha_2 - \frac{e}{\sqrt{p}}\sin f_2\right) \tag{21}$$

$$y_1 = \frac{\sqrt{\mu}}{r_1}\left(\sqrt{p} - \sqrt{p_1}\cos\gamma_1\right)$$

$$y_2 = \frac{\sqrt{\mu}}{r_2}\left(\sqrt{p_2}\cos\gamma_2 - \sqrt{p}\right) \tag{22}$$

$$z_i = \frac{\sqrt{\mu p_i}}{r_i}\sin\gamma_i \tag{23}$$

$$h_i = \left(y_i^2 + z_i^2\right)^{1/2} \tag{24}$$

$$V_i = \left(x_i^2 + h_i^2\right)^{1/2} \tag{25}$$

$$S_i = \frac{x_i}{V_i} \tag{26}$$

$$T_i = \frac{y_i}{V_i} \tag{27}$$

$$W_i = \frac{z_i}{V_i} \tag{28}$$

$$q_i = \frac{p}{r_i} \tag{29}$$

$$E_i = \arccos\left(\frac{e + \cos f_i}{1 + e \cos f_i}\right) \tag{30}$$

$$\sin E_i = \frac{\sqrt{1-e^2}\sin f_i}{1 + e\cos f_i} \tag{31}$$

$$M_i = E_i - e\sin E_i \tag{32}$$

$$T = \sqrt{\frac{a^3}{\mu}}\left(M_{I_2} - M_{I_1} + 2\pi N\right) \tag{33}$$

$$\begin{aligned} X_1 &= \frac{S_1\cos\Delta - S_2}{\sin\Delta} + T_1 \\ X_2 &= \frac{S_1 - S_2\cos\Delta}{\sin\Delta} + T_2 \end{aligned} \tag{34}$$

$$Y = \frac{1}{\left(1-e^2\right)\sin\Delta}\left[3e^2 T\sqrt{\frac{\mu}{p^3}} - 2e\left(\frac{1}{q_2\sin f_2} - \frac{1}{q_1\sin f_1}\right) + \operatorname{cotg} f_2 - \operatorname{cotg} f_1\right] \tag{35}$$

$$Z = \frac{q_2 X_2 - q_1 X_1 + \left(S_1 + S_2\right)\operatorname{tg}\Delta/2}{\operatorname{cotg} f_1 - \operatorname{cotg} f_2 + Y\left[\left(1+e^2\right)\sin\Delta + 2e\left(\sin f_2 - \sin f_1\right)\right]} \tag{36}$$

Therefore, we have an equation system composed by equations 9, 13 and 14. Solving this equation system by Newton Raphson Method (cf. Press et al. 1992), we obtain the transfer orbit that performs the maneuver spending a minimum fuel consumption but with a specific time.

4- RESULTS

The Figures 9 to 26 present some of the results obtained in Rocco (1997, 1999) with the software developed. They not only show the tendency of the parameters, but they quantify the evolution of the variables studied. The graphs were obtained through the variation of the total time spent in the maneuver. Thus, each point was obtained executing the software to the specific time. The points were joined by a line that shows the behavior of that orbital element.

We utilized, as a first example (Figures 9 to 14, 25 and 26), a maneuver of small amplitude between an initial orbit with semi-major axis of 12030 km, eccentricity 0.02, inclination 0.00873 rad, longitude of the periapse 3.17649 rad, longitude of the ascending node zero and a final orbit with semi-major axis of 11994.7 km, eccentricity 0.016, inclination 0.00602 rad, longitude of the periapse 3.05171 rad, longitude of the ascending node 0.15568. This maneuver shows that the method applied here is suitable for maneuver of stationkeeeping. In this example we utilized as initial values l = 12033.55 km, α_1 = 4.03575149 rad, and α_2 = 5.94012897 rad.

As a second example (Figures 15 to 24) we utilized, the maneuver between an initial orbit with semi-major axis of 15000 km, eccentricity 0.05, inclination 0.08726646 rad, longitude of the periapse 0.52359878 rad, longitude of the ascending node 0.17453293 rad, and a final orbit with semi-major axis of 20000 km, eccentricity 0.06, inclination 0.17453293 rad, longitude of the periapse 0.78539816 rad, longitude of the ascending node 0.34906585 rad. This maneuver shows that the method can be applied in transfers involving larger amplitudes. In this example we utilized as initial values l = 17500 km, α_1 = 0.78539816 rad, and α_2 = 1.04719755 rad.

The Figures show the behavior of some orbital elements versus time spent in the maneuver in seconds. Figures 9 and 15 show the behavior of the semi-major axis of the transfer orbit in km. Figures 10 and 16 show the behavior of the eccentricity of the transfer orbit. Figures 11 and 17 show the transfer angle in degrees. Figures 12 and 18 show the behavior of the inclination of the transfer orbit in degrees. Figures 13 and 19 show the plane change angle resulted from the first and second impulses in degrees. Figures 14 and 20 show the velocity increment V_1 and V_2, and the total velocity increment V in km/s. Figures 21 and 22 show the plane change angle generated by the impulses for the second example in more detail. Figures 23 and 25 show de resultant of the plane change ($\gamma_1 + \gamma_2$), and Figures 24 and 26 show the total plane change ($|\gamma_1| + |\gamma_2|$) in degrees for the first and second example, respectively.

5- CONCLUSIONS

From the Figures 9 to 26 and from other examples studied in Rocco (1997), we can verify, in a general way, that when the maneuver spends less time the semi-major axis and the eccentricity of the transfer orbit increases, when the maneuver spends more time the velocity increment decreases and the transfer angle increases. These behaviors occur because when the maneuver is performed with less time the transfer orbit approaches to a hyperbolic orbit, so the semi-major axis and eccentricity tend to increase. When the maneuver is performed with more time the transfer angle can be greater and the impulse directions approach to the movement directions. However, the impulse directions never will be in the movement directions because we always have a component orthogonal to the orbital plane. This component provides a plane change as shown in Figures 13 and 19. We can verify in Figure 13 that when the maneuver spends less time the plane change angle increases. In the second example, shown in Figure 19, this also occur, as we can see in more details in Figures 21, 22, 23 and 24. Figure 21 and 22 show the plane change angle resulted from the first and second impulse, respectively. In these figures it is easy to verify that there is a small variation in the value of the plane change angle along the time. In the Figure 21 we have a curve that initially decreases with the time, but around 2950 seconds the curve turns to increase. However Figure 22 show that the value of the plane change angle result of the second impulse has a clear tendency to decrease with the time. Therefore the resultant of the plane change, shown in Figure 23 and the total plane change angle, shown in Figure 24, decrease along the time. In the first example, although the resultant of plane change increases along the time, as shown in Figure 25, the total plane change angle, shown in Figure 26, presents the same tendency of Figure 24 regarding the second example. The small increase in the value of the total plane change angle, observed in the Figure 26 is due to a change of geometry of the maneuver that happens about 2600 seconds. Comparing these figures with Figures 14 and 20 is possible to verify that for a high plane change there is a high value of the velocity increment. This is expected because changes in inclination, in general, spend more fuel. The increase in the plane change angle occur because when the time fixed is small the spacecraft have to perform the maneuver very fast, then there is a great plane change in the first impulse and another great plane change in the second impulse. But the sum of the plane changes angles almost remains constant because the second impulse undo part of

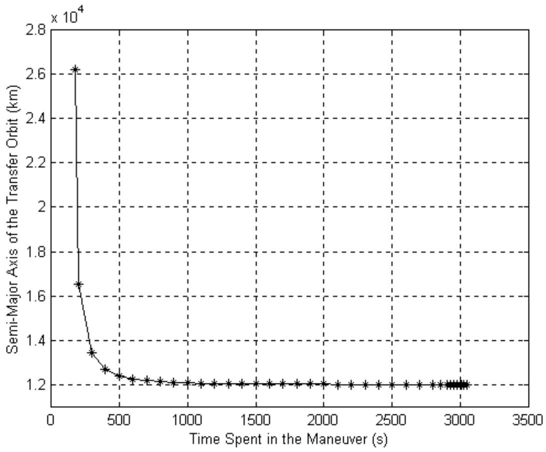

Fig. 9 – Semi-Major Axis vs. Time Spent in the Maneuver (1^o example).

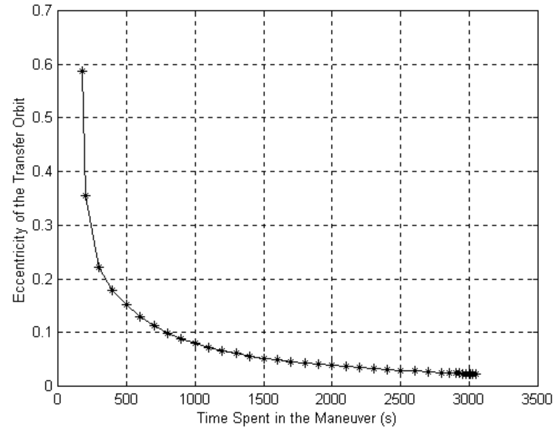

Fig. 10 – Eccentricity vs. Time Spent in the Maneuver (1^o example).

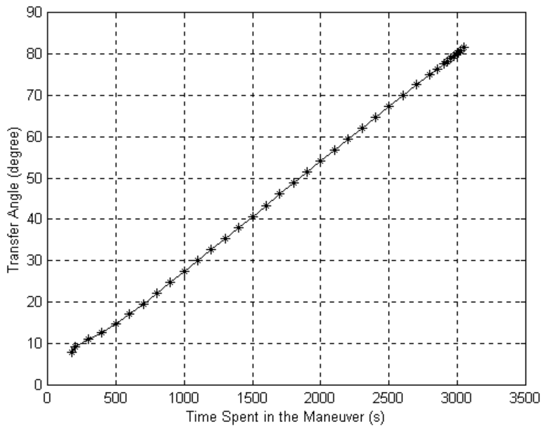

Fig. 11 – Transfer Angle vs. Time Spent in the Maneuver (1^o example).

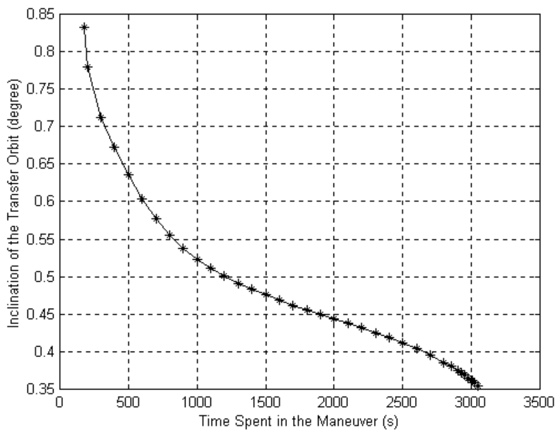

Fig. 12 – Inclination vs. Time Spent in the Maneuver (1^o example).

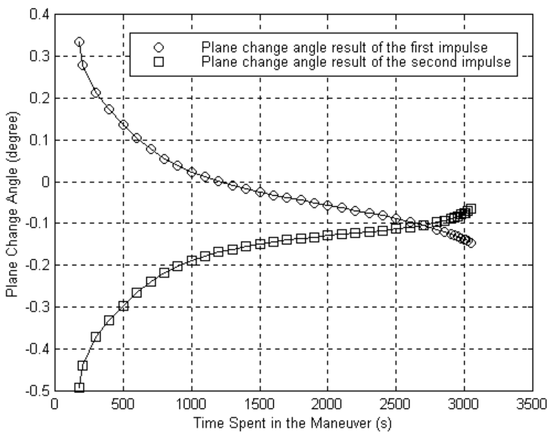

Fig. 13 – Plane Change Angle vs. Time Spent in the Maneuver (1^o example).

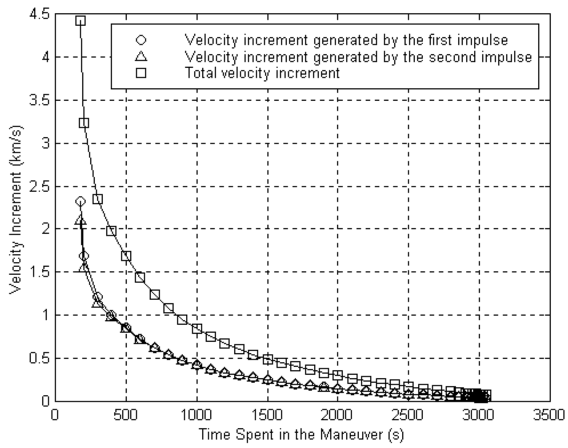

Fig. 14 – Velocity Increment vs. Time Spent in the Maneuver (1^o example).

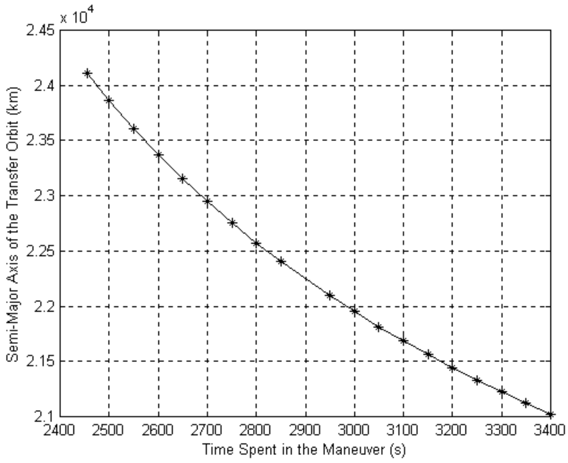

Fig. 15 – Semi-Major Axis vs. Time Spent in the Maneuver (2^o example).

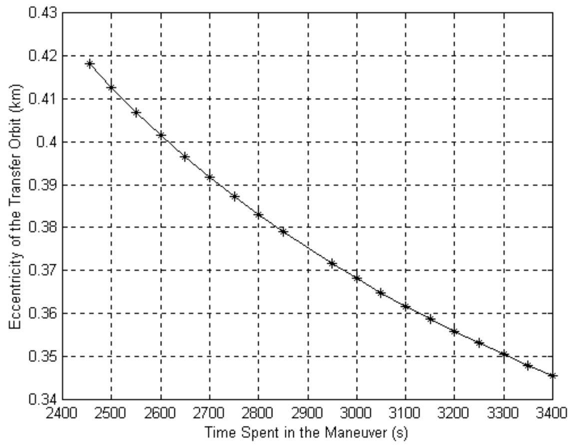

Fig. 16 – Eccentricity vs. Time Spent in the Maneuver (2^o example).

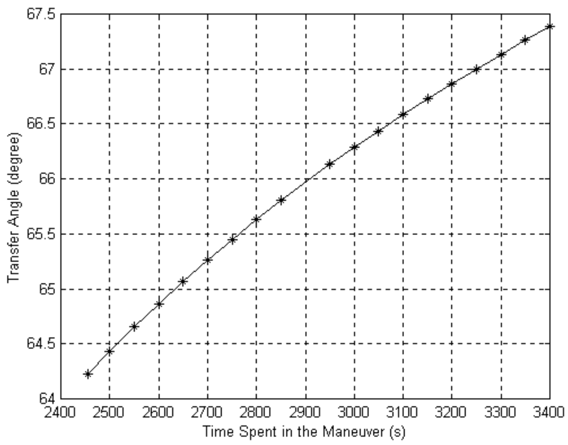

Fig. 17 – Transfer Angle vs. Time Spent in the Maneuver (2^o example).

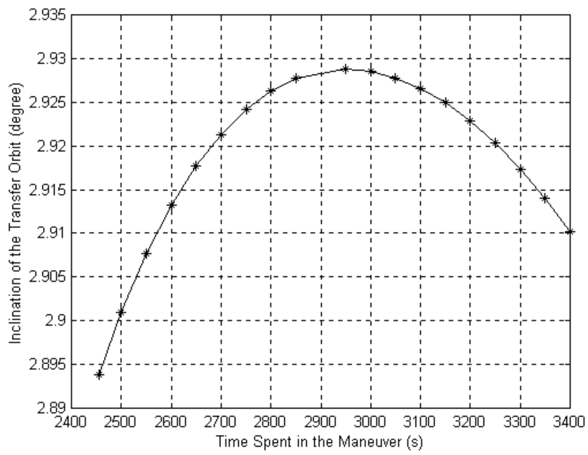

Fig. 18 – Inclination vs. Time Spent in the Maneuver (2^o example).

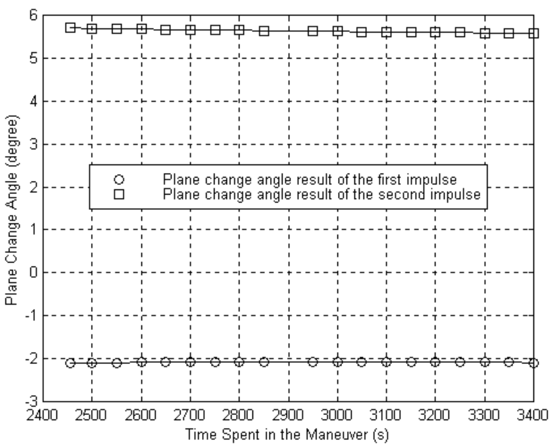

Fig. 19 – Plane Change Angle vs. Time Spent in the Maneuver (2^o example).

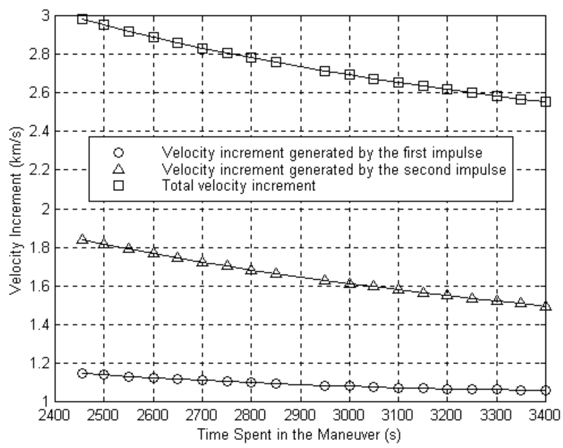

Fig. 20 – Velocity Increment vs. Time Spent in the Maneuver (2^o example).

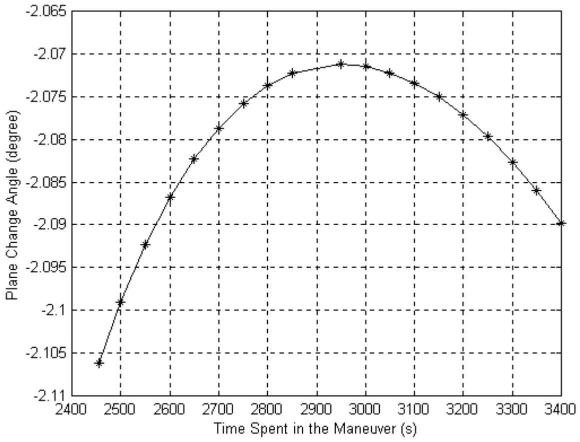

Fig. 21 – Plane Change Angle (γ_1) vs. Time Spent in the Maneuver (2^o example).

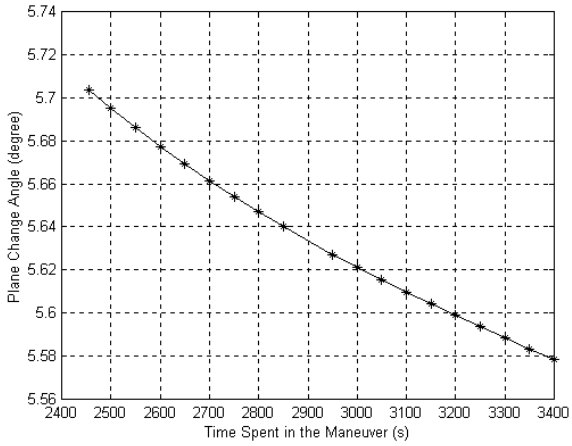

Fig. 22 – Plane Change Angle (γ_2) vs. Time Spent in the Maneuver (2^o example).

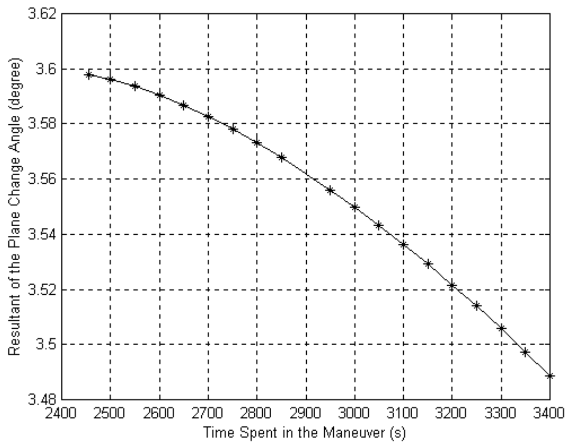

Fig. 23 – Resultant of the Plane Change vs. Time Spent in the Maneuver (2^o example).

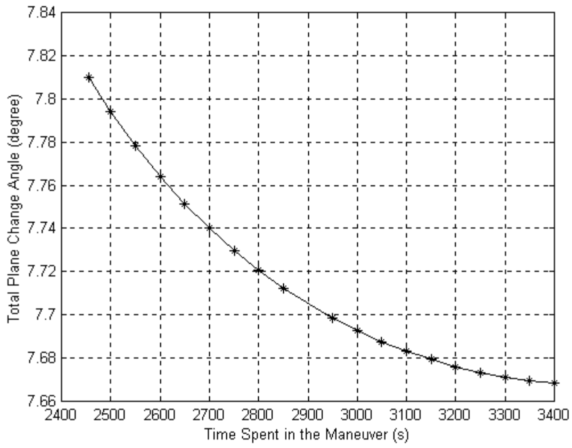

Fig. 24 – Total Plane Change Angle vs. Time Spent in the Maneuver (2^o example).

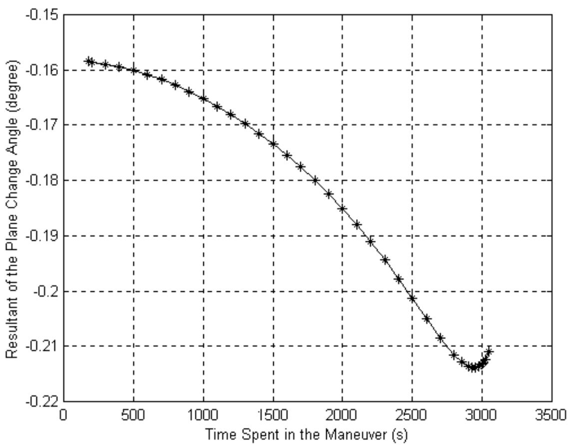

Fig. 25 – Resultant of the Plane Change vs. Time Spent in the Maneuver (1^o example).

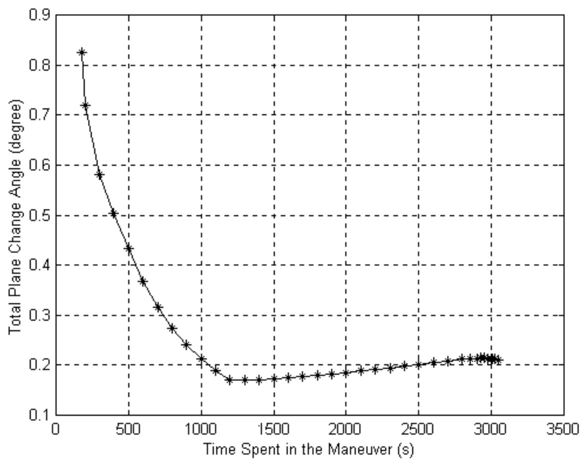

Fig. 26 – Total Plane Change Angle vs. Time Spent in the Maneuver (1^o example).

the plane change angle result of the first impulse. But the resultant of the plane change is not equal of the difference among the inclinations of the initial and final orbits, because these orbits usually do not cross because the maneuver does not correct just the difference in inclination, but also correct the eccentricity, the longitude of the periapside, the longitude of the ascending node and the semi-major axis. Beyond this, we can verify that for the first example the semi-major axis and the eccentricity of the transfer orbit, shown in Figures 9 and 10, stabilized quickly. Therefore inclination changes prevailed in the maneuver, thus most of the velocity increment applied in the maneuver is owed by the plane change. In the second example, the semi-major axis and the eccentricity, shown in Figures 15 and 16, present a high variation along the time without stabilizing around a value, in this way, the changes in the semi-major axis and in the eccentricity dominate the maneuver prevailing in relation with inclination changes. But, in both examples the behavior of the velocity increment is the same. The velocity increment decreases with time. However, there is a limit that occur when the transfer angle is greater than 180°. After this limit the increment velocity increases with time because, in this case, the impulse directions departs from the direction of the spacecraft motion. But there is another kind of solutions that consider one or more complete revolutions in the transfer orbit before the injection in the final orbit, as foreseen in Equation 33. In this kind of solutions the time specified for the maneuver can be very high, in fact, this is recommended to cases when the time specified is greater than the period of the initial and final orbits.

Besides that, we should advise that the developed program can not supply the solution for all combinations of the input parameters. For certain values of time it can be impossible to obtain one solution because for a very small or very large values of the time spent in the maneuver the solution can not exist, or the numerical algorithms used in the program do not converge for the solution, because the initial values used can be too far from the solution. So, it is recommended a physical analysis of the problem, that takes into account the periods of the initial and final orbits, to find the range of values for the time spent in the maneuver which is possible to accomplish the maneuver.

Another question to be solved is if the solution is a local or global minimum. Up to where we verified, the solution obtained seems to be a global minimum because for the same input parameters, but using different initial values, it was not possible until the moment, to obtain better results. It is important to notice that the software tests automatically all the results, verifying if the maneuver obtained is just a mathematical solution or if it can really be implemented. When we use numerical methods there are some solutions, which satisfy the equations, however, in practice, they are impossible. Due to this fact, all the points shown in the graphs were tested and they represent solutions capable of being implemented. Thus, the developed software was tested with success.

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