A DISCUSSION ON THE EFFECTS OF THRUST MISALIGNMENTS ON ORBIT TRANSFERS

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ABSTRACT

This paper presents a discussion on the effects of thrust misalignments on orbit transfers based on the open literature and on our researches. Despite being a topic of theoretical and practical importance, the open literature is poor of works that explicitly deal with it and its related ones. Among them, we will discuss the works of: Porcelli and Vogel, Adams and Melton, Rao, Howell and Gordon, Junkins et al., Junkins, Carlton-Wippern, and Alfriend. To add to or to clarify them, we will also discuss our researches presented in the works of Rodrigues, Santos-Paulo, Jesus, Rocco, and others, under the supervision of the two first authors of this paper. Finally, we will suggest some research directions.

INTRODUCTION

Most space missions need trajectory/orbit transfers to reach their goals. These trajectories/orbits are reached sequentially through transfers between them by changing at least one element of the vehicle velocity vector, or equivalently, of its keplerian elements, by firing jets, apogee motors, or other sources of force. These have misalignments (Figure 1) with respect to their nominal lines of action, caused by many unpredictable reasons like: linear and angular assembly displacements; center of mass-CM displacements due to movable parts like solar panels, antennas, booms, tethers, etc. or fuel consumption, specially during the firing periods, cf. Tandon (1988); many and nonsymmetrical jets firing at the same time; plume impingement of some jets on the vehicle structure; errors in orbit and attitude determination; timing errors; etc. Schwende and Strobl (1977) mention as reasonable an angular misalignment $\delta < 0.002$ rad = 6.8755' for a typical apogee motor. Longuski, Kia & Breckenridge (1989) present a control proposal which eliminates the damage caused by this misalignment during propulsive maneuvers. Their proposal consists of splitting it in two parts, intercalated by a time interval without propulsion. The spin stabilization is a strategy that has also shown very much use in apogee motors, due to the fact of canceling the torque of the transverse misalignment.These misalignments cause wrong forces in wrong times with resultant off the vehicle CM, and then, undesirable torques that turn the vehicle and the attached jets, deviating them even more from their nominal directions, in a positive feedback. They must be fast compensated by the attitude control system, or they will cause catastrophic orbit and attitude effects. Even so, they produce wrong orbit transfers and undesired attitude changes, that demand posterior corrections spending more time, fuel, on-ground evaluations, telemetry, and telecommands, delaying the mission, increasing its cost, and reducing its life. This paper presents a discussion on them.

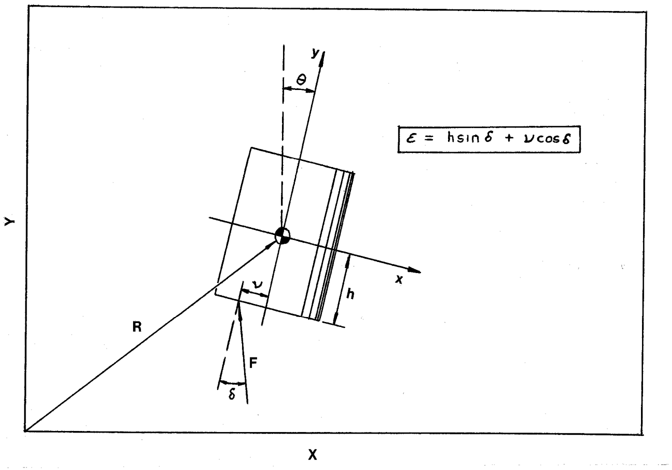


Figure 1: The misalignments of a system in planar motion. Source: Rodrigues (1991).

DISCUSSION OF SOME SIGNIFICANT WORKS

The literature on trajectory/orbit transfers is abundant of significant works, as summarized in Table 1, adapted and extended from Rodrigues (1991). In this table, they are compared through the criteria of: vehicle model, external forces and torques, thrust model, system/error dynamics, solution approach, contribution. We can see that most of them assume punctual mass, newtonian central field, n-impulsive thrust without torques, or misalignments, nonlinear (keplerian) system dynamics but no error dynamics, optimization methods, respectively. According to Souza et al. (1998), we highlight that:

Porcelli and Vogel (1980), assumed punctual mass, Newtonian central field, noncoplanar 2-impulsive apogee-perigee thrust without torques but with magnitude and angle errors, nonlinear (Kepler) system dynamics but linearized error dynamics via covariance analysis, fixed time between the two impulses. They built a semi-analytic algorithm to find the orbit insertion/transfer errors from the covariance matrices of the source errors. They propagated the covariance matrices error sources. From the errors at the first and second impulses, initially given at the orbit perigee and apogee, they got the covariance matrix of the final orbital elements.

Adams and Melton (1986) assumed punctual mass, newtonian central field, non-impulsive multiple finite perigee thrust without torques but with magnitude and angle errors, nonlinear (keplerian) system dynamics but linearized error dynamics via covariance analysis. They modeled the finite thrust as a sequence of n-impulses and extended the Porcelli and Vogel (1980) semi-analytic algorithm to it. They developed an algorithm which calculates the propagation of navigation and directional errors along the nominal trajectory, involving finite perigee burns. This work represented an important and repeated extension of the Porcelli and Vogel (1980) work, because it included finite thrusts, i.e., the non-impulsive hypothesis was used. These two works developed semi-analytic algorithms, using a covariance matrix error analysis.

Rodrigues (1991) assumed nonpunctual mass, newtonian central field, non-impulsive apogee-perigee noncoplanar thrust with torques, with linear and angular misalignments, nonlinear (keplerian) system dynamics, nonlinear error dynamics due to fuel consumption and CM displacements, attitude control or not. He built a software to simulate the orbit and the attitude motions, to evaluate the orbital element errors, the fuel consumption, and the mission success or failure under fuel constraints. His work is unique, since it is the only one in Table 1 and in most of the open literature to explicitly and fully consider: the nonpunctual dimensions and actual characteristics of an on-going satellite project (the first Brazilian Remote Sensing Satellite-RSS1), the change in its center of mass position and on its thrust intensity due to opening solar panels and to fuel consumption, the induced torques and changes in the attitude motion, thrust direction, and orbital motion, under 3 types of attitude control system, the additional maneuvers, time/fuel spent to reach the final desired orbit, and the transfer success/failure to do so.

Figure 2 shows the agreement and the parabolic behavior for small t of the analytical and the numerical simulations of the CM trajectory for the system in Figure 1 with F = 10 N, ν = 0.01 m, δ = 0 rad, ε = 0.01 m, m = 5 kg, r = 1 m, h = 3 m, I = 5 kg.m^2, $R(0) = 0$ m, $\dot{R}(0) = 0$ m/s, $\theta(0) = 0$ rad, $\dot{\theta}(0) = 0$ rad/s.

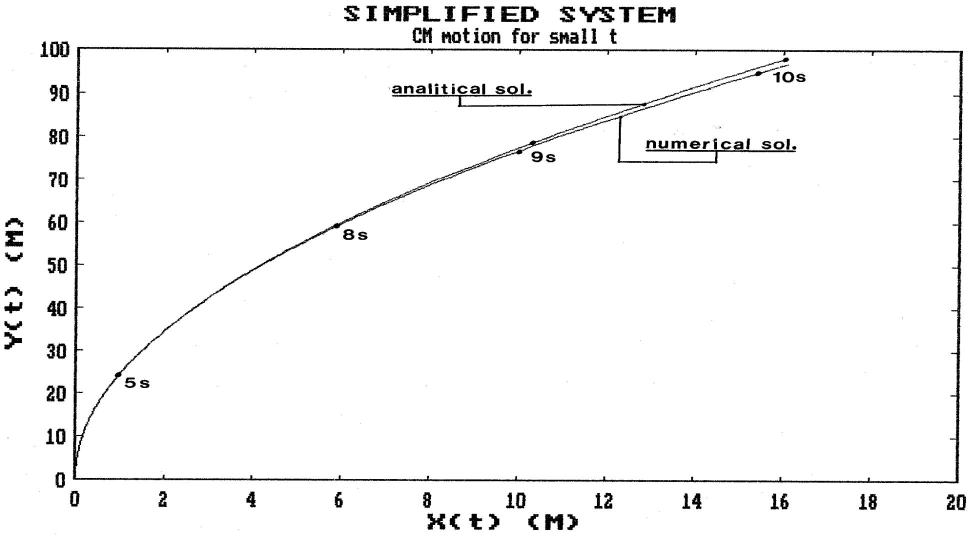


Figure 2: CM motion for small t of a system in planar motion. Source: Rodrigues (1991).

Rodrigues (1991) showed that the CM velocity components $\dot{X}(t), \dot{Y}(t)$ have damped oscillatory behavior whose values converge to the interval of 12-14 m/s. And that $\dot{Y}(t)$ reaches its first maximum $\dot{Y}^*$ at time t*. From his analytical solution he got the convergence values $\dot{X}_\infty = \dot{Y}_\infty = 12.53$ m/s, t* = 12.53 s, $\dot{Y}^*$ = 19.59 m/s, respectively. The numerical value for $\dot{Y}^*$ is 19.55 m/s, indicating a relative error in the analytic model less than 0.5% (for this example). It is interesting to note that the "ideal" value of $\dot{Y}^*$, given by (F/m)t*, is 25.07 m/s. Thus, the application of the force during t* seconds assures propulsive efficiency greater than 75% in that case.

Rao (1993), assumed punctual mass, actual gravitational field, nonlinear (keplerian) system dynamics but linearized error dynamics via covariance analysis. He built a semi-analytic theory to extend such analysis to long-term errors on elliptical orbits.

Howell and Gordon (1994) assumed punctual mass, actual gravitational field, nonlinear (keplerian) system dynamics but linearized error (injection, tracking, maneuver) dynamics via covariance analysis, minimization of a weighted squared velocity increment. They analyzed several orbit determination error methods and they developed a station-keeping strategy applicable to Sun-Earth L1 libration point orbits using impulsive forces with errors at discrete time intervals.
Junkins et al. (1996), and Junkins (1997) in one section, assumed punctual mass, newtonian central field, nonlinear (keplerian) system dynamics; linearized error dynamics via covariance analysis and nonlinear error dynamics via numeric integration. He discussed the progressive imprecision of the propagation of the orbital position covariance matrix due to initial condition errors through nonlinear coordinate transformations. He compared the distributions of the satellite position errors: zero-mean gaussian covariance ellipsoids versus negative mean non-gaussian covariance "bananoids" that grow with time.

Carlton-Wippern (1997), assumed punctual mass, Newtonian central field, first order stochastic forces specially drag, nonlinear system (keplerian) dynamics but perturbation error dynamics via Langevin equation of Statistical Mechanics. He took expectations of the variables involved to relate their means, but this zeroed the stochastic forces involved.

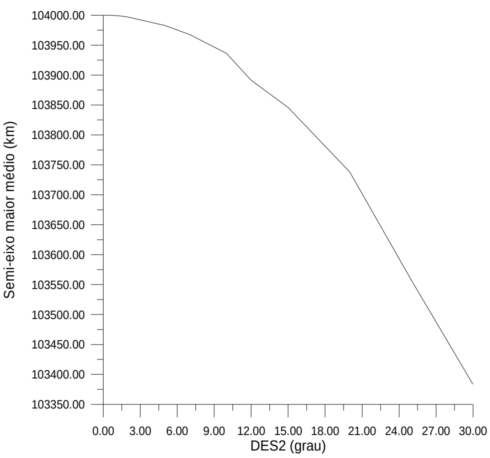
Santos-Paulo (1998) assumed punctual mass, newtonian central field, bi-impulsive noncoplanar thrust without torques, with magnitude and angular errors, nonlinear (keplerian) system dynamics, no error dynamics, fuel minimization. She built a software to simulate the orbit and to evaluate the orbital elements errors, the corrections needed, and the extra transfer time and velocity change as functions of the adopted errors, as in Tables 2 and 3.

Table 2 shows the keplerian elements of the desired and of the reached final orbits, the correspondent extra transfer time Δt and extra velocity change ΔV, the number of iterations needed N, under different hypotheses A, B, ..., H of magnitude error $\delta\Delta V$, pitch error $\delta\alpha$, or yaw error $\delta\beta$, for Case 2 (initial orbit with a = 7500 km, e = 0, i = 10^o, Ω = 0^o, $\omega = \forall$, f = $\forall$). Table 3 shows similar data for Case 7 (initial orbit with a = 7500 km, e = 0.1, i = 0^o, Ω = 0^o, ω = 10^o, f = $\forall$).

Alfriend (1999) assumed punctual mass, newtonian central field, nonlinear (kepler) system dynamics; linearized error dynamics via covariance analysis and nonlinear error dynamics via numeric integration. He discussed the progressive imprecision of the propagation of the orbital position covariance matrix due to initial condition errors and a random drag.

Jesus (1999) assumed punctual mass, newtonian central field, non-impulsive noncoplanar thrust without torques, with magnitude and angular random errors, nonlinear (keplerian) system dynamics, noise or random bias error dynamics, fuel minimization. He built a software to evaluate numerically the means, standard deviations and correlations of the final orbital elements errors, and plotted them against the standard deviations of the magnitude and angular random errors, finding some near parabolic relations as in Figures 3 and 4. It also evaluates the corrections needed, the extra time and fuel comsumptions. He also presented algebraic expressions for some relations in the planar case as in Figure 3.

Figure 3 shows the behavior of the mean final semi-major axis E{a(t_2)} as function of the standard deviation σ = DES2 in the in plane thrust angle $\alpha(t)$ from the zero mean of the gaussian distribution error $\Delta\alpha(t)$ for a theoretical case used by Biggs (1978, 1979), Prado (1989), and Jesus (1999) (a planar, high orbit, low thrust transfer from an initial orbit a = 99000 km, e = 0.7, i = 10.0^o, Ω = 55^o, ω = 105^o, f = -105^o; to a final orbit a = 104000 km, e = 0.714, i = 10.0^o, Ω = 55.006^o, ω = 104.917^o, f = 21.213^o, with thrust intensity T= 1 N, fuel mass m=2.448 kg, ejection speed c= 2.5 km/s, burn time = 1.700 h).



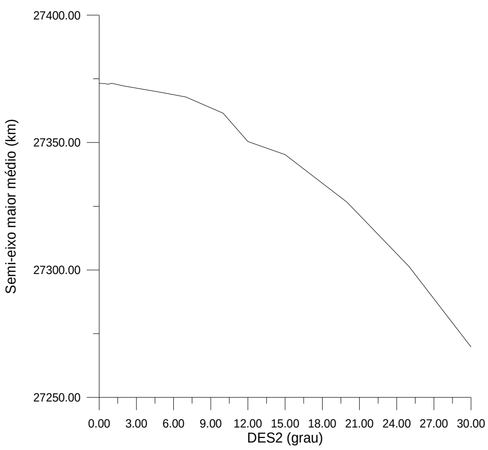


Figure 3: E{a(t_2)} x DES2, th. case. Figure 4: E{a(t_2)} x DES2, pr. case. Source: Jesus (1999).

Figure 2 shows similar data as function of the maximum deviation $\Delta\alpha_{max} = \sqrt{3}\,\sigma$ = DES2 in the in plane thrust angle $\alpha(t)$ from the zero mean of the uniform distribution error $\Delta\alpha(t)$ for a practical case used by Kuga et al. (1996) (a nonplanar middle orbit high thrust Eutelsat transfer from an initial orbit a = 24387.984 km, e = 0.730044, i = 6.9948°, Ω = 277.4743°, ω = 178.1326°, f = 200.1568°, to a final orbit a = 27373.907 km, e = 0.542, i = 3.457°, Ω = 276.265°, ω = 177.004°, f = 189.210°, with thrust intensity T= 407.3 N, fuel mass m= 289.9867 kg, ejection speed c= 3.013 km/s, burn time = 0.622 h).

Rocco (2000) is assuming punctual mass, newtonian central field, non-impulsive noncoplanar thrust without torques, with magnitude and angular random errors, nonlinear (kepler) system dynamics, pink noise or random bias error dynamics, fuel minimization but with a time limit for the transfer. He may extend what Jesus (1999) did.

SUGGESTIONS OF SOME RESEARCH DIRECTIONS

So, we find that the open literature lacks works in many research directions including:

a) to study the effects of thrust misalignments for a nonpunctual mass, in an actual non-Newtonian central field with geopotential and other perturbations, under non-impulsive noncoplanar thrust with torques, with magnitude, linear and angular random errors/misalignments, nonlinear system attitude and orbit dynamics, due to fuel consumption and CM displacements, attitude control or not, pink noise or other error dynamics, fuel minimization.
b) to present algebraic expressions to explain the remaining relations in the planar transfer, and all of them in the non-planar case; and to compare their predictions with numerical and experimental results.
c) to repeat all this analysis with a min-max approach, instead of the deterministic or the stochastic approaches.

CONCLUSIONS

In this work we discussed the effects of thrust misalignments on orbit transfers based on the open literature and on our researches. Despite being a topic of theoretical and practical importance as pointed out above, the open literature is poor of works that explicitly deal with it and its related ones. Among them, we discussed the works of: Porcelli and Vogel (1980), Adams and Melton (1986), Rao (1993), Howell and Gordon (1994), Junkins et al. (1996), Junkins (1997), Carlton-Wippern (1997), and Alfriend (1999). To add to or to clarify them, we discussed our researches presented in the works of Rodrigues (1991), Santos-Paulo (1998), Jesus (1999), Rocco (1999), and others, under the supervision of the two first authors of this paper. Finally, we suggested some research directions.

AKNOWLEDGEMENTS

The authors thanks hartly: 1) CAPES/PICDT, Brasil for supporting this work via a 4 year doctoral scholarship to the 1st. author; 2) INPE, S.J.Campos, Brasil, for employing the 2nd and 3rd authors during the work.

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TABLE 3. Results for Case 7 under errors. Source: Santos-Paulo (1998).

KEPLERIAN ELEMENTS OF THE DESIRED FINAL ORBIT	a (km)	e	i (degrees)	Ω (degrees)	ω (degrees)	Δt_{IDEAL}	ΔV_{IDEAL}
	8000	0.2	5	0	10	45 minutes	0.7545 km/s

ERROR HYPOTHESES→ KEPLERIAN ELEMENTS OF THE REACHED FINAL ORBIT ↓	A $\delta\alpha = \delta\beta = 0°$ $\delta\Delta V = 3\%$	B $\delta\alpha = \delta\beta = 0°$ $\delta\Delta V = -3\%$	G $\delta\alpha=\delta\beta=-2.5°$ $\delta\Delta V = 0\%$	H $\delta\alpha=\delta\beta= 2.5°$ $\delta\Delta V = 0\%$
a (km)	8045	7981	8072	7982
e	0.2	0.2	0.2	0.2
i (degrees)	5	5	5	5
Ω (degrees)	358	359	359	358
ω (degrees)	11	12	10	24
Δt_{TOTAL}	43’	45’	2h 50’	60’
ΔV_{TOTAL} (km/s)	0.7352	0.7347	0.7609	1.0100
Increment in Δt_{TOTAL}	-2’	Zero	2h 05’	15’
Increment in ΔV_{TOTAL} (km/s)	-0.0194	-0.0198	0.0064	0.2555
Number of iterations	3 (7)	3 (7)	7 (7)	5 (7)

TABLE 1 - Summary of some significant works on trajectory/orbit transfers. Source: Rodrigues (1991) with additions.

AUTHOR (YEAR)	VEHICLE MODEL	EXTERNAL FORCES & TORQUES	THRUST MODEL	SYSTEM/ERROR DYNAMICS	APPROACH	CONTRIBUTION
HOHMANN(1925)	PUNCTUAL MASS	NEWTONIAN CENTRAL FIELD	2 IMPULSIVE $T = \delta$, $M = 0$	NONLINEAR/ NONE	GEOMETRIC /ENERGY	PIONEER TRANSFER USED UNTIL NOW
TSIEN(1953)	PUNCTUAL MASS	NEWTONIAN CENTRAL FIELD	NONIMPULSIVE $T = k$, $M = 0$	NONLINEAR/ NONE	ORBITAL	ANALYTICAL STUDY WITH RADIAL THRUST
LAWDEN(1955)	PUNCTUAL MASS	NEWTONIAN CENTRAL FIELD	NONIMPULSIVE $T = k$, $M = 0$	NONLINEAR/ NONE	SEMIANALYTIC OPTIMIZATION	PRIME VECTOR METHOD MUCH USED IN OPTIMIZATION
ECKEL(1962)	PUNCTUAL MASS	NEWTONIAN CENTRAL FIELD	N IMPULSIVE $T = \delta$, $M = 0$	NONLINEAR/ NONE	SEMIANALYTIC OPTIMIZATION	GENERALIZATION TO N IMPULSES TRANSFER
ZEE(1963)	PUNCTUAL MASS	NEWTONIAN CENTRAL FIELD	NONIMPULSIVE $T = k$, $M = 0$	NONLINEAR/ NONE	ORBITAL	ANALYTICAL SOLUTION TO A GIVEN THRUST
ROBBINS(1966)	PUNCTUAL MASS	NEWTONIAN CENTRAL FIELD	BOTH $T = \delta$ & $T = k$, $M = 0$	NONLINEAR/ NONE	COMPARATIVE	COMPARATIVE ANALYTICAL STUDY IMPULSIVE X NONIMPULSIV. THRUST
PORCELLI& VOGEL (1980)	PUNCTUAL MASS	NEWTONIAN CENTRAL FIELD	2 IMPULSIVE $T = \delta+\varepsilon$, $M = 0$	NONLINEAR/ LINEAR	SEMIANALYTIC PROBABILISTIC	LINEAR COVARIANCE ANALYSIS OF ERRORS DUE TO ε
FLURY(1985)	NONPUNCTUAL MASS	NEWTONIAN CENTRAL FIELD	NONIMPULSIVE $T = k$, $M = 0$	NONLINEAR/ NONE	ORBITAL & OPTIMIZATION	WITH ATTITUDE & STABILIZATION DURING TRANSFER
ADAMS&MELTON (1986)	PUNCTUAL MASS	NEWTONIAN CENTRAL FIELD	N IMPULSIVE $T = \delta+\varepsilon$, $M = 0$	NONLINEAR/ LINEAR	SEMIANALYTIC PROBABILISTIC	LINEAR COVARIANCE ANALYSIS OF ERRORS DUE TO ε
VINH ET AL.(1987)	PUNCTUAL MASS	NEWTONIAN C.FIELD+DRAG	NONIMPULSIVE $T = k$, $M = 0$	NONLINEAR/ NONE	SEMIANALYTIC OPTIMIZATION	WITH DRAG IN REENTRY TRANSFER
FERNANDES& MORAES (1989)	PUNCTUAL MASS	NEWTONIAN C.FIELD+OBLAT.	IMPULSIVE $T = \delta$, $M = 0$	NONLINEAR/ NONE	SEMIANALYTIC OPTIMIZATION	WITH EARTH OBLATNESS
CHATERJEE(1989)	PUNCTUAL MASS+FLEX	NEWTONIAN CENTRAL FIELD	NONIMPULSIVE $T = \delta$, $M = 0$	NONLINEAR/ NONE	SEMIANALYTIC OPTIMIZATION	ORBITAL TRANSFER OF FLEXIBLE STRUCTURES
PRADO (1989)	PUNCTUAL MASS	NEWTONIAN CENTRAL FIELD	NONIMPULSIVE $T = \delta$, $M = 0$	NONLINEAR/ NONE	SEMIANALYTIC OPTIMIZATION	3-DIMENSIONAL MINIMUM FUEL NON IMPULSIVE ORBIT TRANSFER
RODRIGUES (1991)	NONPUNCTUAL MASS	NEWTONIAN CENTRAL FIELD	NONIMPULSIVE $T = \delta+\varepsilon$, $M\varepsilon \neq 0$	NONLINEAR/ NONLINEAR	SEMIANALYTIC DETERMINISTIC	COUPLING OF ORBIT &ATTITUDE CHANGES X ATITUDE CONTROL
RAO (1993)	PUNCTUAL MASS	NEWTON. C. FIELD+PERT.	NONE $T = 0$, $M = 0$	NONLINEAR/ LINEAR	SEMIANALYTIC	NUMERICAL &ANALYTICAL ERROR PROPAGATION DUE TO PERTURBAT.
JUNKINS(1997)	PUNCTUAL MASS	NEWTONIAN CENTRAL FIELD	NONE $T = 0$, $M = 0$	NONLINEAR/ LIN.&NONLIN.	NUMERICAL PROBABILISTIC	COMPARISON OF LIN. & NONLINEAR INIT. COND. NUMER.ERROR PROPAG.
CARLTON-WIPPERN(1997)	PUNCTUAL MASS	NEWTONIAN CENTRAL FIELD	IMPULSIVE $T = \delta+\varepsilon$, $M = 0$	NONLINEAR/ NONLINEAR	ANALYTIC PROBABILISTIC	ANALYTICAL MEAN ERROR PROPAGATION DUE TO ε
SANTOS-PAULO (1998)	PUNCTUAL MASS	NEWTONIAN CENTRAL FIELD	2,3 IMPULSIVE $T = \delta+\varepsilon$, $M = 0$	NONLINEAR/ NONLINEAR	NUMERICAL DETERMINISTIC	NUMERICAL ERROR PROPAGATION DUE TO ε
JESUS(1999)	PUNCTUAL MASS	NEWTONIAN CENTRAL FIELD	NONIMPULSIVE $T = k+\varepsilon$, $M = 0$	NONLINEAR/ NONLINEAR	NUM.&ANALYT PROBABILISTIC	MONTE CARLO & SOME ANALYTICAL NONLINEAR ERROR PROPAGATION
ROCCO(1999)	PUNCTUAL MASS	NEWTONIAN CENTRAL FIELD	NONIMPULSIVE $T = k+\varepsilon$, $M = 0$	NONLINEAR/ NONLINEAR	NUM.&ANALYT. PROBABILISTIC	NUMERICAL &ANALYTICAL NONLINEAR ERROR PROPAGATION
OPEN	NONPUNCTUAL MASS	NEWTONIAN C. FIELD +PERT.	REALISTIC MOD. ($T = k+\varepsilon$, $M\varepsilon \neq 0$)	NONLIN.,T.VAR NONLIN.,T.VAR	NUM.,ANALYT., DET, PROB,WR.C.	ANALYSIS OF MORE REALISTIC SITUATIONS

TABLE 2. Results for Case 2 under errors. Source: Santos-Paulo (1998).

KEPLERIAN ELEMENTS OF THE DESIRED FINAL ORBIT	a = 9000 km		e = 0		i = 10°		Ω = 45°		ω = undefined		Δt_{IDEAL} 1 hour		ΔV_{IDEAL} 1,1261 km/s	
$\delta\alpha = \delta\beta$ $\delta\Delta V$	A 0° 3%	B 0° -3%	C 0° 5%	D 0° -5%	E 0° 10%	F 0° -10%	G -2.5° 0%	H 2.5° 0%	I - 5° 0%	J 5° 0%	L - 2.5° -5%	M 2.5° -5%	N -5° -5%	P 5° -5%
KEPLERIAN ELEMENTS OF THE REACHED FINAL ORBIT														
a (km)	8993	8999	8794	8982	8933	8977	8951	8937	8988	8970	8952	8992	8959	8985
E	0.0005	0.0006	0.06	0.003	0.009	0.002	0.006	0.006	0.005	0.002	0.02	0.001	0.04	0.004
i (degrees)	10	10	10	10	10	10	10	10	10	10	9	10	10	9
Ω (degr.)	45	45	34	43	42	45	45	45	45	45	28	45	34	42
ω (degr.)	312	211	125	37	295	350	55	37	260	341	14	39	24	320
Δt_{TOTAL}	5h 32’	3h 28’	9h 09’	2h 45’	5h 40’	3h 18’	4h 30’	2h 07’	5h 29’	3h 31’	5h 36’	4h 17’	4h 33’	4h 29’
ΔV_{TOTAL} (km/s)	2.9430	1.1537	2.1116	1.1214	1.9987	1.1668	1.8287	1.2610	1.9849	1.2255	1.4388	1.1753	1.6288	1.0946
Increment in Δt_{TOTAL}	4h 32’	2h 28’	8h09’	1h 45’	4h 40’	2h 18’	3h 30’	1h 07’	4h 29’	2h 31’	4h 36’	3h 17’	3h 33’	3h 29’
Increment in ΔV_{TOT}(km/s)	1.8169	0.0276	0.9855	- 0.0047	0.8726	0.0407	0.7026	0.1349	0.8588	0.0994	0.3127	0.0492	0.5027	- 0.0315
Number of iterations	13 (13)	7 (7)	15 (19)	7 (7)	9 (9)	7 (7)	9 (9)	7 (7)	9 (9)	7 (7)	9 (9)	7 (7)	9 (9)	7 (7)