Gravity inversion using entropic regularization

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Summary

We present a new gravity inversion method, which produces an apparent density contrast mapping on the horizontal plane by combining the minimization of the first-order entropy with the maximization of the zeroth-order entropy of the estimated density contrasts. The maximization of the zeroth-order entropy is similar to the global smoothness constraint whereas the minimization of the first-order entropy favors solutions presenting sharp borders, so a judicious combination of both constrains may lead to solutions characterized by regions where the estimated density contrasts are virtually constant (in the case of homogeneous bodies), separated by sharp discontinuities. The method is applied to synthetic data simulating the intrusive bodies in sediments. By comparing our results with those obtained with the smoothness inversion we show that both methods produce good and equivalent locations of the source positions, but the entropic regularization delineates the contour of the bodies with greater resolution, even in the case of 100 m wide bodies separated by a distance as small as 50 m. Both, the proposed and the global smoothness inversions, have been applied to real data produced by the Lands' End batholithic intrusion from England. The entropic regularization inversion delineates a batholith with horizontal and nearly flat top being consistent with the known geological information.

Introduction

To estimate a spatial density contrast distribution we assume that a nonnull density distribution depends only on *x* and *y* and it is confined to the interior of a horizontal slab. To obtain stable solutions we combine the maximization of the zero-order entropy with the minimization of the first-order entropy of the density distribution. Unlike other classical regularization schemes, which maximize the smoothness of the density contrast distribution, the proposed method constrains the class of possible solutions to a restricted set of low-entropy geological models, consisting of locally smooth regions separated by sharp discontinuities (Campos Velho and Ramos, 1997). In this way, the gravity entropic regularization improves the resolution of the horizontal edges of the gravity sources.

Methodology

Assume a set of multiple, homogeneous gravity sources (geologic units) presenting different densities, flat tops and flat bottoms with known depths (Figure 1). The geological problem is to estimate, from gravity data, the vertical boundaries between the geologic units, producing an approximate geological map.

Let $\mathbf{g}^o$ be an *N*-dimensional vector of gravity anomalies (Figure 2a) produced by multiple gravity sources (Figure 2b). To estimate the boundaries of these gravity sources, we presume that: i) the region containing the sources can be approximated by an interpretation model consisting of a set of *M* 3D vertical juxtaposed prisms distributed along both horizontal directions (Figure 2b); ii) the dimensions of each prism along the x- and y-directions are equal; iii) the depths to the top and bottom of each prism coincide with the depths to the top and bottom of the true gravity sources; and iv) the density contrast of each prism is constant and unknown.

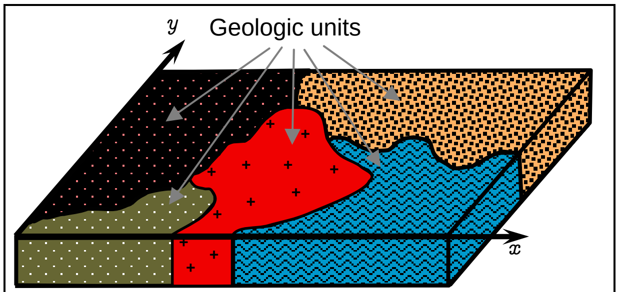

Figure 1: Schematic representation of the assumed geological sources.

The problem of estimating an *M*-dimensional vector $\mathbf{p}$ containing the prisms' density contrasts from $\mathbf{g}^o$ can be formulated as the minimization of the functional

$$\frac{1}{N}\left\|\mathbf{g}^o - \mathbf{g}(\mathbf{p})\right\|^2 , \qquad (1)$$

where $\mathbf{g}(\mathbf{p})$ is an $N \times 1$ vector containing the computed gravity anomaly produced by the set of prisms and $\| \, . \, \|$ is the Euclidean norm. Estimating a vector of parameters $\mathbf{p}^*$ that minimizes functional (1) is an ill-posed problem, characterized by unstable solutions. To transform this ill-posed problem into a well-posed one, one might introduce the additional prior information about the smoothness of the estimated parameters via first-order Tikhonov regularization to estimate a stable distribution of density contrasts of elementary prisms. This constraint imposes that the *i*th density contrast estimate be as close as possible to the estimates of spatially adjacent prisms in both x- and y-directions, subject to the misfit between the observed and the fitted gravity anomalies be within the observation error. Mathematically, the first–order Tikhonov regularization minimizes the functional

$$\mathbf{p}^{T}\mathbf{R}^{T}\mathbf{R}\;\mathbf{p} \qquad (2)$$

subject to

$$\frac{1}{N}\left\|\mathbf{g}^{\mathbf{o}} - \mathbf{g}(\mathbf{p})\right\|^{2} = \delta^{2}, \qquad (3)$$

where δ^2 is the expected mean-square of the noise realizations in the gravity data, $\boldsymbol{R}$ is a matrix representing the first-order discrete differential operator, and the superscript T stands for transposition. Incorporating the constraint (3) into the minimization problem (2) can be done by minimizing the unconstrained functional

$$\frac{1}{N}\left\|\mathbf{g}^{\mathbf{o}} - \mathbf{g}(\mathbf{p})\right\|^{2} + \mu\;\mathbf{p}^{T}\mathbf{R}^{T}\mathbf{R}\mathbf{p}\;, \qquad (4)$$

where μ is a regularization parameter. Despite being unique and stable, estimates minimizing the functional (4) show a smooth spatial density variation, so they will be referred to as global smoothness solution.

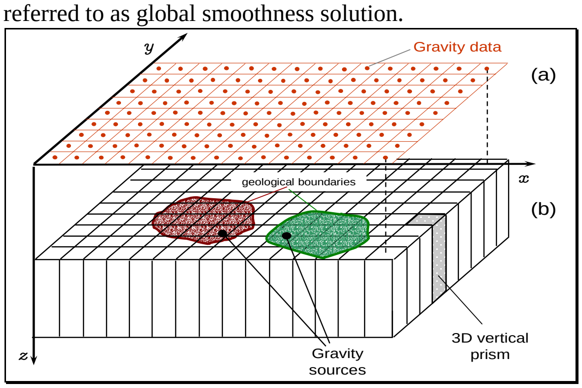

Figure 2: (a) Grid of observed gravity data (dots). (b) The interpretation model consists of a grid of 3D vertical prisms in both horizontal directions. The density contrasts of the *M* prisms are the parameters to be estimated from the gravity data.

The maximization of the zero-order entropy measure, $Q_o(\mathbf{p})$, introduces the same prior information incorporated by the first–order Tikhonov regularization, i.e., the prior information of overall smoothness constraint in the spatial density contrast variations.

In this paper we minimize the first-order entropy measure $Q_1(\mathbf{p})$ of the vector of first-differences of **p** (Campos Velho and Ramos, 1997; Ramos et al., 1999). This approach allows estimating a density contrast distribution, which enhances discontinuities in the density contrast distribution as compared with the first–order Tikhonov regularization method (equation 4). Because the minimization of $Q_1(\mathbf{p})$ alone may not stabilize the solution, it was combined with the maximization of the zero-order entropy of the estimated density contrasts, subject to the observed gravity anomaly be fitted within the experimental errors. Mathematically, this problem is expressed by:

$$\min[-\gamma_0 Q_0(\mathbf{p})/Qo_{\max} + \gamma_1 Q_1(\mathbf{p})/Q_{1\max}], \qquad (5)$$

subject to

$$\frac{1}{N}\left\|\mathbf{g}^{\mathbf{o}} - \mathbf{g}(\mathbf{p})\right\|^{2} = \delta^{2}, \qquad (6)$$

where $Qo_{\max}$ and $Q_{1\max}$ are normalizing constants and

$$Q_\alpha(\mathbf{p}) = -\sum_{k=1}^{L} S_k \log(S_k), \quad \alpha = 0 \text{ or } 1, \qquad (7)$$

are entropy measures of zeroth order if $\alpha = 0$ and of first order if $\alpha = 1$, respectively where

$$S_k = r_k \Big/ \sum_{i=1}^{L} r_i\,, \qquad (8)$$

$$r_k = \begin{cases} p_k & \text{if} \quad \alpha = 0 \\ |p_k - p_{k-1}| + \varepsilon & \text{if} \quad \alpha = 1 \end{cases}, \qquad (9)$$

L is equal to *M* for α=0, and equal to *M*-1 for α=1, and ε is a small positive number guaranteeing the numerical evaluation of the first-order entropy (Campos Velho and Ramos, 1997). Incorporating the constraint (6) into the minimization problem (5) can be done by minimizing the unconstrained functional

$$\tau(\mathbf{p}) = \left\|\mathbf{g}^{\mathbf{0}} - \mathbf{g}(\mathbf{p})\right\|^{2} - \gamma_0 Q_0(\mathbf{p})/Qo_{\max} + \gamma_1 Q_1(\mathbf{p})/Q_{1\max} \qquad (10)$$

where γ_o and γ_1 are positive regularization parameters. Parameters μ (equation 4) and γ_o control the solution stability, by degrading the solution resolution. So, the optimum values for them are the smallest positive numbers still producing stable solutions (Silva et al., 2001). Parameter γ_1, on the other hand, favors discontinuous density contrast distributions. A small value assigned to γ_1, combined with larger values of γ_o, tends to produce a diffuse estimation of the source edges, similar to the one obtained via the first–order Tikhonov regularization [smoothness inversion formulated by equation (4)]. Consequently, the optimum value for γ_1 must be the largest positive value still producing a stable solution and do not introducing discontinuities beyond those expected for the sources to be interpreted.

Application to Synthetic Data

Figures 3a and 3c show the noise-corrupted Bouguer anomaly map (black dashed lines) produced by a simulated sedimentary terrain intruded by two intrusive bodies closely separated to each other and whose horizontal projections of their abrupt boundaries are shown in white lines (Figures 3b and 3d). The uniform density contrasts of the intrusive sources relative to the sediments are 0.3 g/cm^3. The observations are measured on plane *z* =0 m, and the top and the bottom of the intrusive sources are at *z* = 0.0105 km and z = 0.2105 km, coinciding with the top and bottom of the prisms defining the interpretation model, which consists of a 16×24 grid of juxtaposed prisms with dimensions of x = 25m, y=25 m, and *z* = 200 m. To estimate the density contrast of each prism from the set of synthetic gravity data we apply the smoothness (equation 4) and the entropic regularization inversions (equation 10).

Figures 3b and 3d show the estimated density contrast distributions (in contour lines and in perspective view) using the smoothness inversion (with μ =0.02) and the entropic regularization inversion (with γ_o=1 and γ_1 = 0.5), respectively. Comparing Figures 3b and 3d, we note that the horizontal location, and the rough source shape of the intrusive sources are similarly estimated by both inversions. However, the sources edges are better delineated by the entropic regularization inversion (Figure 3d) which combines the maximum of the zero-order entropy with the minimum of the first-order entropy of the density distribution. Graphically, this can be verified by comparing the horizontal projection of the true sources (white lines) with the gradient of the estimated density contrast distribution inferred from the sequence of closely spaced contour lines (black lines) in both inversions (Figures 3b and 3d). On the other hand, the smoothness inversion does not delineate the sources boundaries, as evidenced in Figure 3b by the smooth gradient changes in the estimated density contrast. In addition, the better performance of the entropic regularization inversion (Figure 3d) can also be validated by the estimated density contrast homogeneity of the two intrusive sources as indicated by the consistent estimates of the density contrast very close to the true one (0.3 g/cm^3) defining, in this way, a flat area where the density contrast estimates present nearly constant values nearly constant values (see the perspective view in Figure 3d). Despite being substantially different, both solutions (Figures 3b and 3c) produce an acceptable fit to the data (green solid lines in Figures 3a and 3c).

Application to Real Data from Land's End Pluton

The Cornubian batholith is located in southwest England running down the axis of the peninsula for more than 200 km. In the onshore portion, the Cornubian batholith outcrops in the form of five major plutons where the Land's End pluton is the westernmost batholith.This pluton has a granitic composition and intrudes low-grade, regionally metamorphosed sediments. Figure 4 shows the gravity anomaly from the southwestern Cornubian batholith. Figures 5a and 5c show the gravity data produced by the Land's End granite (black dashed lines) within the selected area (black rectangle in Figure 4). From the available geological information, it is known that this pluton outcrops and has a flat top, which is an important constraint in estimating its thickness. This estimate is obtained by selecting the depth to the bottom of the prisms used to define the interpretation model (Figure 2b), which produces a geologically meaningful estimate of the density contrast. The region containing the batholith was discretized into a 29 × 34 grid of 0.833 km × 0.833 km prisms, along the x- and y-directions, with top and bottom

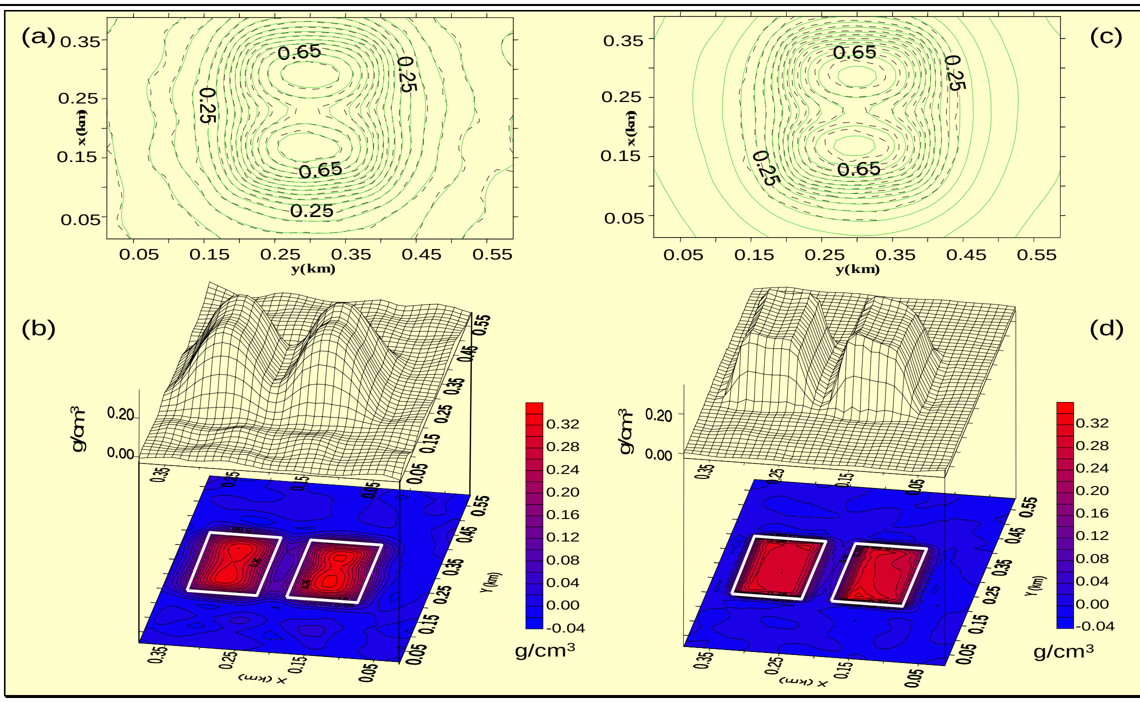

Figure 3: Synthetic example. Noise-corrupted Bouguer (black dashed lines) and fitted (green solid lines) anomalies using the smoothness inversion (a) and entropic regularization inversion (c). Simulated intrusive bodies (white thick lines), smoothness inversion solution (b), and entropic regularization solution (d). The solution is the estimated density contrast distribution shown in contour lines and in perspective views.

at 0 km and 20 km, respectively. We assume the thickness of 20 km for the batholith because it produces density contrast estimates around 0.2 g/cm^3, which approximately coincides with the geological model proposed by Willis-Richards and Jackson (1989). Figures 5b and 5d display the estimated density contrast distributions (in contour lines and in perspective view) produced, respectively, by the smoothness and the entropic regularization inversions. Although, in both inversions the observed anomaly is fitted with a reasonable precision (black solid lines in Figures 5a and 5c), the estimated density contrast distributions differ. Figure 5b shows that the smoothness constraint is insufficient to produce solutions displaying density contrast homogeneity for the batholith, evidenced by the roughness of the surface representing the density contrast distribution in the perspective view of Figure 5b. In contrast, the entropic regularization inversion (Figure 5d) leads to a reliable estimation of a homogeneous batholith having a nearly uniform density contrast of 0.2 g/cm^3, evidenced by the smoothness of the surface representing the density contrast distribution in the perspective view of Figure 5d.

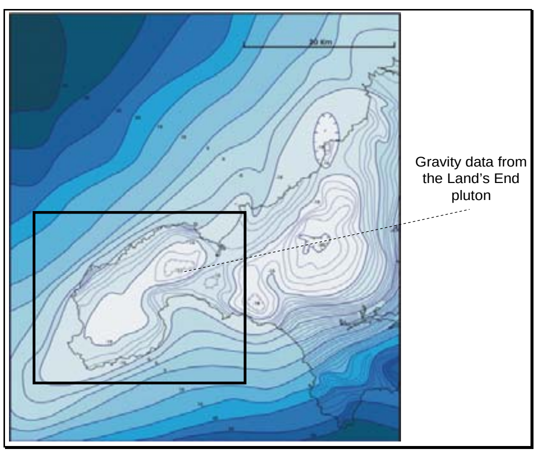

Figure 4: Gravity anomaly from the onshore and offshore of the southwestern Cornubian batholith (Bott et al, 1958).

Conclusions

We proposed a new stable gravity inversion method, which allows estimating the spatial distribution of density contrast on an x-y grid. The method is suited to map sectionally homogeneous gravity sources with abrupt edges. This geologic a priori information is introduced in the inverse problem by combining two constraints: minimum of the first-order entropy of the parameters, and maximum of the zero-order entropy of the parameters. The entropic regularization inversion establishes that the estimated density contrasts be overall smooth and constant, but may present local sharp discontinuities. Tests with synthetic and real data indicate that it may be used as an auxiliary tool in mapping geological contacts between sectionally homogeneous units with different densities.

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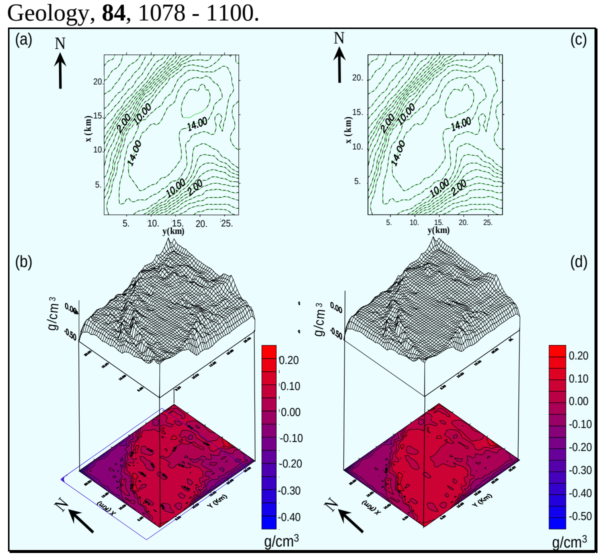

Figure 5: Lands' End Batholith. Bouguer (black dashed lines) and fitted (green solid lines) anomalies using the smoothness (a) and entropic regularization (c) inversions. Estimated density contrast distributions (in contour liles and in perspective views) using: smoothness inversion (b) with $\mu = 10^{-5}$, and entropic regularization inversion (d) with $\gamma_o = 10^{-3}$ and $\gamma_1 = 0.7$.