

ESTIMATE OF THE RENDEZVOUS TRAJECTORY THROUGH THE DYNAMICS OF THE RELATIVE MOTION - FORMATION FLYING

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Abstract. *The objective of this work is the estimation of an optimal Rendezvous maneuver between keplerian orbits with a Kalman filter, using the dynamic equations of "Formation Flying". To occur an encounter between two space vehicles it is necessary that the instant of its arrivals and the place (point) be the same for both the two Bodies involved in the maneuver. The reference measurements are obtained from the routine of Gooding to solve the Problem of Lambert, representing a bi-impulsive transfer between the orbits (Prado, 1993).*

Keywords: *Astrodynamics, Rendezvous, Kalman Filter, Control, Mechanics Celestial.*

1. Introduction

The description of the dynamics of the relative motion between two bodies, in arbitrary elliptic orbits, can be seen in many references in the literature (Carter et. al., 1987). Tschauer-Hempel developed a group of equations to describe the relative motion of a spacecraft (M_3) going around a primary M_1 , in an arbitrary elliptic orbit, with a fixed coordinate system in the primary M_2 .

A priori, the only hypothesis done in the formulation of the problem is that the distance between the bodies M_2 and M_3 is small compared with the distances between the reference point (M_2) and the body M_1 (center of gravity). The methodology of trajectory estimation used a numeric implementation of the Kalman filter together with the analysis of the results.

2. Dynamics of the Relative Motion

M_1 and M_2 , are the two primaries with mass $(1 - \mu)$ and μ , respectively. M_2 is in a circular orbit around M_1 . The space vehicle M_3 leaves M_2 from a point P ($t = \psi_0$). It follows a trajectory around M_1 and meets again with M_2 in a point Q ($t = \psi_f$), where $\psi_0, \psi_f \in [0, 2\pi]$. The values of ψ_0 and ψ_f are not necessarily symmetrical [4, 5]. The problem will be modeled using the dynamics of two bodies, i. é., $\mu = 0$ will be considered, implying in the reduction of the three bodies problem to the two bodies problem. Kepler equations can be used in the development of the solutions. It is assumed that the three bodies involved are mass points and do not suffer external disturbances (figure 1).

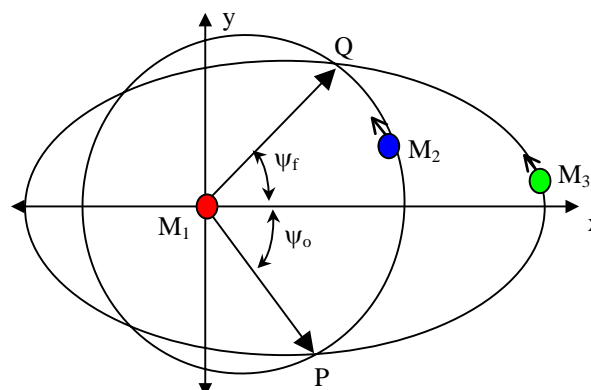


Figure 1. Relative Distance

2.1. Mathematical equations

The Tschauner-Hempel equations, that describe the relative motion, in the relative orbit coordinate system, in the torques of and external forces are (Carter et al., 1987):

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} 2kw^{\frac{3}{2}}x + 2w\dot{y} + \dot{w}y + w^2x \\ -kw^{\frac{3}{2}}y - 2w\dot{x} - \dot{w}x + w^2y \\ -kw^{\frac{3}{2}}z \end{bmatrix} \quad (1)$$

Where w is the angular velocity of the reference orbit, that is the orbit of the primary M_2 around M_1 .

2.2. Kalman Filter

The Kalman Filter discovered by Rudolf E Kalman in 1960, consists of an optimal recursive algorithm, used to estimate states of a dynamic system, based on the measurements of the reliability of the data observed, starting from a covariance uncertainty matrix, and processing of each new measurement available.

Starting from a well-known initial state, this algorithm combines the available data, adds the previous knowledge of the system and of its measurement devices, to process and produce an estimate of the desired variables, so the error is reduced along the time.

The Kalman Filter uses a control process feedback, i. e., it is considered the state of the process in a given instant, and then they are obtained the feedback, under the form of noisy measurements. Combining the measurements, a priori, the knowledge of the system dynamics, statistics of the noise of the system dynamic and the errors of measurements, besides information of the initial condition it produces an estimate of the state, in such way that the statistical error is minimized.

2.3. Problem Model

- The unit of distance is the distance between M_1 and M_2 ;
- The angular velocity (ω) of the motion of M_1 and M_2 is assumed to be unitary;
- The mass of the smaller primary (M_2) is given by $\mu = \frac{m_2}{m_1 + m_2}$, (where m_1 and m_2 are the real masses of M_1 and M_2 , respectively) and the mass of M_2 is $(1-\mu)$, to make the total mass of the system unitary;
- The gravitational constant is one;
- Runge-Kutta of 4th order, fixed step to solve the dynamical equations;
- Utilization of software MATLAB 6.5 to code a Kalman filter to estimate the rendezvous point, given the measurements. The function LQE of Matlab 6.5 was used, that is a Kalman filter for continuous systems in the time;
- It is noticed that all the units were normalized (see a, b, c, d).

Given a dynamical system:

$$\begin{cases} \dot{x} = Ax + Bu + G\omega \\ Y = Cx + Du + v \end{cases} \quad (2)$$

with a white unbiased gaussian process ω and a gaussian noise v in the measurements, both of zero mean and covariances given by:

$$\begin{cases} E\{\omega\omega^T\} = Q \\ E\{vv^T\} = R \end{cases} \quad (3)$$

where Q and R are the corresponding power spectral density.

The function LQE feeds back the Kalman Gain (K_f) and the associated solutions of the Riccati differential equation (equation 4), as in figure 2 and 3:

$$\dot{\bar{P}} = A\bar{P} + \bar{P}A^T + GQG^T \quad (4)$$

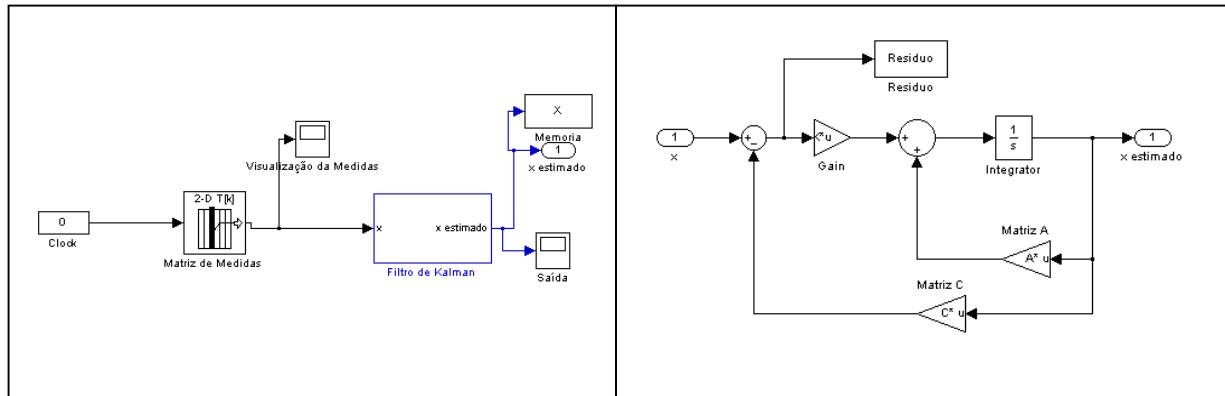


Figure 2. Plant - Kalman Filter

Applying the dynamical equations of the relative motion and assuming that the orbital motion is planar:

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 2kw^{\frac{3}{2}}x + 2w\dot{y} + \dot{w}y + w^2x \\ -kw^{\frac{3}{2}}y - 2w\dot{x} - \dot{w}x + w^2y \end{bmatrix} \quad (5)$$

Then, reducing the system of differential equations for first order:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = 2kw^{\frac{3}{2}}x_1 + 2wx_4 + \dot{w}x_3 + w^2x_1 \\ \dot{x}_3 = x_4 \\ \dot{x}_4 = -kw^{\frac{3}{2}}x_3 - 2wx_2 - \dot{w}x_1 + w^2x_3 \end{cases} \quad (6)$$

In the matrix form it is given explicitly by:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 2kw^{\frac{3}{2}} + w^2 & 0 & \dot{w} & 2w \\ 0 & 0 & 0 & 1 \\ -\dot{w} & -2w & w^2 - kw^{\frac{3}{2}} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (7)$$

Substituting the values for the problem:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 3 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (8)$$

Follows the initial conditions of reference trajectory: $\psi_o = -1$ rad, $\psi_f = 1,2$ rad, $e = 0,001$ (eccentricity of M_3 orbit). The used dynamic noise and measurement noise covariance matrices were:

$$Q = \begin{bmatrix} 10^{-7} & 0 & 0 & 0 \\ 0 & 10^{-7} & 0 & 0 \\ 0 & 0 & 10^{-7} & 0 \\ 0 & 0 & 0 & 10^{-7} \end{bmatrix}; R = \begin{bmatrix} 10^{-1} & 0 & 0 & 0 \\ 0 & 10^{-1} & 0 & 0 \\ 0 & 0 & 10^{-1} & 0 \\ 0 & 0 & 0 & 10^{-1} \end{bmatrix} \quad (9)$$

After solving the Riccati equation through MATLAB, it was obtained the steady state invariant covariance matrix P and Kalman gain Kf's, with the following regime values:

$$P = \begin{bmatrix} 0.001 & 0 & -0.0004 & -0.0002 \\ 0 & 0.0001 & 0.0002 & 0 \\ -0.0004 & 0.0002 & 0.0113 & 0.0006 \\ -0.0002 & 0 & 0.0006 & 0.0005 \end{bmatrix}; Kf = \begin{bmatrix} 0.0013 & 0 & -0.0042 & -0.0025 \\ 0 & 0.001 & 0.002 & 0 \\ -0.0042 & 0.002 & 0.1131 & 0.0064 \\ -0.0025 & -0 & 0.0064 & 0.0047 \end{bmatrix} \quad (10)$$

2.4. Results and Simulations

The results below are based on simulated real data and shows the measurement residues of the Kalman filter in the radial and axial components. These results show a preliminary attempt to use the Kalman filter to estimate in real time a relative orbital motion in a rendezvous problem.

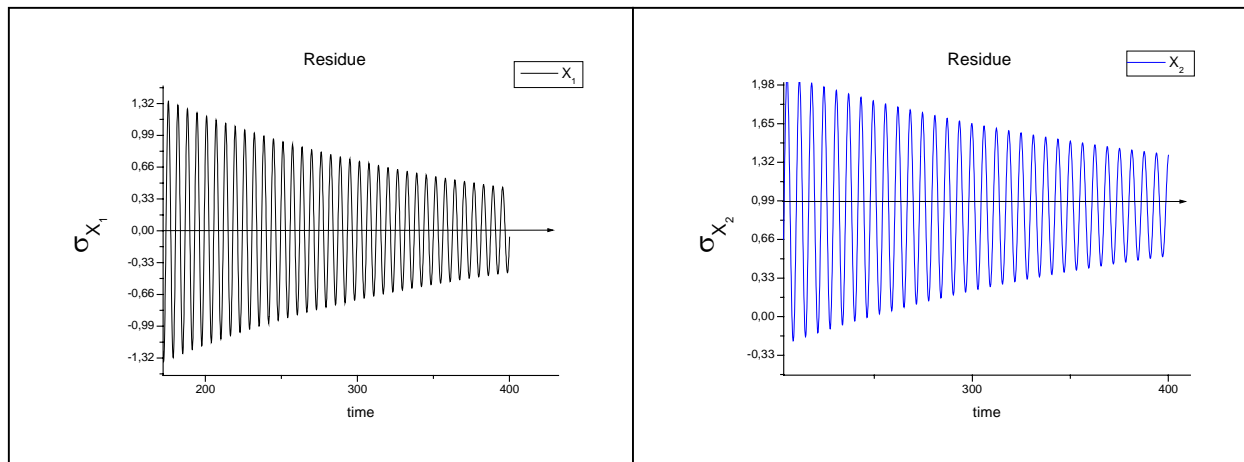


Figure 3. Residues of position X (X_1) and Velocity V_X (X_2)

It is noticed that the residues of the states converge for a band of values that is comprised between $+\sigma$ and $-\sigma$ (standard deviation). The results in the figures 3 and 4 are functions of the respective states. Remember that the components of velocity should tend to the unity value for they are the traverse axis projections.

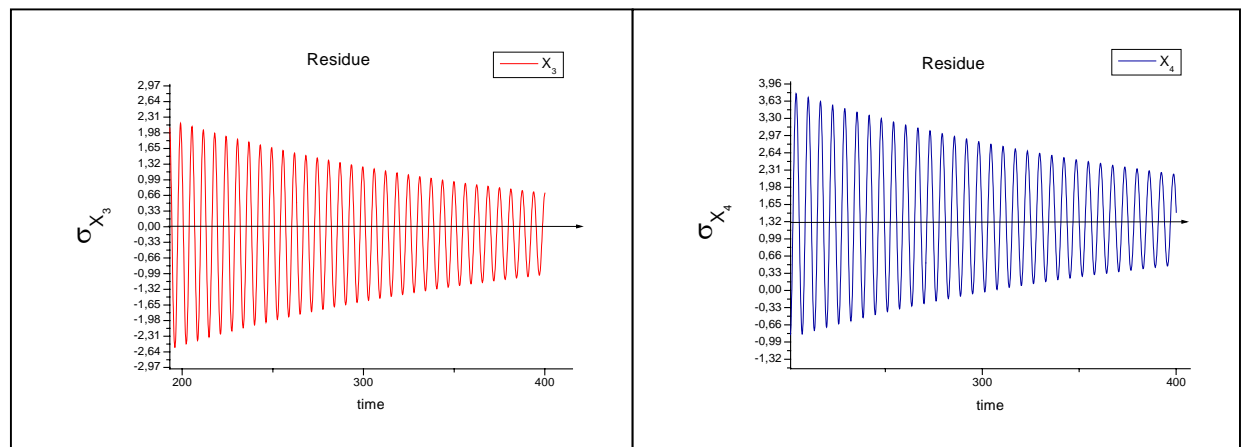


Figure 4. Residues of Position Y(X_3) and Velocity V_Y (X_4)

The graphs of the estimate errors in the states (figure 7 and 8), show a slow convergence along the time of simulation and the values oscillate inside of a band that in a long term (longer time in the filter parameters) satisfy the expected solution, that are the final states (position and velocity) of the Rendezvous.

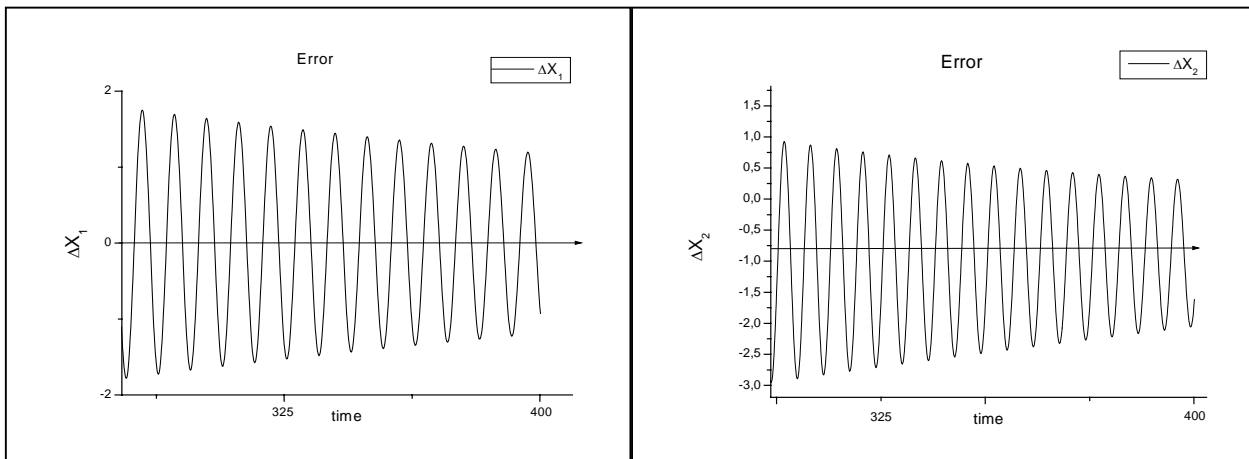


Figure 7. Error of Position X (X_1) and Velocity V_x (X_2)

The models for the problem of estimating Rendezvous trajectories through the Tschauner-Hempel dynamical equations ("Formation Flying ") are not commonly used. Then using this formulation, it should be taken into account the error in the trajectories and consequently the error in the point of encounter of the Rendezvous. When necessary, the error introduced by the use of a model should be compensated for by the use of larger amount of measurements, that will be processed by the state estimator, viz. the Kalman filter.

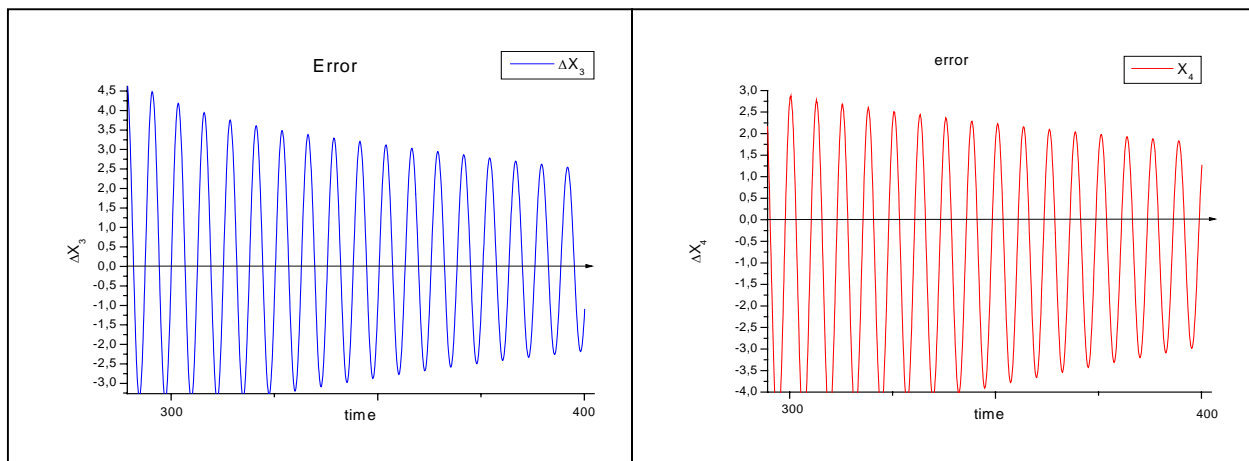


Figure 7. Errors of Position Y (X_3) and Velocity V_y (X_4)

2.5. Conclusion

Based on the analysis of the results obtained by the Kalman filter, it could be verified that it is an efficient tool for error reduction and estimation of the Rendezvous trajectory through the Tschauner-Hempel dynamical equations ("Formation Flying"). Such use is recommended for processing as much as possible the available measurements. It is of fundamental importance for the improvement of the Kalman filter estimates in the case of Rendezvous trajectories.

The benefits expected by this type of procedure are: economy (cost and energy), alternative procedure of orbit determination on board, autonomy on board, and researches in the development of processing algorithms.

As future works it could be researched the use of other filtering schemes to obtain enhanced response times, when the rendezvous time is critical.

3. Acknowledgements

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4. References

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