Proceedings of COBEM 2005 18th International Congress of Mechanical Engineering

November 6-11, 2005, Ouro Preto, MG

NUMERICAL METHODS TO MANEUVER A SATELLITE USING A LOW THRUST ENGINE

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Abstract. *In this paper, the problem of spacecraft orbit transfer with minimum fuel consumption is considered, in terms of testing numerical solutions. The objective is to obtain the control laws to maneuver a satellite using a low thrust engine. A sub-optimal approach will be used for those maneuvers. The optimal control problem will also be solved numerically, in order to compare the results. The spacecraft is supposed to be in a Keplerian motion controlled only by the thrusts, whenever they are active. An initial and a final orbit around the Earth is completely specified for the maneuver. There is no time restriction involved here and the spacecraft can leave and arrive at any point in the given initial and final orbits. The mechanism, time and fuel consumption to change the direction of the thrust is not considered in this paper. The final goal is to use techniques like those ones shown here to maneuver satellites that will make orbit determination and maneuvers in an autonomous way, based in on-board calculations.*

Keywords: *Orbital maneuvers, astrodynamics, low thrust maneuvers*

1. Introduction

R. H. Goddard (1919) was one of the first researchers to work on the problem of optimal transfers of a spacecraft between two points. He proposed optimal approximate solutions for the problem of sending a rocket to high altitudes with minimum fuel consumption.

After that, there is the very important work done by Hohmann (1925). He solved the problem of minimum ΔV transfers between two circular coplanar orbits. His results are largely used nowadays as a first approximation of more complex models and it was considered the final solution of this problem until 1959.

The Hohmann transfer would be generalized to the elliptic case (transfer between two coaxial elliptic orbits) by Marchal (1965). Smith (1959) showed results for some other special cases, like coaxial and quasi-coaxial elliptic orbits, circular-elliptic orbits, two quasi-circular orbits. A numerical scheme to solve the transfer between two generic coplanar elliptic orbits was presented by Bender (1962).

Hohmann type transfers between non-coplanar orbits are discussed in several papers, like McCue (1962), that studied a transfer between two elliptic inclined orbits including the possibility of rendezvous; or Eckel and Vinh (1984), that solved the same problem with time or fuel fixed.

The three-impulse concept was introduced in the literature by Hoelker and Silber (1959) and Shternfeld (1959). They showed that a bi-elliptical transfer between two circular orbits has a lower ΔV than the Hohmann transfer, for some combinations of initial and final orbits. After that, Ting (1960) showed that the use of more than three impulses does not lower the ΔV for impulsive maneuvers.

In this paper, from the analyses of the alternatives of solutions available (Prado, 1989; Prado & Rios-Neto, 1993), results of a new implementation for PC computers and tests of one method selected to solve the problem of sending a space vehicle from one orbit to another with minimum fuel expenditure are shown. The method can be used either for large orbit transfers (as a geosynchronous satellite launched by the Space Shuttle in a low parking orbit) or for small orbit corrections (as the maneuvers required for station-keeping of a space station or a remote sensing satellite). The objective is to find the best way (in terms of minimum fuel expenditure) to accomplish the maneuvers required.

2. Definition of the problem

The objective of this problem is to modify the orbit of a given spacecraft. The problem is to find how to transfer the spacecraft between two given orbits in a such way that the fuel consumed is minimum. There is no time restriction and the spacecraft can leave and arrive at any point in the given initial and final orbits. The maneuver is performed with the use of an engine that is able to deliver a thrust with constant magnitude and linearly variable direction.

3. Model used

The spacecraft is supposed to be in Keplerian motion controlled only by the thrusts. This means that there are two types of motion:

i) A Keplerian orbit. This motion occurs when the thrusts are not firing;

ii) The motion governed by two forces: the Earth's gravity field and the force delivered by the thrusts. This motion occurs during the time the thrusts are firing.

The thrusts are assumed to have the following characteristics:

i) Fixed magnitude;

ii) Constant Ejection Velocity;

iii) Constrained angular motion. This means that the direction of the force given by the thrusts can be modified during the transfer. This direction can be specified by the angles α and β, called pitch (the angle between the direction of the thrust and the perpendicular to the line Earth-spacecraft) and yaw (the angle with the orbital plane). The motion of those angles are constrained (constant, linear variations, forbidden regions for firing the thrusts, etc.);

iv) Operation in on-off mode: It means that intermediate states are not allowed. The thrusts are either at zero or maximum level all the time.

The solution is given in terms of the constants that specifies the control to be applied and the fuel consumed. Several numbers of "thrusting arcs" (arcs with the thrusts active) can be used for each maneuver.

Instead of time, the "range angle" (the angle between the radius vector of the spacecraft and an arbitrary reference line in the orbital plane) is used as the independent variable.

4. Formulation of the optimal control problem

Objective Function to be minimized: $J = m_0 - m_f$, the difference between the initial and the final mass of the spacecraft, that represents the fuel consumed.

This objective function has to be minimized with respect to the control u(.), that is the time to start and to stop the engine and the pitch and yaw angles of the thrust at every instant of time, since the magnitude of the thrust is assumed to be constant.

This system is subject to the following equations of motion (Biggs, 1978 and Prado, 1989):

$$dX_1/ds = f_1 = SiX_1F_1 \tag{1}$$

$$dX_2/ds = f_2 = Si\{[(Ga+1)cos(s)+X_2]F_1+\nu F_2 sin(s)\} \tag{2}$$

$$dX_3/ds = f_3 = Si\{[(Ga+1)sin(s)+X_3]F_1-\nu F_2 cos(s)\} \tag{3}$$

$$dX_4/ds = f_4 = Si\,\nu F(1-X_4)/(X_1W) \tag{4}$$

$$dX_5/ds = f_5 = Si\,\nu(1-X_4)m_0/X_1 \tag{5}$$

$$dX_6/ds = f_6 = -\,SiF_3[X_7cos(s)+X_8sin(s)]/2 \tag{6}$$

$$dX_7/ds = f_7 = SiF_3[X_6cos(s)-X_9sin(s)]/2 \tag{7}$$

$$dX_8/ds = f_8 = SiF_3[X_9cos(s)+X_6sin(s)]/2 \tag{8}$$

$$dX_9/ds = f_9 = SiF_3[X_7sin(s)-X_8cos(s)]/2 \tag{9}$$

where:

$$Ga = 1 + X_2cos(s) + X_3sin(s) \tag{10}$$

$$Si = (\mu X_1^4)/[Ga^3m_0(1-X_4)] \tag{11}$$

$$F_1 = F cos(\alpha)cos(\beta) \tag{12}$$

$$F_2 = F sin(\alpha)cos(\beta) \tag{13}$$

$$F_3 = F sin(\beta) \tag{14}$$

and F is the magnitude of the thrust, W is the velocity of the gases when leaving the engine, ν is the true anomaly of the spacecraft.

In those equations the state was transformed from the Keplerian elements (a = semi-major axis, e = eccentricity, i = inclination, Ω = argument of the ascending node, ω = argument of periapsis, ν = true anomaly of the spacecraft), in the variables X_i, to avoid singularities, by the relations:

$$X_1 = [a(1-e^2)/\mu]^{1/2} \tag{15}$$

$$X_2 = e\cos(\omega-\phi) \tag{16}$$

$$X_3 = e\sin(\omega-\phi) \tag{17}$$

$$X_4 = (Fuel\ consumed)/m_0 \tag{18}$$

$$X_5 = t = time \tag{19}$$

$$X_6 = \cos(i/2)\cos((\Omega+\phi)/2) \tag{20}$$

$$X_7 = \sin(i/2)\cos((\Omega-\phi)/2) \tag{21}$$

$$X_8 = \sin(i/2)\sin((\Omega-\phi)/2) \tag{22}$$

$$X_9 = \cos(i/2)\sin((\Omega+\phi)/2) \tag{23}$$

$$\phi = \nu + \omega - s. \tag{24}$$

and s is the range angle of the spacecraft.

The reference system used here is the usual equatorial system. This system is also subject to the constraints in state, because five of the the Keplerian elements of the initial and the final orbit are fixed: a, e, i, ω, Ω. All the parameters (gravitational force field, initial values of the satellite, etc...) are assumed to be known.

In the suboptimal approach used here (Prado, 1989; Biggs, 1978), a linear parametrization is used for the control law (angles of pitch (α) and yaw (β)):

$$\alpha = \alpha_0 + \alpha' * (x - x_S) \tag{25}$$

$$\beta = \beta_0 + \beta' * (x - x_S) \tag{26}$$

where α_0, β_0, α', β' are parameters to be found, x is the instantaneous range angle and x_S is the range angle when the motor is turned-on.

Considering these assumptions, there is a set of six variables to be optimized (start and end of thrusting and the parameters α_0, β_0, α', β') for each "burning arc" in the maneuver. Note that this number of arcs is given "a priori" and it is not an "output" of the algorithm.

By using parametric optimization, this problem is reduced to one of nonlinear programming, which can be solved by several standard methods.

5. Numerical method

To solve the nonlinear programming problem, the gradient projection method was used (Bazarra & Sheetty, 1979; Luemberger, 1973). It means that at the end of the numerical integration, in each iteration, two steps are taken:

i) Force the system to satisfy the constraints by updating the control function according to:

$$\boldsymbol{u}_{i+1} = \boldsymbol{u}_i - \nabla \mathbf{f}^{\mathbf{T}}.\left[\nabla \mathbf{f}.\nabla \mathbf{f}^{\mathbf{T}}\right]^{-1}\mathbf{f} \tag{27}$$

where f is the vector formed by the active constraints;

ii) After the constraints are satisfied, try to minimize the fuel consumed. This is done by making a step given by:

$$\boldsymbol{u}_{i+1} = \boldsymbol{u}_i + \bar{\alpha}\,\frac{\mathbf{d}}{|\mathbf{d}|} \tag{28}$$

where:

$$\bar{\alpha} = \gamma\,\frac{\mathrm{J}(\boldsymbol{u})}{\nabla\mathrm{J}(\boldsymbol{u}).\mathbf{d}} \tag{29}$$

$$\mathbf{d} = -\left(\mathbf{I} - \nabla\mathbf{f}^{\mathrm{T}}\left[\nabla\mathbf{f}.\nabla\mathbf{f}^{\mathrm{T}}\right]^{-1}\mathbf{f}\right).\nabla\mathrm{J}(\boldsymbol{u}) \tag{30}$$

where **I** is the identity matrix, **d** is the search direction, J is the function to be minimized (fuel consumed) and γ is a parameter determined by a trial and error technique. The possible singularities in Eqs. (27) to (30) are avoided by choosing the error margins for tolerance in convergence large enough.

This procedure continues until $|\boldsymbol{u}_{i+1} - \boldsymbol{u}_i| < \varepsilon$ in both Eqs. (27) and (28), where ε is a specified tolerance.

6. Results

The maneuvers used to validate the method and the software developed are the same ones used in Biggs, 1978 and Prado (1989), with the goal of having the possibility to compare the solutions obtained.

Maneuver 1:

Initial orbit: Semi-major axis: 99000 km, eccentricity: 0.7, inclination: 10 deg, longitude of the ascending node: 55 deg, argument of periapsis: 105 deg.

Initial data of the spacecraft: Total mass: 300 kg, Thrust magnitude: 1.0 N, Initial position: 0, True anomaly: -105 deg, Ejection velocity of the gas: 2.5 km/s.

Condition imposed in the final orbit: Semi-major axis = 104000 km.

Propulsion: 1 arc.

Table 1. Solutions for maneuver 1.

Variable	**Reference Solution (Biggs, 1988)**	**Reference Solution (Prado, 1989)**	**Solution**
$\mathbf{S_0}$ (degree)	78.0	78.0	76.0
$\mathbf{S_f}$ (degree)	132.5	132.6	130.6
$\boldsymbol{\alpha_0}$ (degree)	-11.3	-8.8	-9.8
$\boldsymbol{\beta_0}$ (degree)	0.,0	0.0	0.0
α' (degree / degree)	0.416	0.467	0.497
β' (degree / degree)	0.0	0.0	0.0
Fuel consumed (Kg)	2.44	2.44	2.42
Duration of burn (s)	---	6113.9	6119.9
FINAL ORBIT OBTAINED			
Semi-major axis (Km)		104000.53	
Eccentricity		0.716	
Inclination (degree)		10	
Longitude of the ascending node (degree)		55	
Argument of periapsis (degree)		105	
True anomaly (degree)		28.4	

Maneuver 2:

Initial orbit: Semi-major axis: 99000 km, eccentricity: 0.7, inclination: 10 deg, longitude of the ascending node: 55 deg, argument of periapsis: 105 deg.

Initial data of the spacecraft: Total mass: 300 kg, Thrust magnitude: 1.0 N, Initial position: 0, True anomaly: -105 deg, Ejection velocity of the gas: 2.5 km/s.

Condition imposed in the final orbit: Semi-major axis = 104000 km.

Propulsion: 1 arc, with restriction in applying thrust between the true anomalies of 120.0 deg and 180.0 deg.

Table 2. Solutions for maneuver 2.

Variable	**Reference Solution (Biggs, 1988)**	**Reference Solution (Prado, 1989)**	**Solution**
S_0 (degree)	25.3	25.3	25.9
S_f (degree)	65.0	65.0	65.5
α_0 (degree)	-31.,9	-26.4	-22.4
β_0 (degree)	0.0	0.0	0.0
α' (degree / degree)	0.396	0.177	0.179
β' (degree / degree)	0.0	0.0	0.0
Fuel consumed (Kg)	2.81	2.81	2.80
Duration of burn (s)	---	7039.3	7034.3
FINAL ORBIT OBTAINED			
Semi-major axis (Km)	104000.08		
Eccentricity	0.715		
Inclination (degree)	10		
Longitude of the ascending node (degree)	55		
Argument of periapsis (degree)	103.1		
True anomaly (degree)	320.1		

The inclusion of these constraints provided a solution with larger fuel consumption after this first solution was obtained.

Maneuver 3:

Initial orbit: Semi-major axis: 9900 km, eccentricity: 0.2, inclination: 10 deg, longitude of the ascending node: 0 deg, argument of periapsis: 25 deg.

Initial data of the spacecraft: Total mass: 300 kg, Thrust magnitude: 2.0 N, Initial position: 0, True anomaly: -10 deg, Ejection velocity of the gas: 2.5 km/s.

Condition imposed in the final orbit: Semi-major axis = 10000 km.

Propulsion: 1 arc.

Table 3. Solutions for maneuver 3.

Variable	**Reference Solution (Biggs, 1988)**	**Reference Solution (Prado, 1989)**	**Solution**
S_0 (degree)	0.0	0.0	0.0
S_f (degree)	179.4	179.5	178.4
α_0 (degree)	4.0	2.5	2.1
β_0 (degree)	0.0	0.0	0.0
α' (degree / degree)	0.035	0.053	0.050
β' (degree / degree)	0.0	0.0	0.0
Fuel consumed (Kg)	3.74	3.74	3.75
Duration of burn (s)	---	4675.6	4679.2
FINAL ORBIT OBTAINED			
Semi-major axis (Km)	10000.01		
Eccentricity	0.2		
Inclination (degree)	10		
Longitude of the ascending node (degree)	55		
Argument of periapsis (degree)	27.3		
True anomaly (degree)	165.1		

Note that the constraints $S_0 \geq 0.0$ is active.

Maneuver 4:

Initial orbit: Semi-major axis: 9900 km, eccentricity: 0.2, inclination: 10 deg, longitude of the ascending node: 0 deg, argument of periapsis: 25 deg.

Initial data of the spacecraft: Total mass: 300 kg, Thrust magnitude: 2.0 N, Initial position: 0, True anomaly: -10 deg, Ejection velocity of the gas: 2.5 km/s.

Condition imposed in the final orbit: Semi-major axis = 10000 km. Propulsion: 2 arcs.

Table 4. Solutions for maneuver 4.

Variable	**Reference Solution (Biggs, 1988)**	**Reference Solution (Prado, 1989)**	**Solution**
Arc 1			
S_0 (degree)	0.0	0.0	0.0
S_f (degree)	81.7	89.5	90.5
α_0 (degree)	-0.3	1.2	1.9
β_0 (degree)	0.0	0.0	0.0
α' (degree / degree)	0.124	0.171	0.175
β' (degree / degree)	0.0	0.0	0.0
Arc 2			
S_0 (degree)	307.5	299.3	289.3
S_f (degree)	434.3	417.7	407.9
α_0 (degree)	-9.8	-8.6	-3.2
β_0 (degree)	0.0	0.0	0.0
α' (degree / degree)	0.152	0.099	0.101
β' (degree / degree)	0.0	0.0	0.0
Fuel consumed (Kg)	3.19	3.20	3.21
Duration of burn (s)	---	4002.9	4009.9
FINAL ORBIT OBTAINED			
Semi-major axis (Km)		10000.02	
Eccentricity		0.206	
Inclination (degree)		10	
Longitude of the ascending node (degree)		0.0	
Argument of periapsis (degree)		25.1	
True anomaly (degree)		45.3	

Maneuver 5:

Initial orbit: Semi-major axis: 4500 km, eccentricity: 0.5, inclination: 8 deg, longitude of the ascending node: -145 deg, argument of periapsis: -20 deg.

Initial data of the spacecraft: Total mass: 11300 kg, Thrust magnitude: 60000 N, Initial position: 0, True anomaly: 170 deg, Ejection velocity of the gas: 4.25 km/s.

Condition imposed in the final orbit: Semi-major axis = 10000 km, eccentricity = 0.122, Inclination = 2.29 deg..

Propulsion: 1 arc and the burn must be completed before the true anomaly of 35.0 deg.

Table 5. Solutions for maneuver 5.

Variable	**Reference Solution (Biggs, 1988)**	**Reference Solution (Prado, 1989)**	**Solution**
S_0 (degree)	6.7	6.5	4.9
S_f (degree)	28.0	27.7	25.7
α_0 (degree)	1.30	0.50	1.5
β_0 (degree)	16.0	16.8	19.7
α' (degree / degree)	-0.017	-0.034	-0.032
β' (degree / degree)	0.007	-0.067	-0.061
Fuel consumed (Kg)	5269.5	5248.9	5258.3
Duration of burn (s)	---	375.6	379.6
FINAL ORBIT OBTAINED			
Semi-major axis (Km)		7435.98	
Eccentricity		0.121	
Inclination (degree)		2.290	
Longitude of the ascending node (degree)		259.2	
Argument of periapsis (degree)		175.0	
True anomaly (degree)		329.2	

This maneuver considers the case where the thrust is large and that there are three keplerian elements to be changed.

7. Conclusions

Suboptimal control was explored to generate algorithms to obtain solutions for the minimum fuel maneuvers for a spacecraft.

A new implementation of a method used in Prado (1989) was made for use in modern PC computers and the simulations realized showed that it worked in all the simulations performed.

By comparing the results obtained with the algorithms developed and those found in the literature (Biggs, 1978 and Prado, 1989) it seems that the suboptimal solutions is very adequate, specially when a large number of "thrusting arcs" is used.

8. Acknowledgments

The authors are grateful to CNPq (National Council for Scientific and Technological Development – Brazil) and FAPESP (Fundação de Amparo a Pesquisa no Estado de São Paulo) for supporting this research.

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