

## EXPERIMENTAL ONE AXIS ATTITUDE DETERMINATION USING GPS CARRIER PHASE MEASUREMENTS

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**Abstract** *This paper presents experimental results for one axis (a base line with two antennas) attitude determination, through the design of an algorithm to estimate the attitude, using two GPS Ashtech Z12 receivers collecting phase measurements in dual frequency mode. The GPS antennas were placed at two IBGE (Brazilian Institute of Geography and Statistics) surveyed landmarks within INPE's ground, and collected carrier phase measurements at 1Hz rate for one hour. In this work, the initialization step, ambiguity resolution, and algorithm convergence are addressed. The assessment of the estimator is performed through comparison of the results with the solution based on the official IBGE landmarks. For terrestrial applications it can provide heading directions for cars, ships and airbornes.*

**Keywords.** *GPS, attitude determination, Ashtech Z12 receiver, integer ambiguity.*

### 1. Introduction

The Global Positioning System (GPS) is poised to revolutionize many fields of application. Extensive applications of GPS are accomplished due to technology development and consequently, reduction of receiver cost, power, size and weight. GPS originally was developed as a means for positioning and navigation. Nowadays, it can be used also as a means of attitude determination using the subcentimeter precision delivered by GPS carrier phase measurements, together with interferometry techniques. These generally involve some combination of the signal phases measured at multiple GPS receiver antennas such that the phase data from the various antennas and simultaneous GPS satellites are combined and processed to estimate the orientation of body axes with respect to some coordinate system. This paper presents experimental results for one axis (a base line with two antennas) attitude determination, through the design of an algorithm to estimate the attitude, using two GPS Ashtech Z12 receivers collecting phase measurements in dual frequency mode. The two GPS antennas were placed at two IBGE (Brazilian Institute of Geography and Statistics) surveyed landmarks within INPE's ground, and collected carrier phase measurements at 1Hz rate for one hour. In this work, the initialization step, ambiguity resolution, and convergence of the designed algorithm will be addressed. The assessment of the estimator will be performed through comparison of the results with the solution based on the official IBGE landmarks. For terrestrial applications it can provide heading directions for cars, ships and airbornes in. This work is a step forward to the development of a full three-axis attitude determination system based on GPS, envisaging future applications on a Brazilian satellite to be flown with on-board GPS receivers. The work is part of a doctor level current research.

This paper is organized as follows: brief description of the GPS system, the observables, introduction of the GPS based attitude determination, case studies description and finally simulation results using the data acquired from the Ashtech Z12 receivers.

### 2. GPS System

GPS is an all-weather, radio-based satellite navigation system established by the U.S. Department of Defense (DoD) to meet the military requirements of worldwide positioning, velocity determination and time keeping. The overall system is composed of three major segments: space segment, control segment and user segment. The space segment consists of 24 satellites which are deployed in six orbital planes inclined at 55 degrees, at 20000 km of altitude. The orbital period is 12 hours and the satellites are positioned in such a manner that at least five of them are normally visible by the user anywhere on the earth at any time. The control segment consists of five ground stations around the world. The main tasks of the control segment are to track and monitor the satellite performance, precisely compute satellite orbits and clock corrections, and periodically upload satellite ephemeris and other system data to all satellite for retransmission to the user segment. The user segment is the collection of all GPS users on the land, sea, air or space (Lu, 1994; Chu and Van Woerkom, 1997; Kaplan, 1996).

GPS is one-way ranging system, i.e., signals are only transmitted by the GPS spacecraft. The fundamental observable is the signal travel time between the transmitting antenna and the receiver antenna. The signal travel time is scaled into a range measurement using the signal propagation velocity (Lu, 1994).

The fundamental navigation principle of GPS is based on the measurement of the so-called pseudoranges between the user and at least four GPS spacecrafts. Starting from known GPS spacecraft position coordinates in a suitable reference frame, the position coordinates of the user antenna can be determined. From a geometrical point of view three range measurements are sufficient. A fourth observation is necessary because GPS uses the one-way ranging technique and the receiver clock is not synchronized with the GPS spacecraft clock. This synchronization error is the reason for the term pseudorange and can be reduced using measurements from four spacecrafts simultaneously (Lu, 1994).

### **2.1. GPS Observables and Data Processing**

Two frequencies are continuously transmitted from each GPS satellite for positioning: L1 at 1575.42MHz and L2 at 1227.60MHz, which are respectively 154 and 120 times the fundamental frequency of 10.23MHz. The pseudo random noise (PRN) and navigation messages are modulated on each frequency. Coarse acquisition (C/A) code is modulated on L1, while Precise (P) code is modulated on both frequencies.

The observations of the GPS are code observation, carrier phase observation and the broadcast ephemerides. The observed phase shift between two sequences of codes is a measure of the signal travel time between the GPS spacecraft and the receiver antennas. This technique is referred to as code phase observation.

The carrier phase observation is derived from a phase comparison between the receiver Doppler shifted carrier signal and the (nominally constant) receiver generated reference frequency (Chu and Van Woerkom, 1997). This observation has one problem called integer ambiguity that has to be solved. The ephemerides of the GPS satellites are in the navigation message that contains: time parameters, Keplerian elements, additional orbit perturbation parameters, health status of the GPS satellite and other information.

### **2.2. Error Sources in attitude determination**

Some error sources in GPS attitude sensing consist of multipath, line bias, receiver noise, phase centre variation and geometry of selected satellites (Purivigraipong et al, 1999).

Multipath is usually considered the dominant error source in GPS attitude determination. A Geometrical Theory Diffraction method can be used to model and verify the differential carrier phase error caused by multipath. However, multipath error can be mitigated by using signal to noise ratio information to correct multipath errors in carrier phase difference measurements or by using a calibration method (Purivigraipong et al, 1999).

Line bias is a phase offset between two antennas chain, caused simply by different length cables or by different RF front ends. Possible methods to remove line bias include the use of double phase differences or calibration signal source (Purivigraipong et al, 1999).

Antenna phase center is one factor that affects the carrier phase measurement. Generally, the geometrical center of the antenna does not coincide with the received point of the GPS signals on the antenna, namely the electrical phase center. Furthermore, the received signal point will vary with the elevation angle, and also from one antenna to another antenna (Purivigraipong et al, 1999).

The geometry of the GPS satellites is another factor in achieving high quality results. The geometry changes with time due to the relative motion of the GPS satellites. A measure for the geometry is generally described as the Dilution Of Precision (DOP) factor (Purivigraipong et al, 1999).

The paper concentrates in the attitude observation from interferometry using double differences of L1 carrier phase. In this context, integer ambiguity was solved by the quite simple antennas' swap method (Hwang, 1991). Multipath mitigation, as well as bias and phase center calibration procedures and integer ambiguity resolution algorithms have been investigated in other works (Lopes et al 2000; Lopes and Milani 2000; Lopes 2002) and shall be incorporated to the present research project in the next steps.

## **3. GPS Based Attitude Determination**

The fundamental principle of attitude determination with GPS and multiple antennas is shown in Fig. (1). The GPS satellite is so distant relative to the antenna separation that arriving wavefronts can be considered as effectively planar ones. A signal traveling at the speed of light arrives at the antenna closer to the satellite slightly before reaching the other. By measuring the difference in carrier phase between the antennas, a receiver can determine the relative range between the pair of antennas. With the addition of carrier phase measurements from multiple satellites using three or more antennas, the receiver can estimate the full three-axis attitude of an object (Parkinson and Spilker, 1996).

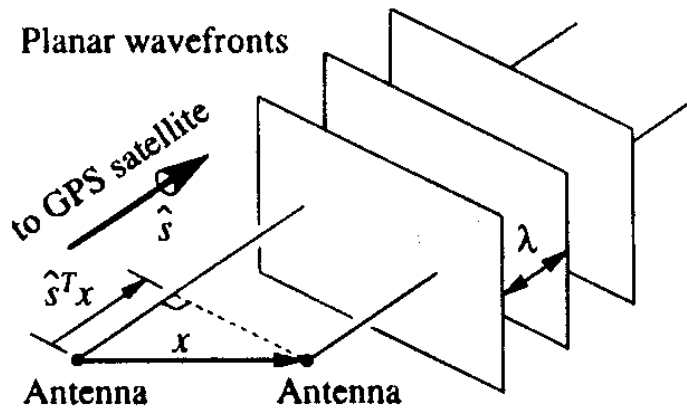


Figure 1 – Fundamental Principle of attitude determination with GPS (Parkinson and Spilker, 1996).

For GPS Based Attitude Determination, the most important measurement is L1 C/A code carrier phase, which is provided by most GPS receivers. The main difficulty related to the carrier phase method is the determination of the integer ambiguity, whereas the observable only determines the phase (and hence distance) within one wavelength. The integer ambiguity is an unknown number of wavelengths in the single difference of carrier phases. The integer ambiguity term has to be determined with appropriate methods. In this work, the strategy to solve the integer ambiguity problem is the antenna swap (Hwang, 1991), which is simple, suited, and compatible with the paper purpose.

### 3.1 Case Studies

The interest in this work is to estimate the one axis attitude that means azimuth ( $\alpha$ ) and elevation ( $\theta$ ), from a baseline vector of two antennas as defined in Fig. (2). The vector between antennas is the baseline vector defined as:

$$\hat{u} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} \quad (1)$$

The data from those antennas are acquired by two isolated GPS receivers Ashtech Z12. Antenna 1 is adopted as the origin of the topocentric reference system and antenna 2 is at the tip of the baseline vector that will be observed. Azimuth is defined as the angle formed by north and the vector projection in the horizon plane.

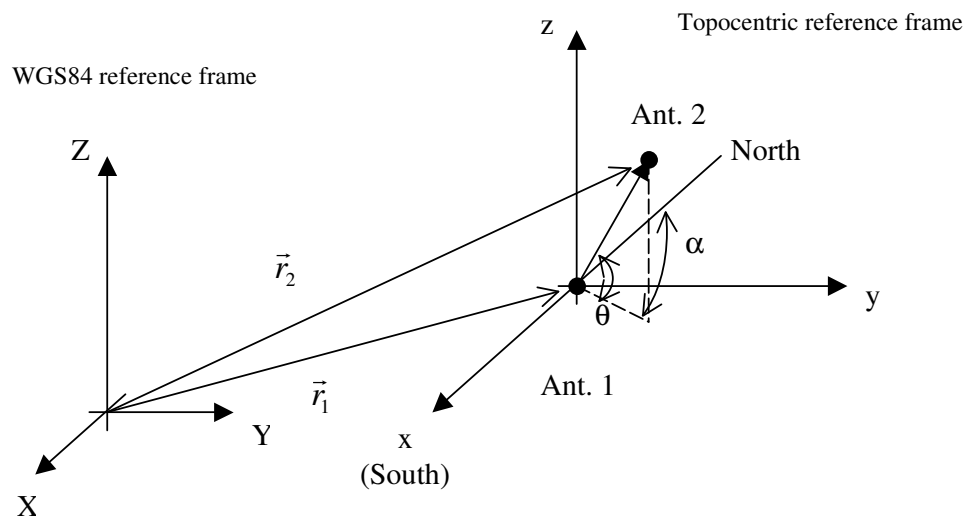


Figure 2 – Reference frames of the case studies.

With this definition, we can now establish the carrier phase measured by each antenna from each GPS satellite,  $\phi_i^n(t_k)$ , as defined by Fig. (3). There,  $n$  is the GPS satellite,  $i$  is the antenna number and  $t_k$  is the instant of the measurement.

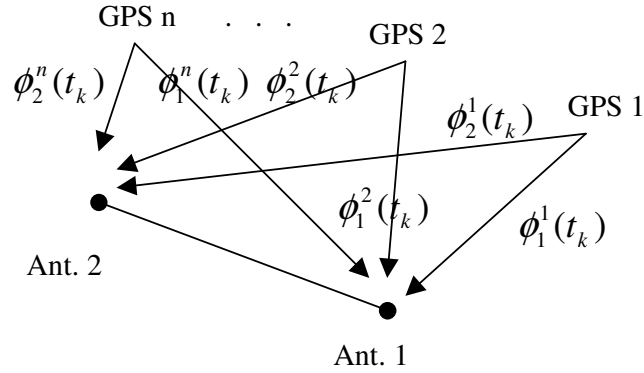


Figure 3 – Carrier phase definition.

Considering the antenna and the receiver configuration, the acquired data are not perfectly synchronized, because receivers do not use the same oscillator. The solution adopted for this condition was to use double differences and compensate for the Doppler on both receivers, resulting in measurements at the same instant.

#### 4. Problem Formulation

##### 4.1 Double difference and integer ambiguity

The carrier phase Single difference for each GPS satellite is defined as:

$$\Delta\phi_{1,2}^n = \phi_2^n - \phi_1^n \quad (2)$$

and the carrier phase Double difference is defined as:

$$\Delta\phi_{1,2}^{n,0} = \phi_2^0 - \phi_2^n - \phi_1^0 + \phi_1^n \quad (3)$$

where  $n$  is the  $n$ -th satellite and 0 means the reference satellite.

The antenna swap method (see for instance Hwang, 1991) to solve for the integer ambiguity problem related with the carrier phase Double difference can be described in two steps: the first step is to take two double differences points close to the antenna swap time, one before and the other after; and the second step is the computation of the integer ambiguity as the half part of the difference between double differences at those points. This solution has to be subtracted from double differences at every point.

##### 4.2 Attitude estimation

Considering that all necessary corrections on data are made, including ambiguity and other correction like cycle slips, the usual formulation to estimate one axis attitude can be established.

The equation that relates the observations (Double difference of L1 carrier phase) and the vector to be estimated is defined as:

$$y_k = h_k(u_k) + v_k \quad (4)$$

where,

$$y_k = \begin{bmatrix} \Delta\phi_{1,2}^{1,0} \\ \Delta\phi_{1,2}^{2,0} \\ \vdots \\ \Delta\phi_{1,2}^{n-1,0} \end{bmatrix}, \quad h_k = \begin{bmatrix} L(\hat{\rho}^1 - \hat{\rho}^0) \cdot u_k \\ L(\hat{\rho}^2 - \hat{\rho}^0) \cdot u_k \\ \vdots \\ L(\hat{\rho}^{n-1} - \hat{\rho}^0) \cdot u_k \end{bmatrix}, \quad (5)$$

$L$  is the distance between antennas,  $\hat{\rho}^n$  is the line of sight of the  $n$ -th GPS satellite in the Topocentric reference frame, and  $v_k$  represents a random measurement error with covariance matrix  $R$ .

No a priori estimate or dynamical model is considered. Independent estimates are obtained at each observation time by applying the least squares criterium. In this case, the error covariance matrix and the unconstrained estimate of the baseline vector are given by:

$$\hat{P}_k = (H_u^t R^{-1} H_u)^{-1} \quad (6)$$

$$\hat{u}_k = \hat{P}_k H_u^t R^{-1} y_k \quad (7)$$

whith  $H_u = \frac{\partial h_k(u_k)}{\partial u_k}$ . The baseline vector should be a unit vector with only two degrees of freedom. This is taken

into account by defining a state vector  $x_k = \{\alpha_k, \theta_k\}^t$ , where  $\alpha_k$  is the azimuth and  $\theta_k$  is the elevation. The baseline vector in the topocentric reference frame South-East-Zenith is a nonlinear function of the state vector:

$$u_k = \begin{Bmatrix} -\cos(\theta_k) \cos(\alpha_k) \\ \cos(\theta_k) \sin(\alpha_k) \\ \sin(\theta_k) \end{Bmatrix} \quad (8)$$

The new equation of the measurements results:

$$\delta y_k = H_k \delta x_k + v_k \quad (9)$$

where  $\delta y$ ,  $\delta x$  and  $H$  are given by:

$$\delta y_k = y_k - h_k(\bar{x}_k), \quad \delta x_k = x_k - \bar{x}_k, \quad H_k = H_u \left. \frac{\partial u_k}{\partial x} \right|_{x=\bar{x}_k}$$

and  $\bar{x}_k$  is the initial guess, obtained from the unconstrained estimate of the baseline vector,  $\hat{u}_k$ .

The equations for covariance and state estimation after linearization are:

$$\hat{P}l_k = (H^t R^{-1} H)^{-1} \quad (10)$$

$$\delta \hat{x}_k = \hat{P}l_k H_k^t R^{-1} \delta y_k \quad (11)$$

with the linearization the state estimate results:

$$\hat{x}_k = \bar{x}_k + \delta \hat{x}_k \quad (12)$$

## 5. Real Data Results

The formulation established before was implemented and the results will be shown next.

Table (1) summarizes the experiment conditions during the campaign to collect the measurements.

Table 1 – Experiment conditions.

Distance Between Antennas (L)	5.2m
Data rate	1 Hz
Data acquisition period	1900s
Antenna Swap epoch	Around instant 1700s
IBGE Reference antenna position (Vector $r_1$ )	Latitude: 23° 12' 40.40928" S Longitude: 45° 51' 38.38152" W Altitude: 612.0274 m

For simplicity and to increase figure visibility, only the latest 500 seconds, covering the swap of antennas, were considered for simulations.

To start the simulation some verifications were made to establish the correct range of measurements, for example: confirm that the same group of satellites are locked by both receivers at the same time and exclude satellites with low signal to noise ratio.

Figures (3) and (4) show respectively the Double Difference without any correction and shifted by an integer such that the first data of any satellite ranges from 0 to 1.

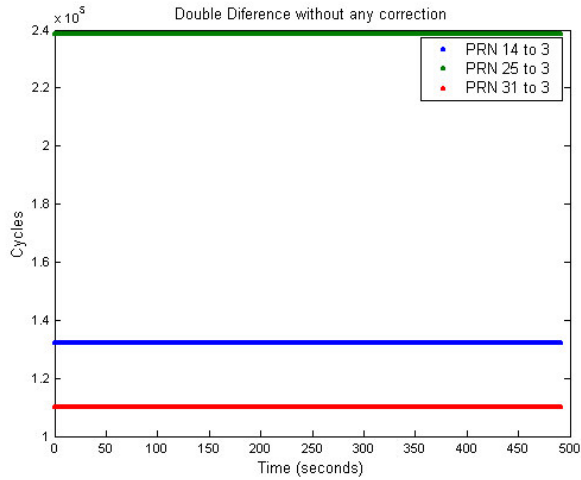


Figure 3 – Double difference without any correction .

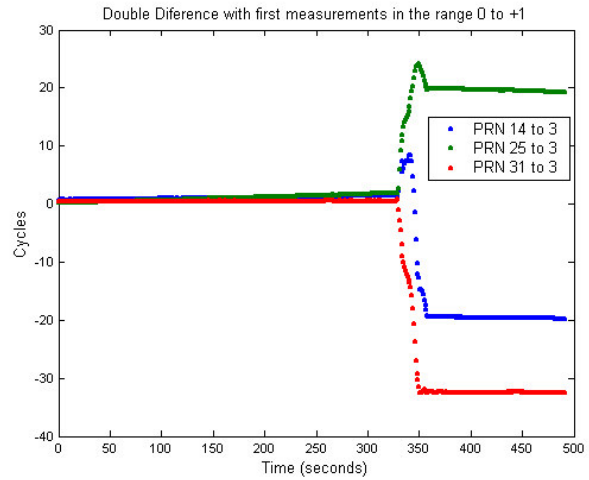


Figure 4 – Double difference in the range 0 to +1.

Figures (5) and (6) shows the double differences after integer ambiguity resolution ( $y$  in Eq. (4)) for the satellites in view: 14, 25 and 31, considering the satellite 3 as the reference satellite. In Fig. (6) it can be observed that the ambiguity was correctly solved, because the double difference curves are continuous. Figure (6) considers a signal inversion after the antennas swap.

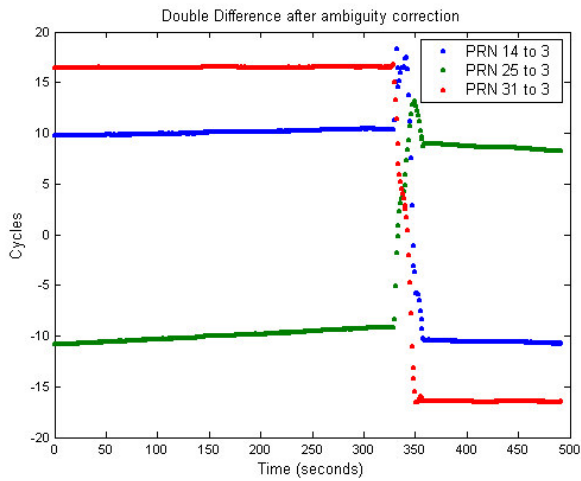


Figure 5 – Double difference.

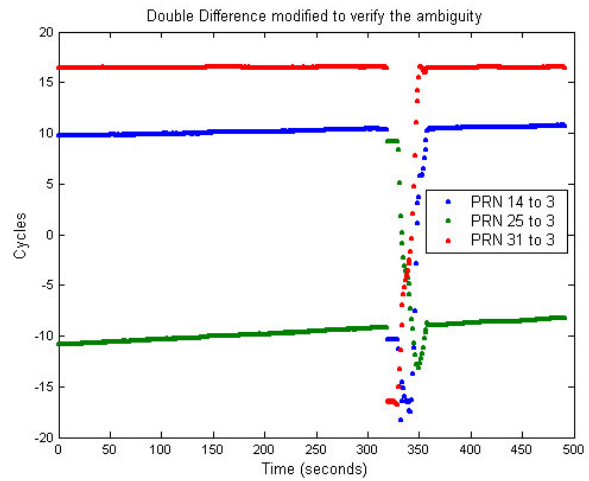


Figure 6 – Double difference (signal inversion after swap).

The magnitude of the unconstrained estimate of the baseline vector can be observed in Fig. (7), and in Fig. (8) it can be seen the correspondent initial guess of the state estimation process (on azimuth and elevation).

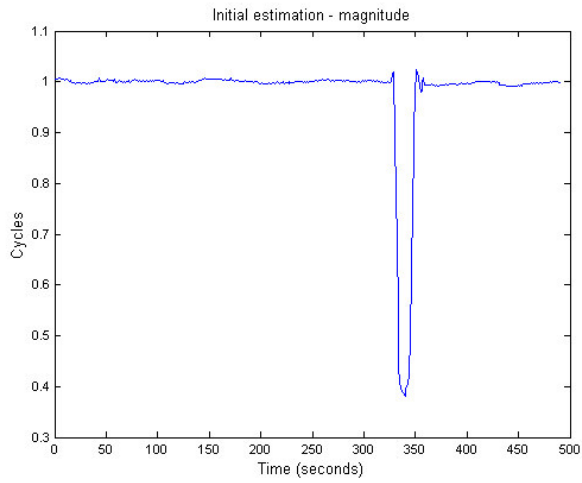


Figure 7 – Initial estimation – magnitude.

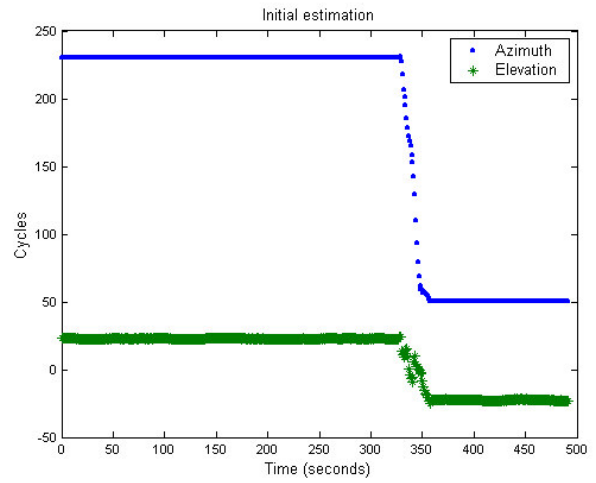


Figure 8 – Initial estimation.

Figure (9) shows the state estimates (Eqs. (9) to (12)). Figures (10) and (11) shows the estimated Azimuth and Elevation, adding 180 degrees to the azimuth and with a signal inversion in elevation after antennas swap, for better visualization. At the same graph it is shown the mean value and limits of one sigma.

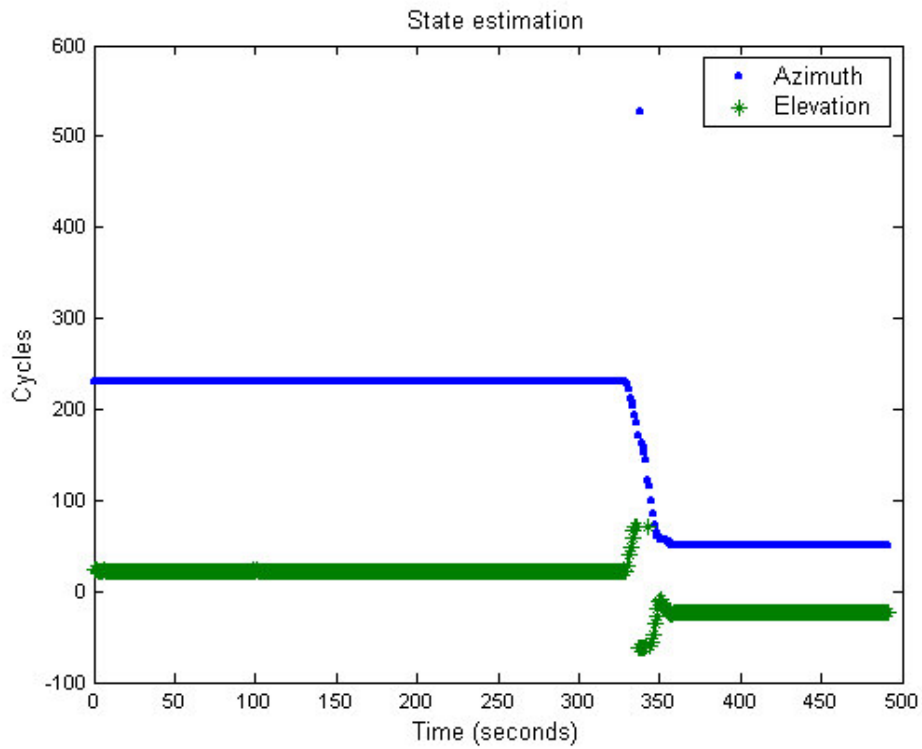


Figure 9 – State estimation.

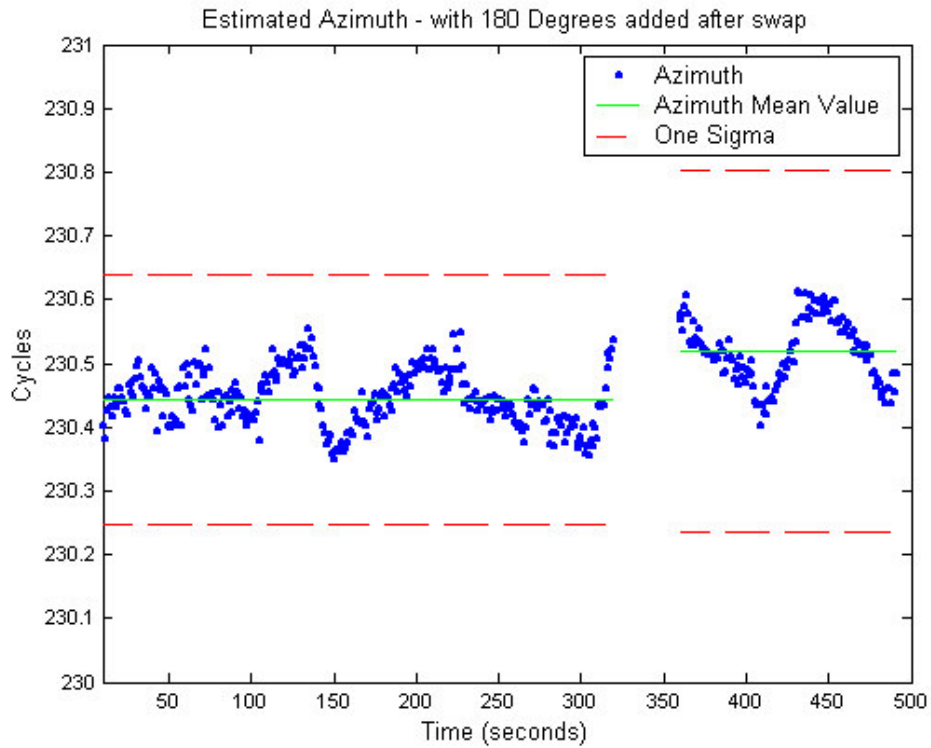


Figure 10 – Azimuth estimated with mean value and standard deviation.

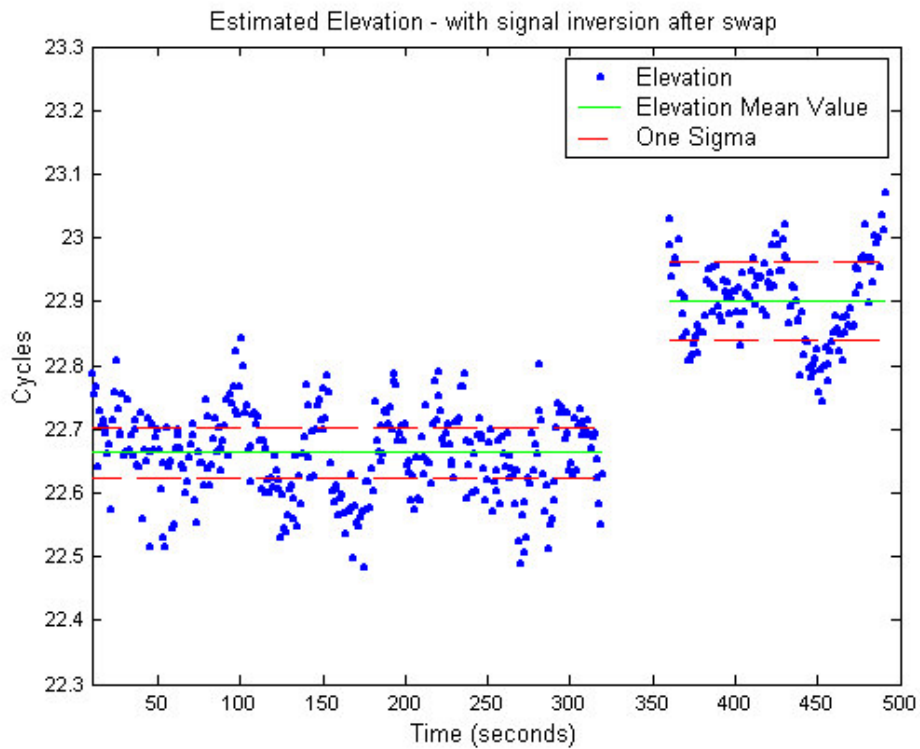


Figure 11 – Elevation with mean value and standard deviation.



## 6. Conclusions and Future Works

The results shown in Fig. (10) and (11) are very close to the expected values. For Elevation the expected value was 22.6 Degrees and for Azimuth there is not one measurement that can define the correct value, but the quadrant is correct comparing the results before and after the antennas swap with local antenna geometry.

In Fig. (10) and Fig (11) it can be seen that Azimuth and Elevation before and after antennas swap are slightly different. This difference probably is because the antennas position after the swap was not exactly interchanged to the same place or is not exactly 180 Degrees for Azimuth or signal change for Elevation. Even a slight inclination of one of the antennas can easily contribute for a fraction of degree offset in elevation. The comparison results also that Azimuth has better precision than Elevation, as elevation variation is not inside one sigma, after swap. The azimuth differs around 0.07° before and after swap, whereas elevation difference is around 0.24°. In any case, these deviations are well within the expected level of accuracy for such a preliminary experiment and represent a promising result to further developments.

This work showed an application of GPS for one axis attitude determination as a first step envisaging future applications on three-axis attitude determination of satellites. Next steps include multipath mitigation, bias calibration, others methods for integer ambiguity resolution and real time processing capability.

## 7. Acknowledgement

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## 8. References

- Chu, Q.P. and Van Woerkom, P.Th.L.M., 1997, "GPS For Low-Cost Attitude Determination", *Acta Astronautica*, Vol. 41, Nos.4-10, pp.421-433.
- Hwang, P. Y. C. 1991, "Kinematic GPS for Differential Positioning: Resolving Integer Ambiguities on the Fly." *Journal of The Institute of Navigation*, Vol. 38, No. 1, pp. 205-219.
- Kaplan, E.D.1996, "Understanding GPS, Principles and applications", Artech House, INC., London, 554p.
- Lopes, R. V. F. "Integer Ambiguity Resolution for Spacecraft Attitude Determination Using GPS." *Proceedings of ION GPS 2002*, pp. 1088-1093. Portland, OR, USA, 24-27 Sept., 2002.
- Lopes, R. V. F.; Carrara, V.; Enderle, W. and Arbinger, C. "Mitigating Multipath by Neural Network." (AAS 00-207) *Advances in the Astronautical Sciences*, Vol. 105, Part II, pp. 1639-1650, 2000.
- Lopes, R. V. F. and Milani, P. G. "Consistent On-Board Multipath Calibration for GPS Based Spacecraft Attitude Determination." *Proceedings of ION GPS 2000*, pp. 2216-2226. Salt Lake City, USA, 19-22 Sept., 2000.
- Lu, G.,1994, "Development of a GPS Multi-Antenna System for Attitude Determination", PhD Thesis, Department of Geomatics Engineering, Calgary, Alberta, Canada, 179 p.
- Parkinson, B.W., Spilker, J.J.,1996, "Global Positioning System: Theory and Applications", Vol. II, American Institute of Aeronautics and Astronautics, INC, Washington, DC, 643p.
- Purivigraipong S., Hodgart, M.S., Hashida, Y., Unwin, M.J., 1999, " New approach to Achieving Stand Alone GPS Attitude Determination Using Dual Short Baselines for Small-Satellite", 13<sup>th</sup> Annual AIAA/USU Conference on Small Satellites, Utah, USA, 11p.