

REAL TIME ESTIMATION OF GPS RECEIVER CLOCK OFFSET BY THE KALMAN FILTER

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Abstract. *The global positioning system (GPS) is a one-way, all-weather, real time, and world-wide radio navigation system whose main purpose is to provide a GPS dedicated receiver with a signal that allows a user to compute accurately and in real-time the longitude, latitude, altitude and precise time. Since precise time is an integral part of GPS, a large community of precise time, time interval, and frequency standard users has come to depend on GPS as primary source of time broadcasting and control. The accuracy to which the GPS receiver clock offsets relative to GPS system time are known to the user, or the accuracy to which satellite-to-user propagation errors are compensated for, are of extreme importance. In this paper, a Kalman filter has been applied to estimate in real time the receiver clock offsets, taking into account that GPS receivers may introduce discontinuous changes (jumps) in the clock time to keep the offsets within prescribed tolerances. For this work, a receiver of geodetic quality provided GPS measurement data with 1Hz of sampling rate, collected continuously along half an hour. The results were analyzed and its performance assessed in terms of accuracy, filter tuning parameters, and the suitability of the jump detection scheme.*

Keywords: *clock offset, GPS receiver, real time, Kalman filter*

Introduction

The Global Positioning System (GPS) is an Earth-orbiting satellite constellation based radio navigation utility, providing users worldwide with twenty-four hour a day accurate three-dimensional position (latitude, longitude, and altitude), velocity and precise time traceable to Coordinate Universal Time (UTC). The satellite constellation, the so-called Space Segment of the GPS, consists of twenty-one active satellites and three in-orbit operating spares. The GPS satellites transmit radio frequency signals toward the Earth, containing the required information for the user equipment to make use of GPS. The user equipment needs receiver/processors specifically designed to receive, decode, and process the GPS satellite ranging codes and navigation data messages. The system was funded, designed and still controlled and operated by US military, but the service is nowadays open to civilian users.

GPS position determination is based on the concept of time of arrival (TOA) ranging (Kaplan, 1996). TOA concepts involves transmitting a signal at a known time and measuring the arrival (reception) of that signal at a later known time. This time interval, the signal propagation time or the TOA value, is then multiplied by the speed of the signal to obtain the emitter-to-receiver range. By measuring the propagation time of signals broadcast from multiple emitters at known locations, the receiver can determine its position. Each measured distance (pseudorange) can be related to three unknown position coordinates by an equation. Given the range measurements to three objects, there are three such equations that can usually be solved for three unknowns. The time of measurements must be very accurate, or positioning accuracy will not be possible.

In order to measure the true transit time of a signal from a satellite to a receiver, the clocks in the satellite and the receiver must be maintained in synchronism. Each satellite typically carries a pair each of cesium and rubidium atomic standards, and receivers uses inexpensive quartz oscillators. The receiver time is determined by keeping a running count of how many cycles are generated by the oscillator.

Synchronization of receiver time with GPS time does not occur until the receiver locks onto its first satellite. The GPS L1 signal has two main streams of data modulated on the carrier. These data are the C/A and P codes. Additionally, a navigation message contains GPS satellite data including the ephemeris, clock corrections and constellation status. This navigation message is encoded on both the C/A and P codes. The navigation message is transmitted via individual subframes. Each subframe contains the transmit time of the next subframe in seconds of GPS time of week. After the first subframe is collected and decoded by the receiver, an approximate calculation of the receiver clock offset can be made. The receiver clock offset is the difference between GPS time and internal receiver time. The calculation is based on subframe transit time and the approximate propagation time from satellite to the receiver. The position of the satellite and receiver clock offset are used to reinitialize the seconds counter on the receiver, resulting in receiver/GPS time synchronization. This initial synchronization is referred to as coarse time set (Novatel, 1997).

Once enough satellites have been acquired to calculate the antenna position, a more accurate estimate of the receiver clock offset is calculated. The new receiver clock offset is used to synchronize the receiver clock even closer to GPS time. This is referred to as fine time set (Novatel, 1997).

The bias in the receiver clock at the instant of the measurements affects the observed transit times for all satellites equally, then it becomes the fourth unknown to be estimated, in addition to the three coordinates of position. A user, therefore, needs a minimum of four satellites in order to estimate it's four-dimensional position: three coordinates of spatial position plus time (Enge and Misra, 1999). If dt is the clock error and c is the speed of light, then every range at specific instant will be wrong by a distance $c\,dt$. The range calculated erroneously is called pseudorange.

Although GPS description is focused as a system to provide precise spatial coordinates and to determine speed, GPS is also a source of precise time, time interval and frequency. GPS can deliver time information anywhere in the world ultimately with up to nanoseconds (ns) of precision (Hewlett Packard), which is more than adequate for most applications.

The accuracy to which the GPS receiver clock offsets are known to the user, or the accuracy to which satellite-to-user propagation errors are compensated for, are of extreme importance. In GPS surveying, the code observations (pseudoranges) are often used to estimate the three coordinates and the timewise sequence of receiver clock offsets (changing with time). In this paper, a Kalman filter has been applied to estimate in real time the receiver clock offsets, taking into account that GPS receivers may introduce discontinuous changes in the clock time to keep the offsets within prescribed tolerances (Strang and Borre, 1997). Certain receivers, like the ones used in this experiment, have their clocks reset when the offset approaches one millisecond. In this case, the Kalman filter technique is proposed to provide a solution for real time clock offset estimation. For this work, GPS measurements data were provided by a GPS receiver of geodetic quality (Ashtech Z12), with 1Hz of sampling rate, collected continuously along half one hour.

Receiver clock error modelling

The receiver clock offset, needs a model of the crystal clock error. A typical model consists of an offset, b, and a drift, d, components (Grewall et alli, 2001). A discrete random ramp can be used to describe the behavior of the receiver clock (Strang and Borre, 1997), with the state defined by $\boldsymbol{x}_k = [b_k \quad d_k]^T$. The drift $\dot{d}_k = w_d$ is modeled as white noise, and the bias $\dot{b}_k = d_k + w_b$, modeled as random walk. The noise vector is composed of $\boldsymbol{w}_k = [w_b \quad w_d]$ where w_b and w_d are independent zero-mean white noise variances.

However, in this paper we adopt a different approach, where the receiver clock error is modeled as a bias described by a sum of white noise components. In this case, the state vector takes the form $\boldsymbol{x}_k = [x_L \quad x_{L-1} \quad \cdots \quad x_1 \quad x_0]$, so that $\dot{x}_L = \dot{x}_{L-1} = \cdots = 0$. This approach is detailed in the sequel.

Kalman Filter Design

The Kalman filter is a set of mathematical equations that provides an efficient computational (recursive) solution for the problem of real time state estimation. In this paper, linear and continuous dynamical model is considered for the GPS receiver clock offset mode. Nonetheless, an adequate implementation of the filter equations is required when a clock reset takes place in the GPS receiver.

Measurement model at time t_k is given by:

$$\boldsymbol{y}_k = \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{v}_k \tag{1}$$

with

$$\boldsymbol{v}_k = \mathrm{N}(0, \boldsymbol{R}_k(\mathrm{t})) \tag{2}$$

where $\boldsymbol{y}_k$ is the actual m measurements vector, $\boldsymbol{H}_k$ is the $m \times n$ observation matrix, $\boldsymbol{x}_k$ is the n-state vector of the system, $\boldsymbol{v}_k$ is the m-white gaussian measurement noises and $\boldsymbol{R}_k$ is the $m \times m$ observation noise covariance matrix, all terms at time t_k.

The proposed model to be tested is governed by a polynomial equation:

$$\boldsymbol{y}_\mathrm{k} = \mathrm{x}_\mathrm{L}\mathrm{k}^\mathrm{L} + \mathrm{x}_\mathrm{L\text{-}1}\mathrm{k}^\mathrm{L\text{-}1} + \cdots + \mathrm{x}_1\mathrm{k}^1 + \mathrm{x}_0\mathrm{k}^0 + \boldsymbol{v}_k \tag{3}$$

Then, according to (1):

$$\boldsymbol{H} = \begin{bmatrix} \mathrm{k}^\mathrm{L} & \mathrm{k}^\mathrm{L\text{-}1} & \cdots & \mathrm{k}^1 & \mathrm{k}^0 \end{bmatrix} \tag{4}$$

$$\boldsymbol{x} = \begin{bmatrix} \mathrm{x}_\mathrm{L} & \mathrm{x}_\mathrm{L\text{-}1} & \cdots & \mathrm{x}_1 & \mathrm{x}_0 \end{bmatrix} \tag{5}$$

The dynamic model for the state is defined as:

$$\dot{\boldsymbol{x}} = \boldsymbol{F}\boldsymbol{x} + \boldsymbol{G}\boldsymbol{w} \tag{6}$$

$$\boldsymbol{w} = \mathrm{N}(0, \boldsymbol{Q}(\mathrm{t})) \tag{7}$$

where $\boldsymbol{x}$ is the time variant continuous state, $\boldsymbol{F}$ is the $n \times n$ matrix, $\boldsymbol{G}$ is the $n \times m$ process noise transformation matrix, $\boldsymbol{w}$ is the white Gaussian process noise, and $\boldsymbol{Q}$ is the $n \times n$ process noise covariance matrix**.**

The time update equations of the Kalman filter are summarized as:

$$\dot{\overline{\boldsymbol{x}}} = \boldsymbol{F}\overline{\boldsymbol{x}} \tag{8}$$

$$\dot{\overline{\boldsymbol{P}}} = \boldsymbol{F}\overline{\boldsymbol{P}} + \overline{\boldsymbol{P}}\boldsymbol{F}^t + \boldsymbol{G}\boldsymbol{Q}\boldsymbol{G}^t \tag{9}$$

where $\boldsymbol{P}$ is the covariance matrix, and the respective initial conditions are coming from the measurement update cycle of the Kalman filter:

$$\overline{\boldsymbol{x}}_\mathrm{k\text{-}1} = \hat{\boldsymbol{x}}_\mathrm{k\text{-}1} \tag{10}$$

$$\overline{\boldsymbol{P}}_{k\text{-}1} = \hat{\boldsymbol{P}}_{k\text{-}1} \tag{11}$$

Equation (9) is referred to as the matrix Riccatti equation. Since the proposed problem modeling is simple ($\boldsymbol{F} = \boldsymbol{0}$), it can be solved analytically, otherwise numerical techniques are available for more complex problems (Gelb, 1974).

For this particular problem, initial estimates for $\hat{\boldsymbol{x}}_o$ and $\hat{\boldsymbol{P}}_o$ are:

$$\hat{x}_o = y_1, \quad \hat{x}_i = 0, \quad i = 1, \ldots, L \tag{12}$$

$$\hat{\boldsymbol{P}}_\mathrm{o} = diag\left\{\sigma^2_{x_o}, \cdots, \sigma^2_{x_L}\right\} \tag{13}$$

where y_1, the first measurement, defines the *a priori* bias term $\hat{x}_o$, and the other state components are null. The initial covariance matrix is initialized diagonal.

The measurement update equations and clock reset detection scheme are now introduced. For $k \in \begin{bmatrix} 1 & \cdots & m \end{bmatrix}$:

$$\Delta_\mathrm{k} = \mathrm{y}_\mathrm{k} - \boldsymbol{H}_\mathrm{k}\overline{\boldsymbol{x}}_\mathrm{k} \tag{14}$$

where Δ_k denotes the residue (innovation sequence) at sampling time t_k. If $\Delta_k > \varepsilon$, with $\varepsilon < 1$ milliseconds then a discontinuity or jump is detected and

$$\bar{x}_o(t_k) = y_k\,, \quad \bar{x}_i(t_k) = \hat{x}_i(t_{k-1}),\ i = 1,\ldots,L$$

$$\overline{\boldsymbol{P}}_k = \begin{bmatrix} \sigma^2_{x_o}(t_o) & 0 & 0 & 0 \\ 0 & X & X & X \\ 0 & X & X & X \\ 0 & X & X & X \end{bmatrix} \tag{15}$$

that is, the bias term estimate is reset to the measurement value y_k, and the other state estimates keep their values. The covariance matrix, keeps the original values except for the bias term variance which is set to the initial value $\sigma^2_{x_o}(t_o)$ of (13), and corresponding row and column correlations are zeroed. Therefore, state and covariance are reset to this new condition. It is noted that at a discontinuity detection the bias error will naturally increase and therefore part of the covariance matrix is re-initialized to reflect this event. Then the measurement update cycle of the Kalman filter follows:

$$\boldsymbol{K_k} = \overline{\boldsymbol{P}}_{\boldsymbol{k}} \boldsymbol{H}^{\boldsymbol{t}}_{\boldsymbol{k}} (\boldsymbol{H_k} \overline{\boldsymbol{P}}_{\boldsymbol{k}} \boldsymbol{H}^{\boldsymbol{t}}_{\boldsymbol{k}} + \boldsymbol{R_k})^{-1} \tag{16}$$

$$\hat{\boldsymbol{P}}_{\boldsymbol{k}} = (\boldsymbol{I} - \boldsymbol{K_k H_k})\overline{\boldsymbol{P}}_{\boldsymbol{k}} \tag{17}$$

$$\hat{\boldsymbol{x}}_{\boldsymbol{k}} = \bar{\boldsymbol{x}}_{\boldsymbol{k}} + \boldsymbol{K_k}(\boldsymbol{\Delta_k}) \tag{18}$$

where $\hat{\boldsymbol{x}}_k$ is the updated estimate state vector, $\hat{\boldsymbol{P}}_{\mathrm{k}}$ is the updated estimate *n x n* error covariance matrix, and $\boldsymbol{K}_{\mathrm{k}}$ is the Kalman gain.

Measurement Error and Covariance Tuning

The measurement noise $\boldsymbol{R}$, defined by Eq.(2), is simply a function of the measurement accuracy and resolution of the receiver being used. It is derived from the Geometric Dilution of Precision (GDOP) parameter from the receiver navigation solution. GDOP is a composite measure reflecting the geometry on the position and time estimates or the relationship between the errors in the pseudorange (Spilker, 1996):

$$\mathrm{GDOP} = \left(\sigma_x^2 + \sigma_y^2 + \sigma_z^2 + \sigma_b^2\right)^{1/2} / \sigma \tag{19}$$

where $\sigma_x, \sigma_y, \sigma_z, \sigma_b$ are the standard deviations in the user position coordinates (*x, y, z*) and the user clock offset *b*, and σ denotes the standard deviation for a typical pseudorange measurement. Unfortunately receivers not necessarily provide the DOP related to time (TDOP). Thus, assuming that the σ_i's have all the same magnitude σ_b, one can perform the following approximation:

$$\sigma_{b_k} = \sigma\,(\mathrm{GDOP})_k / 2 \tag{20}$$

i.e., the standard deviation of the measurement is taken from the value of GDOP at time t_k. Therefore the noise covariance matrix may be formed stepwise by:

$$\boldsymbol{R} = \begin{bmatrix} \sigma^2_{y_1} & 0 & \cdots & 0 \\ 0 & \sigma^2_{y_2} & \cdots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma^2_{y_m} \end{bmatrix} \tag{21}$$

where σ_{y_i} is computed at each instant by (20).

The process noise covariance matrix $\boldsymbol{Q}$ is used to represent the modeling errors. A “good” estimate of the matrix is made, and this value is then adjusted (tunned) until the best performance is obtained. The matrix $\boldsymbol{Q}$ has the following form:

$$\boldsymbol{Q} = diag\{q_{ii}\},\ i = 0, \ldots, L \tag{22}$$

Kalman Filter Results

Data were provided by twin Ashtech Z12 GPS receivers, called *base* and *user*. They were positioned 5.2 m apart and both data were collected at 1Hz rate for half an hour. No peculiar feature was noticed during the campaign, and both receivers were exposed to the same environmental condition. At first glance a linear trend seemed enough to model the drift history of the clock offset. However it was seen that one of our twin receivers did not present a characteristical linear trend and thus first and second order polynomials were used as measurements model.

The Kalman filter initial parameters were set to:

$$\hat{\boldsymbol{x}}_o = (0,\ y_1) \quad \textit{for}\ \text{linear case}$$
$$\hat{\boldsymbol{x}}_o = (0,\ 0,\ y_1) \quad \textit{for}\ \text{parabolic case}$$

$$\hat{\boldsymbol{P}}_o = diag\left\{(100\text{m/s})^2,\quad (100\text{m})^2\right\} \quad \textit{for}\ \text{linear case}$$
$$\hat{\boldsymbol{P}}_o = diag\left\{(100\text{m/s}^2)^2,\quad (100\text{m/s})^2,\quad (100\text{m})^2\right\} \quad \textit{for}\ \text{parabolic case}$$

The process noise covariance matrix $\boldsymbol{Q}$ were chosen diagonal, on a trial and error basis, and all diagonal terms were set to $(10^{-5})^2$.

Results are presented in Fig. 1-3, where dashed lines denote the standard deviations of measurement error, and solid lines denote the clock offset residuals (measured – estimated values).

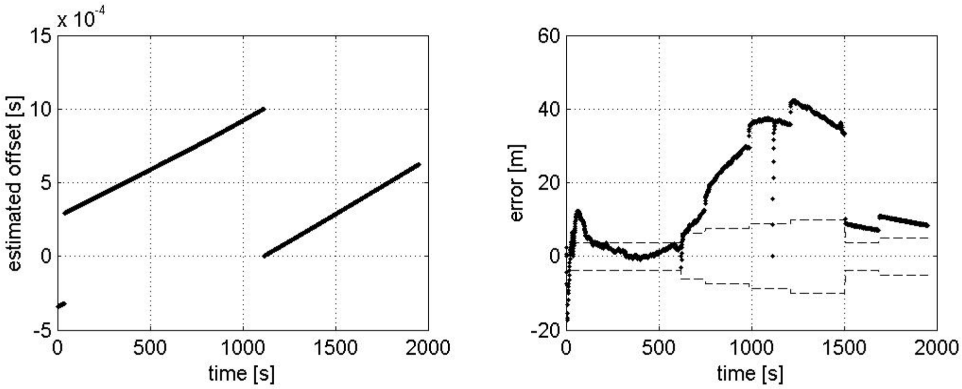

Figure 1: First order polynomial model results (*base receiver*): on left the estimates of the clock offset, on right the residuals and standard deviation of measurements

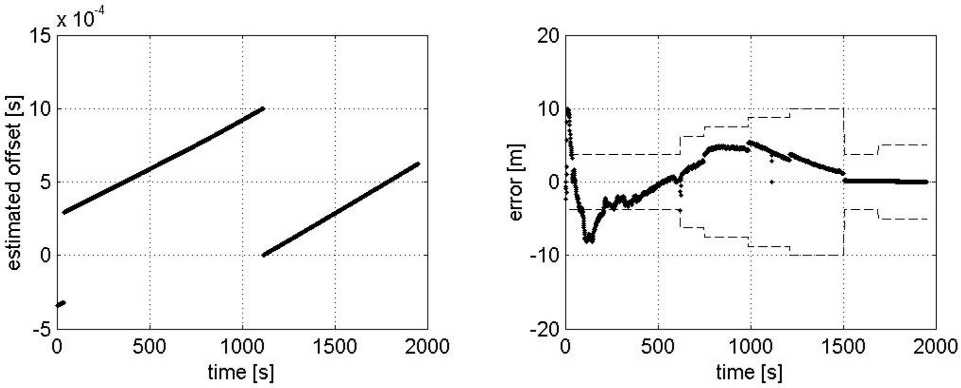

Figure 2: Second order polynomial model results (*base receiver*): on left the estimates of the clock offset, on right the residuals and standard deviation of measurements

Figures 1 and 2, on the left, shows the estimated offset discontinuities of approximately 1 ms on the base receiver. The first order model presents errors with magnitudes greater than the values predicted by measurement standard deviation (dashed lines). The second order model presents better results with errors confined to less than 10 m, which corresponds to an accuracy better than 30 ns. In Figures 1 and 2, it is noted that after the jump around 1100 s, the residue goes close to zero and then back to former levels. This is due to the way the state and covariance right after the jump was reset. In the state, only the first coefficient $\hat{x}_o$ (bias term) is changed; whereas in the covariance matrix, only the variance corresponding to bias term is set to its initial original value, and the correlations are zeroed, according to (15).

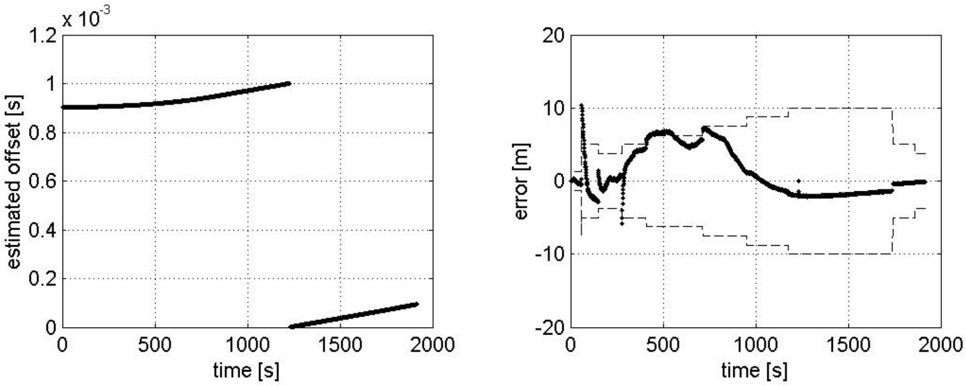

Figure 3: Second order polynomial model results (*user receiver*): on left the estimates of the clock offset, on right the residuals and standard deviation of measurements

Figure 3 shows the results of another set of data from the twin receiver (user receiver). On the left, the estimated offset is characterized by a curve slightly parabolic and shows the discontinuity, also of approximately 1 ms. By using the same set of Kalman filter tuned parameters the residuals were also confined to less than 10 m.

In addition there were discontinuities steaming from sources other than the clock reset. For example, at around 1500 s in Figures 1 and 2 on the right, the GPS constellation change caused a rather large discontinuity. Mostly constellation change, but also weak/strong signal to noise ratio, low/high GPS satellite elevation above horizon, and multipath noise amplification induce not smooth (discontinuous) clock offset estimate, which can be clearly seen at several points of the data collection interval.

At 1 Hz sampling rate, the filter did not have any problems to accomplish real time processing. The scheme developed assures also that the filter is quite insensitive to jumps, converging and preserving the level of accuracy through the discontinuities. It is remarked that the second receiver (user) presented a non-linear trend in the clock offset (Figure 3 on left) which could not be suitably modeled by a linear model. A second order polynomial provided a better performance for both receivers without any significant computer overload, being considered a rather simple and direct way of delivering quick real time clock offset estimates.

Conclusions

One approach to estimate receiver clock offset combining Kalman filtering and GPS receiver clock offset reset detection scheme were presented. A linear model and a second order model were tested for two sets of data taken from two GPS receivers. The simulation results show better performance with the second order polynomial model, where the offset errors were limited by 1 standard deviation most of the time. In both sets of distinct receivers data (base and user), the maximum errors occur in the beginning of the estimation, with magnitudes around 10m or $3.3x10^{-8}$s. In the whole time interval, the procedure delivered smooth accuracy through the discontinuities with accuracy better than 30 ns, when using the second order model. Unfortunately a longer batch of data without jumps could not be exercised in this work. It is proposed a future investigation using this long batch, with in turn would provide us with a sample large enough to extract meaningful statistics of the measurement residuals as well as depicting a long term filter behavior.

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References

Enge P., Misra P., 1999, “Scanning the Issue/Technology: Special Issue on Global Positioning System”, Proceedings of IEEE, vol. 87, n°. 1, january 1999.

Gelb, A., 1974, “Applied Optimal Estimation”, MIT Press, Cambridge, MA,USA.

Grewal, M. S.; Weill, L. R.; Andrews A. P., 2001, “Global Positioning Systems, Inertial Navigation, and Integration”, John Wiley & Sons.

Hewlett Packard, “GPS and Precision Timing Applications”, Application Note 1272.

Kaplan, Elliot D., 1996, “Understandig GPS: principles and applications”, Artech House, Inc., Norwood, MA, USA.

Novatel Technical Bulletin, 1997, “Receiver Time, GPS Time, Clock Steering and the 1 PPS Strobe”, Novatel OEM series APN-015 rev 1.

Spilker, J.J., 1996, “Satellite Constellation and Geometric Dilution of Precision. In: Parkinson, B.W., Spilker, J.J., Global Positioning System: Theory and Applications”, American Institute of Aeronautics and Astronautics, vol.I.

Strang G., Borre K., 1997, “Linear Algebra, Geodesy, and GPS”, Wellesley-Cambridge Press, MA, USA.