Comparison of Bootstrap Current Models in a Self-Consistent Equilibrium Calculation for Tokamak Plasmas

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1. Introduction. Different bootstrap current formulations are employed in a self-consistent equilibrium for tokamak plasmas where the total plasma current profile is supposed to have contributions of the diamagnetic, Pfirsch-Schlüter, and the neoclassical ohmic and bootstrap currents [1]. A comparison among the different bootstrap current models is performed for a variety of plasma parameters of the small aspect ratio tokamak ETE (*Experimento Tokamak Esférico*) [2], placed at the Associated Plasma Laboratory of INPE, in Brazil. The calculations described here were performed using the Mathematica package in a PC type computer.

2. Equilibrium and Self-Consistent Calculation. A self-consistent equilibrium calculation, valid for arbitrary aspect ratio tokamaks, is obtained through a direct variational technique where the extremum of the functional given by the internal plasma energy represents the flux surface averaged Grad-Shafranov equation (GS) being, in this way, a solution to the plasma equilibrium. This extremum establishes a relation between the total poloidal current, $I(\rho)$, between the symmetry axis and a given magnetic surface, denoted by the radial coordinate ρ, and the total plasma current enclosed by this surface, $I_T(\rho)$, as follows [1]:

$$I_T(\rho)\frac{dI_T}{d\rho} = -K(\rho)\frac{dL}{d\rho}I(\rho)\frac{dI}{d\rho} - K(\rho)\frac{dV}{d\rho}\frac{dp}{d\rho}\ , \tag{1}$$

where $p(\rho)$ is the total plasma pressure profile and $L(\rho)$, $V(\rho)$ and $K(\rho)$ are, respectively, the inductance of the toroidal solenoid coincident with a given flux surface, the volume involved by this surface and the inverse kernel used to calculate the self-inductance of the plasma loop. The self- consistent calculation requires that the current profile which satisfies Eq. (1), has also to be given by the sum of all current components, described by:

$$-I\frac{dI}{d\rho} = \frac{I_T(\rho)}{K(\rho)}\left[\frac{\left\langle \vec{J}_{oh}\cdot\vec{B}\right\rangle}{\left\langle B^2\right\rangle}I + \frac{\left\langle \vec{J}_{bs}\cdot\vec{B}\right\rangle}{\left\langle B^2\right\rangle}I\right] + \frac{\mu_0 I^2}{\left\langle B^2\right\rangle}\frac{dp}{d\rho}\ , \tag{2}$$

where the first term on the *rhs* of Eq.(2) refers to the ohmic current (oh) modified by neoclassical effects through the plasma conductivity, the second term to the bootstrap current (bs) calculated through different models as described in the next section and the last term results from a combination of the diamagnetic (dia) and the Pfirsch-Schlüter (ps) currents. The brackets in Eq.(2) refer to the usual flux surface average and $\vec{B}$ is the total induction in the plasma. The self-consistent equilibrium is obtained when Eq.(2) is solved iteratively with Eq.(1) as described in [1]. The loop voltage that enters in the ohmic current calculation is determined consistently in order to reproduce the prescribed value of the total plasma current.

3. Bootstrap Current Models. Different bootstrap current models are listed in Table 1 and classified according their validity regarding aspect ratio, collisionality or plasma impurity level. The full matrix Hirshman-Sigmar (H-S) formulation for the bootstrap current estimate is based on the solution of the system formed by the flux surface average of the parallel momentum and heat flow balance equations for each species in the plasma, as follows [3]:

$$\mu_{a1}\left\langle u_{a//}\,B\right\rangle-\mu_{a1}\,V_{a1}\,B+\frac{2}{5}\mu_{a2}\left\langle\frac{q_{a//}}{p_a}B\right\rangle-\mu_{a2}\,V_{a2}\,B=\sum_b\left(l_{11}^{ab}\left\langle u_{b//}B\right\rangle-\frac{2}{5}l_{12}^{ab}\left\langle\frac{q_{b//}}{p_b}B\right\rangle\right) \quad (3)$$

$$\mu_{a2}\left\langle u_{a//}\,B\right\rangle-\mu_{a2}\,V_{a1}\,B+\frac{2}{5}\mu_{a3}\left\langle\frac{q_{a//}}{p_a}B\right\rangle-\mu_{a3}\,V_{a2}\,B=\sum_b\left(-l_{21}^{ab}\left\langle u_{b//}B\right\rangle+\frac{2}{5}l_{22}^{ab}\left\langle\frac{q_{b//}}{p_b}B\right\rangle\right) \quad (4)$$

The solution of this system provides the parallel fluid $<u_{a//}B>$ and heat $<2q_{a//}/5p_a>$ flows for each species in terms of the thermodynamic flows (described through V_{a1} and V_{a2}) and determines the bootstrap current through the expression $\langle J_{bs}\cdot B\rangle=\sum_a n_a e_a \langle u_{a//}\,B\rangle$, where n_a and e_a are respectively the density and charge of species a. The Hirshman-Sigmar model is therefore suitable for a multi-species plasma treatment since the equations in the system above are added accordingly as the number of plasma species increases. l_{ij}^{ab} are the friction coefficients, which are independent of the magnetic field configuration and so, valid in all collision frequency regimes. Their formulae are found in [3]. The problem is then to calculate the viscosity coefficients μ_{aj}, since they depend on the plasma geometry and collisionality. We will make a comparison among the H-S [3], Shaing [4] and Sauter´s [5] proposals regarding this calculation.

In order to compute the viscosity coefficients, the Hirshman-Sigmar model solves the linearized drift kinetic (ldk) equation asymptotically in each collisionality regime and proposes a formula continuously valid throughout the various regimes. However, in the plateau regime the result is valid only for large aspect ratios. Shaing solves the ldk equation where the mass velocity appears explicitly. The asymptotic values of his coefficients converge to the same values given by the Hirshman-Sigmar model in the Pfirsch-Schlüter and in the banana regimes, when the H-S banana coefficient is corrected by a $1/f_c$ factor in order to reproduce the correct limit in the finite aspect ratio banana regime (f_c is the fraction of circulating particles in the plasma). In the plateau regime Shaing´s coefficients are different from those derived by Hirshman-Sigmar. Both the H-S and Shaing´s models consider, however, an approximated Coulomb collision operator which can introduce errors up to 20% in the viscosity coefficients in the Pfirsch-Schlüter regime [3], which could be a problem mainly in trying to model the plasma edge. Finally, Sauter solves ldk´s equations for electrons and a single-ion species without considering plasma flows. In order to solve these ldk´s equations, Sauter uses an adjoint formalism and does not make any assumption regarding different collisionality regimes being, in this respect, completely general. Moreover, Sauter considers the full Coulomb collision operator and solves the adjoint equations varying aspect ratio, collisionality and ion charge Z. In this way, he proposes fitted formulae for the bootstrap current and for the neoclassical conductivity, whose expressions depend basically on these three parameters. For more accurate formulas regarding the ion charge dependence, Sauter should solve the adjoint equations for electrons and for each ion species as it is stressed in his paper. Multi-species effects are therefore approximately treated in his formulation by substituting the ion charge Z by the effective ion charge in the plasma (Z_{eff}). All the equations describing these models are found respectively in [3,4,5].

Models	Aspect Ratio	Collisionality	Impurities
Hinton-Hazeltine	large	all	Single-ion (Z_{eff})
Hirshman	all	banana regime	Single-ion (Z_{eff})
Hirshman-Sigmar	large	all	Multi-Species
Shaing	all	all	Multi-Species
Sauter	all	all	Single-ion (Z_{eff})

Table 1: Different formulations for the bootstrap current estimate

4. Results and Discussions. Bootstrap current estimates provided by the full matrix Hirshman-Sigmar model with the viscosity coefficients given by Shaing (H-S/Shaing model) and by the fitted formulae proposed by Sauter, both for the bootstrap current and for the neoclassical conductivity, are presented for the ETE low aspect ratio tokamak [2]. For Shaing´s model, we compute the neoclassical conductivity according to the Hirshman, Hawryluk and Birge (HHB) formulation [6]. In both cases, the trapped particle fraction is estimated according to Lin-Liu and Miller [7]. The main ETE parameters correspond to aspect ratio $A=R_0(a)/a=1.5$, major radius $R_0(a)=0.3$ m, plasma current $I_p=200$ kA, vacuum toroidal field $B_T=0.4$ T (at $R_0(a)=0.3$ m), and plasma beta during the ohmic phase $\beta=4$-10%. The pressure, electron and ion temperature profiles were taken as gaussian shaped functions as in $f(\rho) = f(0) \exp[-\alpha_f (\rho/(w_f-\rho))^2]$, where $f(0)$ determines the profile value at the axis, whereas α_f and w_f determine the value at the boundary. We have also considered $T_e(0) = T_i(0)$, $T_e(a) = T_i(a) = 0.03$ keV and $\alpha_{Te} = \alpha_{Ti} = 0.2$. As the pressure profile is given as a fixed input in our code, the density profiles were derived from the fact that the total pressure in the plasma is $p = n_e T_e + \sum_k n_k T_k$ and from the quasi-neutrality condition ($n_e = \sum_k n_k Z_k$), with the summations taken over all ion species. In Table 2, results provided by the self-consistent calculation for hydrogen plasmas in the ETE tokamak, lead to bootstrap current fractions roughly ranging from 10 to 35% for both models analysed according to the optimization level of the plasma parameters. We can also observe, by comparing cases II and V, that plasmas with higher shaped factors (higher elongation $\kappa(a)$ in our case) generate higher fractions of bootstrap current and that the presence of 2% of carbon in the plasma ($Z_{eff}=1.54$) causes a slight decrease in the bootstrap current fraction as noticed from cases II and III. Sauter predicts slightly higher fractions of bootstrap current in relation to the H-S/Shaing model in all cases analysed. The highest difference between the two models occurs for case IV (more collisional with ($T_e(0) = 200$ eV)). This may be due a combined effect resulting from the fact that Sauter uses the complete Coulomb collision operator whereas Shaing uses the approximated one, and that Shaing´s viscosity coefficients are derived from a different drift-kinetic equation which may result in differences in relation to Sauter´s, mainly in the transition banana-plateau or plateau-Pfirsch-Schlüter regimes.

	p(0) (kPa)	**$T_{e,i}(0)$ (keV)**	**α_p**	**$\kappa(a)$**	**$I_T(a)$(kA)**	**Z_{eff}**	**β_I**	**β_0**	**$I_{bsShaing}/I_T(a)$**	**$I_{bsFIT}/I_T(a)$**
I	15	0.5	3.0	2.0	200	1.54	0.54	0.09	**0.36**	**0.37**
II	10	0.4	3.0	2.0	200	1.00	0.36	0.06	**0.23**	**0.25**
III	10	0.4	3.0	2.0	200	**1.54**	0.36	0.06	**0.22**	**0.24**
IV	10	**0.2**	3.0	2.0	200	1.00	0.36	0.06	**0.14**	**0.20**
V	10	0.4	3.0	**1.7**	200	1.00	0.31	0.06	**0.17**	**0.19**
VI	10	0.4	3.0	**1.7**	**180**	1.54	0.38	0.06	**0.22**	**0.23**

Table 2: Plasma parameters for bootstrap current estimate provided by the self-consistent calculation

Figure 1 shows the total equilibrium current density profiles and all their different components for case II in Table 2, both for the bootstrap current obtained from the H-S/Shaing model (1a) and from the fitted formula proposed by Sauter (1b). For the H-S/Shaing model the neoclassical conductivity used in the ohmic current calculation is obtained from the HHB formulation [6] whereas in (1b) we have used Sauter´s fitted fomula [5]. Figure 2 shows the comparison of the bootstrap current profiles obtained from different models for cases II, III and IV. We can observe that, as the plasma becomes less collisional (higher temperatures in the cases shown) all the models approach each other. For figures 2a and 2b, the Hirshman collisionless model [8] always predicts more bootstrap current since it does not take properly into account the collisionality dependence on the viscosity coefficients. Still comparing figures 2a and 2b we see that as the Z_{eff} increases the H-S/Shaing and Sauter´s models approach each other. The strongest difference between the two predictions (fig. 2c) occurs for the most collisional case ($T_e(0) = 200$ eV) as previously mentioned. For the less collisional plasma (fig.2d) all the models agree quite well. The self-consistent calculation generated when the bootstrap current is determined by the fitted formulae is approximately four times faster to compute than that provided when the H-S/Shaing model is used. Finally, figure 3 shows the neoclassical and Spitzer conductivities calculated for the HHB formulation and from the fitted formula, for case II, after the self-consistent calculation is achieved and obtained for the bootstrap current calculated according to Sauter. The figure also shows the reduction of the neoclassical conductivity in relation to Spitzer´s.

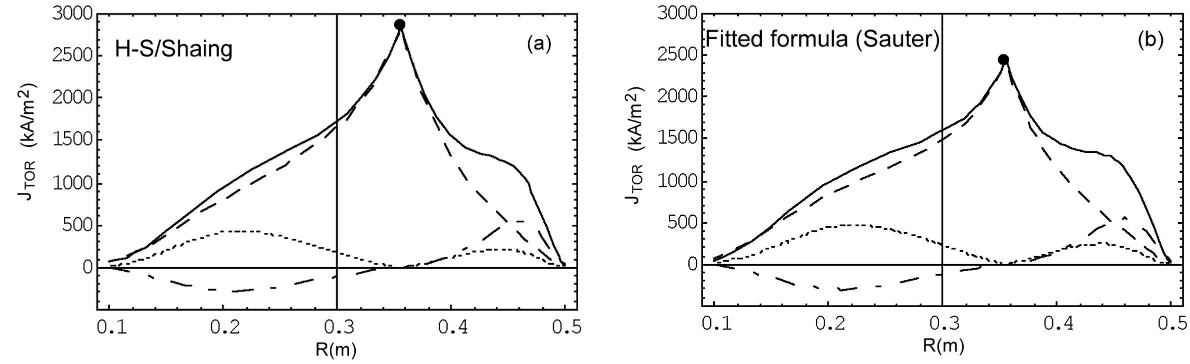

Figure 1: Current density profiles for case II in Table 2.The full line corresponds to the total equilibrium current density, the dashed line is the ohmic contribution, the dash-dotted line is the sum of the diamagnetic and Pfirsch-Schlüter components and the dotted line represents the boostrap current contribution.

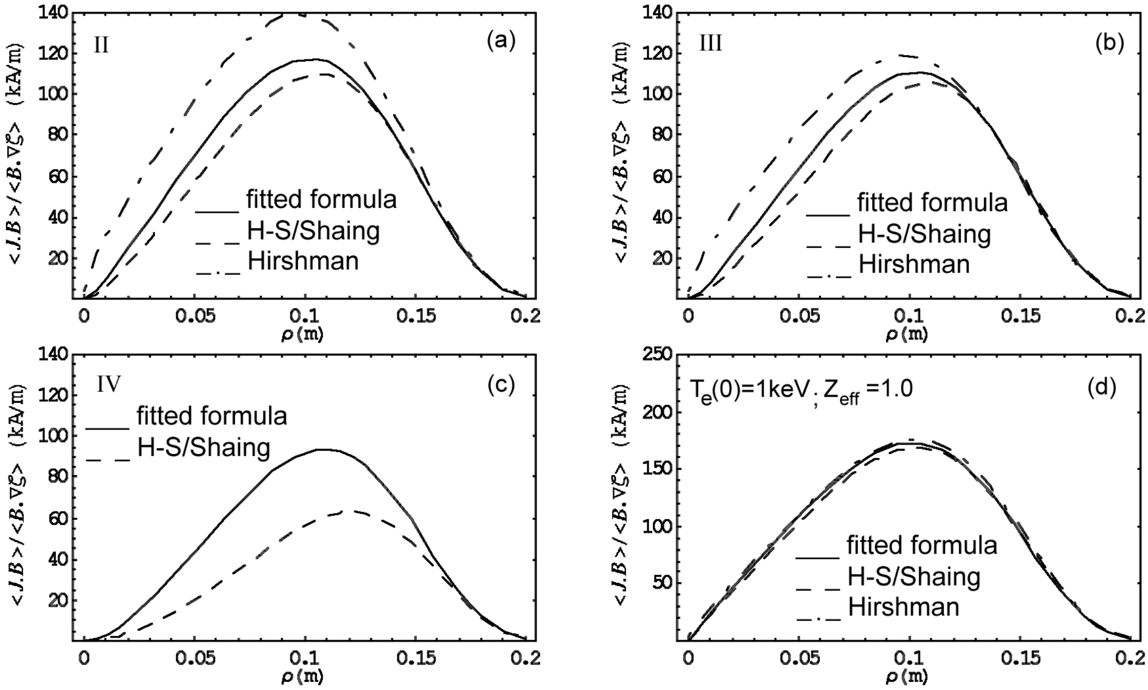

Figure 2: Bootstrap current profiles obtained from different models

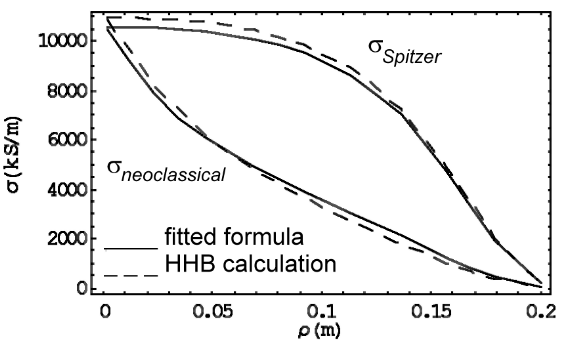

Figure 3: Neoclassical and Spitzer conductivities

References

[1] Andrade M.C.R., Ludwig G. O. and Camargo S.J., *Plasma. Phys. and Control. Fusion* **42**, 1269 (2000).
[2] Del Bosco E. et al, *Overview of the ETE Spherical Tokamak* in this workshop.
[3] Hirshman S.P, Sigmar, D. J., *Nuclear Fusion* **21**, 1079 (1981).
[4] Shaing K. C. et al, *Phys.Plasmas* **3**, 965 (1996).
[5] Sauter O. et al, *Phys. Plasmas* **6**, 2834 (1999).
[6] Hirshman S. P. et al, *Nuclear Fusion* **17**, 611 (1977).
[7] Lin-Liu Y. R. and Miller R. L., *Phys. Plasmas* **2**, 1666 (1995).
[8] Hirshman, S.P., *Phys. Fluids* **31**, 3150 (1988).