RADIO FREQUENCY WAVE DISSIPATION BY ELECTRON LANDAU DAMPING IN ELONGATED SPHERICAL TOKAMAKS

N.I. Grishanov[1], C.A. de Azevedo[2], G.O. Ludwig[3], J.P. Neto[1,2]

[1]*Laboratório Nacional de Computação Científica, 25651-070 Petrópolis, Brazil*
[2]*Universidade do Estado do Rio de Janeiro, 20550-013 Rio de Janeiro, Brazil*
[3]*Instituto Nacional de Pesquisas Espaciais, 12221-010 São José dos Campos, Brazil*

Spherical Tokamaks (or Low Aspect Ratio Tokamaks) represent a promising alternative route to magnetic thermonuclear fusion. In order to achieve fusion conditions in these devices additional plasma heating must be employed. Effective schemes of heating and current drive in tokamak plasmas can be realized using radio frequency waves. As is well known, the kinetic wave theory of any toroidal plasma should be based on the solution of the Vlasov-Maxwell's equations. However, this problem is not simple even in the scope of linear theory since to solve the wave (or Maxwell's) equations it is necessary to use the correct dielectric (or wave conductivity) tensor valid in the given frequency range for the realistic two- or three-dimensional plasma model. In this paper, the longitudinal permittivity elements are derived for radio frequency waves in a two-dimensional axisymmetric tokamak with elliptic magnetic surfaces, for arbitrary elongation and arbitrary aspect ratio. A high-temperature collisionless plasma model is considered. The drift-kinetic equation is solved separately for untrapped and usual *t*-trapped particles as a boundary-value problem, in the case when the so-called [6] *d*-trapped particles are absent in the plasma, using an approach developed [7] for Low Aspect Ratio Tokamak (LART) with circular magnetic surfaces. To describe an elongated tokamak we use the variables (r, θ') instead of quasi-toroidal co-ordinates (ρ, θ) [7]:

$$r = \rho \sqrt{\frac{a^2}{b^2} \sin^2\theta + \cos^2\theta}, \qquad \theta' = 2\arctan\left[\sqrt{\frac{1-\varepsilon}{1+\varepsilon}} \tan\left(\frac{1}{2}\arctan\left(\frac{a}{b}\tan\theta\right)\right)\right], \qquad (1)$$

transforming the initial elliptic cross-sections of the magnetic surfaces to circles with the maximum radius *a* of the external magnetic surface. In the (r, θ')-coordinates, the magnetic field lines become "straight", and the modulus of the equilibrium magnetic field, $H = |\mathbf{H}|$, is

$$H(r, \theta') = \sqrt{H_{\phi 0}^2 + H_{\theta 0}^2}\, g(r, \theta'), \qquad g(r, \theta') = \frac{\sqrt{(1 - \varepsilon\cos\theta')^2 + \lambda(\varepsilon - \cos\theta')^2}}{1 - \varepsilon^2}, \qquad (2)$$

where $\qquad \varepsilon = \dfrac{r}{R}, \qquad \lambda = h_\theta^2\left(\dfrac{b^2}{a^2} - 1\right), \qquad h_\phi = \dfrac{H_{\phi 0}}{\sqrt{H_{\phi 0}^2 + H_{\theta 0}^2}}, \qquad h_\theta = \dfrac{H_{\theta 0}}{\sqrt{H_{\phi 0}^2 + H_{\theta 0}^2}},$

$H_{\varphi 0}(r)$ and $H_{\theta 0}(r)$ are the toroidal and poloidal projections of $\mathbf{H}$ for a given magnetic surface at the points $\theta = \pm \pi/2$; R is the major tokamak radius; *b* and *a* are the major and minor semiaxes of the elliptical cross-section of the external magnetic surface. In this model, all magnetic surfaces are similar to each other with the same elongation equal to *b/a*.

To solve the drift-kinetic equation for plasma particles we use the standard method of switching to new variables associated with conservation integrals of energy, $v_\parallel^2 + v_\perp^2 = const$, and magnetic moment, $v_\perp^2 / 2H = const$. Introducing the variables *v* (particle energy) and μ (nondimensional magnetic moment) in velocity space instead of $v_\parallel$ and $v_\perp$:

$$v^2 = v_\parallel^2 + v_\perp^2, \qquad\qquad \mu = \frac{v_\perp^2}{v_\parallel^2 + v_\perp^2}\frac{\sqrt{H_{\phi 0}^2 + H_{\theta 0}^2}}{H(r, \theta')}, \qquad (3)$$

the perturbed distribution function of plasma particles (any kind of ions and electrons) can be found as

$$f(t,r,\theta',\phi,v_{\parallel},v_{\perp}) = \sum_{s=\pm 1} f_s(r,\theta',v,\mu)\exp(-i\omega t + in\phi),$$

where we have taken into account that the problem is uniform in both time t and toroidal angle ϕ. In zero order over the magnetization parameters (i.e., neglecting finite Larmor radius effects), the linearized drift-kinetic equation for harmonics f_s can be written as

$$\frac{(1-\varepsilon\cos\theta')^2}{(1-\varepsilon^2)^{1.5}}\frac{\sqrt{1-\mu\cdot g(r,\theta')}}{g(r,\theta')}\left(\frac{\partial f_s}{\partial\theta'}+inqf_s\right)-i\frac{sr\omega}{h_\theta v}f_s = 2\frac{erE_{\parallel}F}{Mh_\theta v_T^2}\sqrt{1-\mu\cdot g(r,\theta')}, \qquad (4)$$

where $\qquad F = \dfrac{N}{\pi^{1.5}v_T^3}\exp\left(-\dfrac{v^2}{v_T^2}\right), \qquad v_T^2 = \dfrac{2T}{M}, \qquad g = \dfrac{\varepsilon\cdot h_\phi}{h_\theta\sqrt{1-\varepsilon^2}},$

$E_{\parallel} = \mathbf{E}\cdot\mathbf{h}$ is the parallel (to $\mathbf{H}$) electric field component; the steady-state distribution function F is given as a Maxwellian with the particle density N, temperature T, charge e and mass M. The index of particles species (ions and electrons) is omitted in Eqs. (4). By the indexes $s=\pm 1$ for f_s, we distinguish the perturbed distribution functions with positive and negative values of the parallel velocity $v_{\parallel} = sv\sqrt{1-\mu\cdot g(r,\theta')}$ relative to $\mathbf{H}$.

Thus, the initial drift-kinetic equation is reduced to a first order differential equation with respect to the poloidal angle θ', where the variables r, v, μ (as well as R, a, b, q, N, T) appear as parameters. After solving Eq. (4), the longitudinal (parallel to $\mathbf{H}$) component of the current density $j_{\parallel} = \mathbf{j}\cdot\mathbf{h}$ can be expressed as

$$j_{\parallel}(r,\theta') = \pi eg(r,\theta')\sum_s^{\pm 1}s\int_0^\infty v^3\int_0^{1/g(r,\theta')}f_s(r,\theta',v,\mu)d\mu dv . \qquad (5)$$

Depending on μ and θ' the phase space of plasma particles must be split in the phase space of untrapped, t-trapped and d-trapped particles according to the following inequalities:

$$0\leq\mu\leq\mu_u \qquad -\pi\leq\theta'\leq\pi \qquad \text{- for untrapped particles} \qquad (6)$$

$$\mu_u\leq\mu\leq\mu_t \qquad -\theta_t\leq\theta'\leq\theta_t \qquad \text{- for } t\text{-trapped particles} \qquad (7)$$

$$\mu_t\leq\mu\leq\mu_d \qquad -\theta_t\leq\theta'\leq-\theta_d \quad \text{- for } d\text{-trapped particles} \qquad (8)$$

$$\mu_t\leq\mu\leq\mu_d \qquad \theta_d\leq\theta'\leq\theta_t \qquad \text{- for } d\text{-trapped particles} \qquad (9)$$

where [analyzing the condition $v_{\parallel}(\mu,\theta')=0$]

$$\mu_u = \frac{1-\varepsilon}{\sqrt{1+\lambda}}, \qquad \mu_t = \frac{1+\varepsilon}{\sqrt{1+\lambda}}, \qquad \mu_d = \varepsilon\frac{1+\lambda}{\lambda+\varepsilon^2},$$

and the reflection points $\pm\theta_t$ and $\pm\theta_d$ for t- and d-trapped particles are, respectively,

$$\pm\theta_t = \pm\arccos\left\{\frac{\varepsilon(1+\lambda)}{\lambda+\varepsilon^2}-\sqrt{\frac{\varepsilon^2(1+\lambda)^2}{(\lambda+\varepsilon^2)^2}-\frac{1}{\lambda+\varepsilon^2}\left[1+\varepsilon^2\lambda-\left(\frac{1-\varepsilon^2}{\mu}\right)^2\right]}\right\}. \qquad (10)$$

$$\pm\theta_d = \pm\arccos\left\{\frac{\varepsilon(1+\lambda)}{\lambda+\varepsilon^2}+\sqrt{\frac{\varepsilon^2(1+\lambda)^2}{(\lambda+\varepsilon^2)^2}-\frac{1}{\lambda+\varepsilon^2}\left[1+\varepsilon^2\lambda-\left(\frac{1-\varepsilon^2}{\mu}\right)^2\right]}\right\} \qquad (11)$$

Now, we solve Eq. (4) for untrapped and usual t-trapped particles only, under the condition when the d-particles are absent, i.e., considering the LART magnetic field configuration as a system with one minimum of $\mathbf{H}(r,\theta')$. In this case, the criterion [6] of d-particle existence, $\varepsilon < \lambda$ or $b/a > \sqrt{1+\varepsilon+q^2(1-\varepsilon^2)/\varepsilon}$, cannot be satisfied.

The solution of Eq. (4) must be found for the specific boundary conditions of the trapped and untrapped particles. For untrapped particles we use the periodicity of f_s over θ'. Whereas, the boundary condition for the t-trapped particles is the continuity of f_s at the corresponding stop-points, Eq. (10). As a result, we seek the perturbed distribution functions of untrapped, f_s^u, and t-trapped, f_s^t, particles as

$$f_s^u = \sum_p^{\pm\infty} f_{s,p}^u \exp\left[i2\pi(p+nq)\frac{\tau(\theta')}{T_u} - inq\theta'\right], \quad f_s^t = \sum_p^{\mp\infty} f_{s,p}^t \exp\left[i2\pi p\frac{\tau(\theta')}{T_t} - inq\theta'\right] \quad (12)$$

where p is the number of resonance bounces,

$$\tau(\theta') = \int_0^{\theta'} \frac{(1-\varepsilon^2)\cdot g(r,\eta)\cdot d\eta}{(1-\varepsilon\cos\eta)^2\sqrt{1-\mu g(r,\eta)}} \quad (13)$$

is the new time-like variable (instead of θ') describing the bounce-periodic motion of untrapped and t-trapped particles along the magnetic field line with the corresponding periods $T_u = 2\tau(\pi)$ and $T_t = 4\tau(\theta_t)$. The Fourier harmonics $f_{s,p}^u$ and $f_{s,p}^t$ for untrapped and t-trapped particles can be readily derived after the corresponding bounce averaging.

To evaluate the dielectric tensor elements we use the Fourier expansions of the current density and electric field over the poloidal angle θ':

$$\frac{j_\parallel(\theta')}{(1-\varepsilon^2)g(r,\theta')} = \sum_m^{\pm\infty} j_\parallel^m \exp(im\theta'), \quad E_\parallel(\theta')\frac{(1-\varepsilon^2)g(r,\theta')}{(1-\varepsilon\cos\theta')^2} = \sum_{m'}^{\pm\infty} E_\parallel^{m'} \exp(im'\theta') \quad (14)$$

As a result, the whole spectrum of electric field, $E_\parallel^{m'}$, is present in the given m-th harmonic $j_\parallel^m$ of the current density:

$$\frac{4\pi i}{\omega} j_\parallel^m = \sum_{m'}^{\pm\infty} \varepsilon^{m,m'} E_\parallel^{m'} = \sum_{m'}^{\pm\infty} (\varepsilon_{\parallel,u}^{m,m'} + \varepsilon_{\parallel,t}^{m,m'}) E_\parallel^{m'} \ , \quad (15)$$

where $\varepsilon_{\parallel,u}^{m,m'}$ and $\varepsilon_{\parallel,t}^{m,m'}$ are the separate contributions of untrapped and t-trapped particles, respectively, to the longitudinal (parallel) permittivity elements:

$$\varepsilon_{\parallel,u}^{m,m'} = \frac{\omega_p^2 r^2}{h_\theta^2 v_T^2 \pi^3} \sum_{p=-\infty}^{\infty} \int_0^{\mu_u} \frac{\tau(\pi) C_p^m C_p^{m'}}{(p+nq)^2}\left[1 + 2u_p^2 + 2i\sqrt{\pi}u_p^3 W(u_p)\right]d\mu , \quad (16)$$

$$\varepsilon_{\parallel,t}^{m,m'} = \frac{2\omega_p^2 r^2}{h_\theta^2 v_T^2 \pi^3} \sum_{p=1}^{\infty} \int_0^{\mu_t} \frac{\tau(\theta_t)}{p^2} D_p^m D_p^{m'}\left[1 + 2v_p^2 + 2i\sqrt{\pi}v_p^3 W(v_p)\right]d\mu . \quad (17)$$

Here we have used the following definitions

$$\omega_p^2 = \frac{4\pi Ne^2}{M} , \qquad u_p = \frac{r\omega\sqrt{1-\varepsilon^2}\,\tau(\pi)}{h_\theta\,|\,p+nq\,|\,v_T\pi} , \qquad v_p = 2\frac{r\omega\sqrt{1-\varepsilon^2}\,\tau(\theta_t)}{h_\theta\,p v_T\pi} ,$$

$$C_p^m = \int_0^{\pi} \cos\left[(m+nq)\eta - (p+nq)\pi\frac{\tau(\eta)}{\tau(\pi)}\right]d\eta , \quad W(z) = e^{-z^2}\left(1 + \frac{2i}{\sqrt{\pi}}\int_0^z e^{t^2} dt\right),$$

$$D_p^m = \int_0^{\theta_t} \cos\left[(m+nq)\eta - p\frac{\pi\tau(\eta)}{2\tau(\theta_t)}\right]d\eta + (-1)^{p-1}\int_0^{\theta_t} \cos\left[(m+nq)\eta + p\frac{\pi\tau(\eta)}{2\tau(\theta_t)}\right]d\eta .$$

It should be noted that Eqs. (16, 17) describe the contribution of any kind of untrapped and trapped particles to the dielectric elements. The corresponding expressions for plasma electrons and ions can be obtained from (16, 17) replacing T, N, M, e by the electron T_e, N_e, m_e, e_e and ion T_i, N_i, M_i, e_i parameters, respectively. To obtain the total expressions of the permittivity elements, as usual, it is necessary to carry out the summation over all species of plasma particles.

One of the main mechanisms of radio frequency plasma heating is the electron Landau damping of waves due to the Cherenkov resonance interaction of $E_{\parallel}$ with the trapped and untrapped electrons. Cherenkov resonance conditions are different for trapped and untrapped particles in the LART plasmas and have nothing in common with the wave-particle resonance condition in cylindrical magnetized plasmas. Another important feature of tokamak plasmas is the contributions of all $E_{\parallel}^{m'}$-harmonics to the given $j_{\parallel}^{m}$-harmonic, Eq. (15). As a result, after averaging in time and poloidal angle, the wave power absorbed, $P = \mathrm{Re}(E_{\parallel} \cdot j_{\parallel}^{*})$, due to trapped and untrapped electrons can be estimated by the expression

$$P = \frac{\omega}{8\pi} \sum_{m}^{\pm\infty} \sum_{m'}^{\pm\infty} \left(\mathrm{Im}\,\varepsilon_{\parallel,u}^{m,m'} + \mathrm{Im}\,\varepsilon_{\parallel,t}^{m,m'} \right) \left(\mathrm{Re}\,E_{\parallel}^{m}\,\mathrm{Re}\,E_{\parallel}^{m'} + \mathrm{Im}\,E_{\parallel}^{m}\,\mathrm{Im}\,E_{\parallel}^{m'} \right), \qquad (18)$$

where $\mathrm{Im}\,\varepsilon_{\parallel,u}^{m,m'}$ and $\mathrm{Im}\,\varepsilon_{\parallel,t}^{m,m'}$ are the contributions of untrapped and t-trapped electrons to the imaginary part of the longitudinal permittivity elements: $\mathrm{Im}\,\varepsilon_{\parallel}^{m,m'} = \mathrm{Im}\,\varepsilon_{\parallel,u}^{m,m'} + \mathrm{Im}\,\varepsilon_{\parallel,t}^{m,m'}$. In the simplest case of Toroidicity-induced Alfvén Eigenmodes (TAEs) [8], describing the coupling of only two harmonics with m_o and m_o-1, the width m_o, m_o-1 should also be accounted in Eq. (15) to estimate the TAEs absorption by the trapped and untrapped electrons. As a result, the dissipated power of TAEs by electron Landau damping is

$$P = \frac{\omega}{8\pi} \sum_{m=m_o-1}^{m_o} \mathrm{Im}\,\varepsilon_{\parallel}^{m,m'}\,|\,E_{\parallel}^{m}\,|^2 + \frac{\omega}{4\pi} \mathrm{Im}\,\varepsilon_{\parallel}^{m_o,m_o-1} \left(\mathrm{Re}\,E_{\parallel}^{m_o}\,\mathrm{Re}\,E_{\parallel}^{m_o-1} + \mathrm{Im}\,E_{\parallel}^{m_o}\,\mathrm{Im}\,E_{\parallel}^{m_o-1} \right) \qquad (19)$$

where $|\,E_{\parallel}^{m}\,|^2 = (\mathrm{Re}\,E_{\parallel}^{m})^2 + (\mathrm{Im}\,E_{\parallel}^{m})^2$. Note that the non-diagonal elements $\varepsilon_{\parallel}^{m,m'}\,|_{m \neq m'}$ are characteristic only of toroidal plasmas. For the one-mode (cylindrical) approximation, when $m=m'=m_o$, the non-diagonal elements vanish, i.e., $\mathrm{Im}\,\varepsilon_{\parallel}^{m,m'}\,|_{m \neq m'} = 0$, and Eqs. (18, 19) reduce to the well-known expression

$$P = \frac{\omega}{8\pi} \mathrm{Im}\,\varepsilon_{\parallel}^{m,m}\,|\,E_{\parallel}^{m}\,|^2 .$$

The longitudinal permittivity elements evaluated in this paper are suitable for both large and low aspect ratio tokamaks with elliptic magnetic surfaces and valid in a wide range of wave frequencies, mode numbers, and plasma parameters. The expressions (16, 17) have a natural limit to the corresponding results [7] for LART plasmas with circular magnetic surfaces, if b=a and $\lambda \to 0$. Since the drift kinetic equation is solved as a boundary-value problem, the longitudinal permittivity elements (16, 17) are suitable for studying wave processes with a regular frequency, such as wave propagation and dissipation during plasma heating and current drive generation, with wave frequency given, e.g., by the antenna-generator system.

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