NUMERICAL AND SEMI-ANALYTICAL PROPAGATION OF A SPACE DEBRIS DISTRIBUTION FOR COLLISION AVOIDANCE WITH CONSTELLATIONS

Marcelo Lopes de Oliveira e Souza, E-mail: marcelo@dem.inpe.br
National Institute for Space Research-INPE
Danton Nunes, E-mail: danton@inexo.com.br
InterNexo Ltda.
Sandro Felgueiras Castro, E-mail: sandro-ita05@bol.com.br
Anderson Patrick Alves Pereira, E-mail: patrick@h8.ita.br
Technological Institute of Aeronautics -ITA

ABSTRACT

This work intends to study, model and simulate the numerical and semi-analytical propagation of a space debris distribution moving around Earth for collision avoidance with constellations. To do that, we simulated numerically the fragmentation of a body and the propagation of its debris under a central gravitational field with a program called KK written in C language, for the operational system UNIX, as a basis for these simulations. Then we adapted the program KK to run under the operational system Windows 2000 with the help of the MS Visual C++ 6.0 program from the MS Visual Studio 6.0 environment. Besides that, we adapted the KK program data outputs to make it compatible with the MATLAB environment, to use its capacity of analysis. On the other hand, we infer some of its fundamental properties that can even be proved analytically. They also allow us to propagate the distribution of debris for more time and with less errors than the standard covariance analysis method. Towards this, we are formulating a sequence of analytical (geometrical, kinematic, etc.) models of propagation and we are comparing their analytical previsions with the numerical results.

In agreement with the specific objectives of this research project, we obtained the following results:

1) To observe and to interpret the basic properties of such process.

2) To calculate the statistics of the space debris distribution, to study its time evolution, beginning with the position of it Center of Mass – CM and the spread around it.

3) To test simple analytical models (geometrical, kinematic, etc.) for a space debris distribution and its evolution.

4) To compare their previsions with the simulated ones to improve the analytical,models.

Based on these data, we intend later, to study the advanced properties of such process to calculate the probability of collision avoidance of such space debris distribution with some object of importance and size. So, it will be possible to analyze better the problems of collisions and/or interferences with other objects found in space as satellites, constellations, the Space Shuttle or the International Space Station.

INTRODUCTION

Most space missions need orbit transfers to reach their operational configuration and begin its useful life. These trajectories/orbits are reached sequentially through transfers between them by changing their Keplerian elements, by firing apogee motors or other sources of force. These thrusters may leave particles, pieces of combustibles, and even burnt stages along its track. Along the lifetime of space vehicles similar phenomena may generate such particles/objects. After its useful life, the vehicle itself or parts of it caused by fragmentation or even explosion of remaining fuels may generate more objects. Part of all these artificially generated objects may remain orbiting for a long time in useful orbits and thus risking collisions with other space vehicles in the same orbits. These and other undesirable objects are called **artificial space debris**. For all those reasons and sources, artificial space debris are becoming an increasingly important field of study, ranging from the most theoretical to the most practical aspects.

Processes that generate particles/objects are not new in Nature. They are natural, well established, ethernal processes and occur in regions so distant of Earth that they usually have no human interest. But they may be important for space missions if there is any chance of interference on these missions. Then, these and other undesirable objects may be called **natural space debris.** Among them, asteroid belts, comet comas, meteoroid fragmentation, etc. are known and studied by astrophysicists, astronomers, etc. Along time, these scientists developed a rich literature to study the naturally generated particles/objects. This literature is an invaluable source of inspiration/adaptation for studying the artificially generated ones. Both families are constituted by particles/objets moving in a gravitational field. However the artificial ones do not need to consider neither their mutual interaction nor a long scale of time. Their similarities and differences require us to review that literature, as done ahead.

One very practical aspect of these phenomena is the propagation of their trajectories: a) forwards (direct problem) to check risk of collision with satellites, constellations, or a big object such as the Space Shuttle or the International Space Station; or b) backwards to find their common origin (inverse problem), if any. This may be done under a number of approaches and considering initial and/or forcing uncertainties. One of them is the probabilistic approach. Its most used version is the (linearized) covariance analysis that, as such, may not always fit.

The first goal of this paper is to show how traditional covariance matrix propagation is not always fit for the purpose of forecasting either the distribution of space debris or (which turns out to be equivalent) the probability of finding a body drifting on a gravitational field with incomplete knowledge of its initial conditions.This is shown via: a) Figures 1-20 for initial uncertainties; b) a theoretical discussion on them. Other goals are: to show some properties and methods of solution.

REVISION OF THE LITERATURE

The mathematical treatment for initial and/or forcing uncertainties is done by complementary approaches (deterministic, probabilistic, minimax, phase space, etc.), in at least 3 specialized literatures:

In the space engineering literature, we highlight: Souza et alli(1998) reviewed it on the 3 first approaches, from deterministic works by Schwende and Strobl(1977), Tandon(1988), Longuski, Kia and Breckenridge(1989), Rodrigues(1991), Santos-Paulo(1998), to probabilistic works, specially in its most used version (linearized covariance analysis), by: Porcelli and Vogel(1980) presented an algorithm for the determination of the orbit insertion errors in biimpulsive noncoplanar orbital transfers (perigee and appogee), using the covariance matrices of the sources of errors. Adams and Melton(1986) extended such algorithm to ascent transfers under a finite thrust, modeled as a sequence of impulsive burns. They developed an algorithm to compute the propagation of the navigation and direction errors among the nominal trajectory, with finite perigee burns. Kuga e Gill(1991) applied UD covariance factorization to orbit de termination and apogee boost maneuver estimation. Rao(1993) built a semi-analytic theory to extend covariance analysis to long-term errors on elliptical orbits. Howell and Gordon(1994) also applied covariance analysis to the orbit determination errors and they develop a station-keeping strategy of Sun-Earth L1 libration point orbits. Junkins et alli(1996) and Junkins(1997) discussed the precision of the error covariance matrix method through nonlinear transformations of coordinates. He also found a progressive deformation of the initial ellipsoid of trajectory distribution (due to gaussian initial condition errors), that was not anticipated by the covariance analysis of linearized models with zero mean errors. Carlton-Wippern(1997) proposed differential equations in polar coordinates for the growth of the mean position errors of satellites (due to errors in the initial conditions or in the drag), by using an approximation of Langevin's equation and a first order perturbation theory. Alfriend(1999) studied the effects of drag uncertainty via covariance analysis.

In the stellar dynamics literature we highlight: Contopoulos(1964) reviewed it and the problems of stellar dynamics so far. He emphasizes that three different techniques have been used in most work in stellar dynamics, namely: a) the N-body problem approach, b) the continuum approach; c) the statistical approach. Chandrasekhar(1942, 1960) set fundamental ideas and methods on the principles of stellar dynamics in his classical book, emphasizing his own work on stellar systems. According Contopoulos(1964) "he finds first a complete formula for the time of relaxation of a stellar system, and then he discusses at lenght the quadratic solutions of Liouville's equation. The Dover edition contains his discussion of dynamical friction and an exposition of his views concerning the statistical mechanics of stellar systems." Hameen-Anttila(1975, 1976) set up a pioneer work on the statistical mechanics of Keplerian orbits, first considering identical particles. Later on, he extended it by considering the dispersion in particle size.

In the space debris literature we highlight: Johnson and McKnight(1991) reviewed it and the field in a pioneer book. Langebartel(1964) "applied Liouville's theorem to the dispersion of particles in Hamiltonian systems and showed the utility of introducing generalized functions into the analysis", according to Heard(1976a). Dasenbrock and Kaufman(1975, 1976) analyzed the satellite disintegration dynamics in complementary ways. Heard(1976a) generalized and extended the work of Langebartel(1964) to deal with the dispersion of ensembles of non-interacting particles. Heard(1976b) analyzed the breakup of satellite 1974-103A (COSMOS 699) applying the previous technique. Heard(1977) analyzed the asymptotic distribution of particles from fragmented celestial bodies, complementing the previous technique. Heard(1978) conducted a survey of a dynamical theory of satellite breakups until then.

Most of these and later works tried to solve the partial differential equation for the debris density function, resulting from applying a theorem by Liouville(1838) to the specific dynamics and boundary conditions. Covariance analysis is an approximate way of characterizing such debris density function., but that may not fit it, as shown in Figures 1-20 for uncertain initial conditions.

The main limitation of covariance matrix propagation comes from the fact that the debris density function is poorly described by the two first moments alone, even if the initial density function is spherical. Given enough time, the chaotic nature of the motion under gravity stretches and bends the initial debris distribution into distorted and growing shapes (called by us **"bananoids"**) that the two lowest moments can no longer model it. To illustrate this fact we analyzę below a very simple model composed of an almost massless body under the action of a central gravitational force to show that, even under such favorable conditions (no atmospheric effects, no non-gravitational forces, no random forces), covariance propagation fails miserably.

MATHEMATICAL FORMULATION AND COORDINATE SYSTEMS

In the 3-dimensional case, we can model a cloud of space debris as an ensemble of N non-interacting particles/objects moving in a central gravitational field, each of them subject to the dynamics in inertial coordinates O X_i, Y_i, Z_i given by Newton's laws, as done by Contopoulos(1964). In the 2-dimensional case ($Z_i \equiv 0$, dropping i) studied here, such dynamics become:$\forall t \in [t_0,t_f]$,

$$m.\dot{v}_t(t) = T.\cos\alpha(t) - m.v_r(t).\dot{f}(t) \qquad (1)$$

$$\dot{f}(t) = \frac{v_t(t)}{r(t)} \qquad (2)$$

$$\dot{r}(t) = v_r(t) \qquad (3)$$

$$m.\dot{v}_r(t) = T.\mathrm{sen}\,\alpha(t) - \frac{\mu.m}{r^2(t)} + m.v_t(t).\dot{f}(t) \qquad (4)$$

which are the transverse and radial components of an external force actuating on each particle, with initial conditions given below. The other variables are: T= magnitude of the applied force; $\dot{v}_t(t), \dot{v}_r(t)$ = transverse and radial accelerations; $v_t(t), v_r(t)$ = transverse and radial velocities; $\dot{f}(t)$ = angular velocity; r(t) = position vector from the center O to the particle with mass m.

To model a radial explosion T(0) =β.R.δ(t), we will take the parent body as an homogeneous rotating disk in the plane X_i, Y_i that is fragmented in N equal particles. Each of these particles has: a) initial position vector (just after the explosion) equal to its final position in the body (just before the explosion); b) initial relative (to the body center of mass = CM) linear velocity in the transverse direction due to disk rotation ω and proportional to relative radius R; and on the radial direction due to explosion with gradient β to relative radius R; for symmetric particles to preserve the linear momentum during the explosion. The simulations were performed with N = 500 particles such to allow us estimate expectations by the arithmetic mean over 500 realizations (mean over the ensemble) later. This value produced convergence of all tested arithmetic means to their steady states.

NUMERICAL SIMULATIONS

Figure 1: Debris at t = 0, 1 τ of explosion 5X.

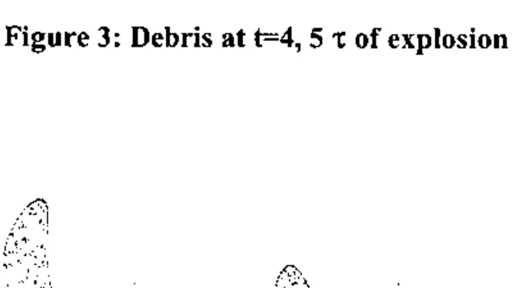


Figure 3: Debris at t=4, 5 τ of explosion 5X.

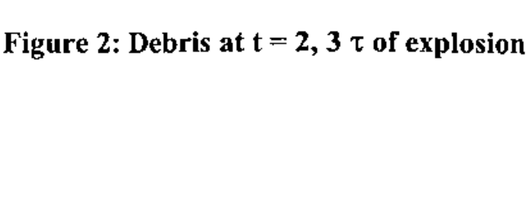


Figure 2: Debris at t = 2, 3 τ of explosion 5X.

Figure 4: Debris at t=6, 7 τ of explosion 5X.

Figure 5: Debris at t=8, 9 τ of explosion 5X.

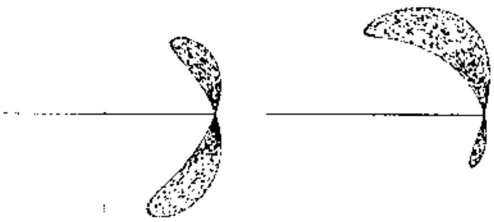


Figure 6: Debris at t=10,11 τ of explosion 5X.

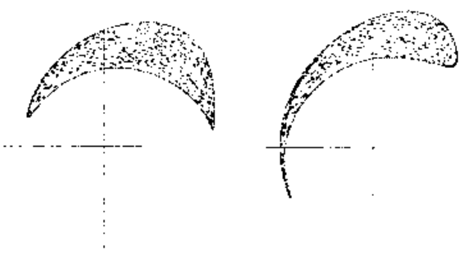


Figure 7: Debris at t=12,13 τ of explosion 5X.

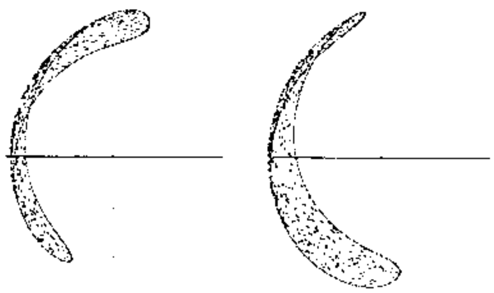


Figure 8: Debris at t=14,15 τ of explosion 5X.

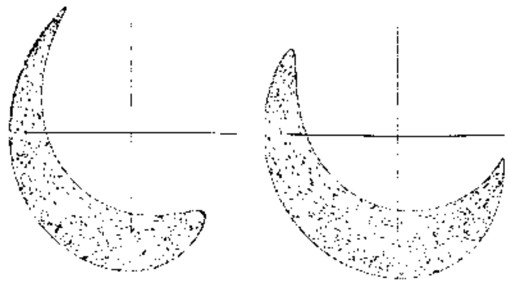


Figure 9: Debris at t=16,17 τ of explosion 5X.

Figure 10:Debris at t=18,19τ of explosion 5X.

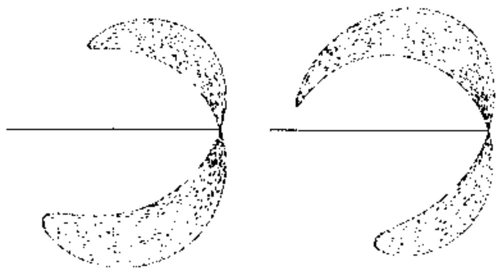


Figure 11:Debris at t=20,21τ of explosion 5X.

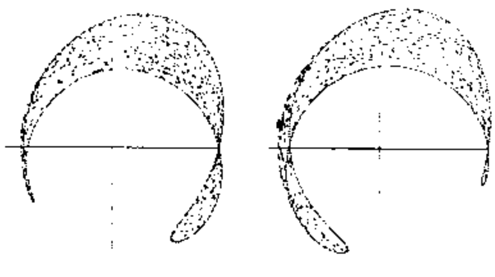


Figure 12:Debris at t=22,23τ of explos. 5X.

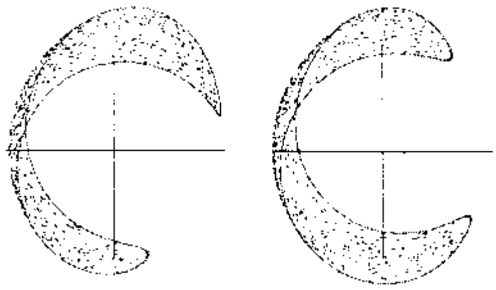


Figure 13:Debris at t=24,25 τ of explos. 5X.

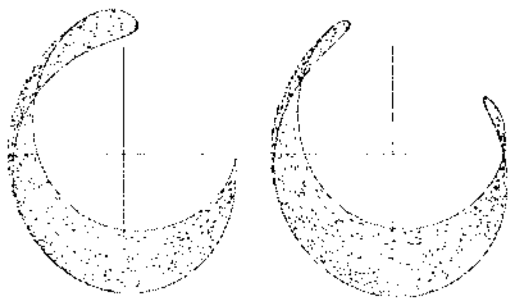


Figure 14:Debris at t=26,27τ of explos. 5X.

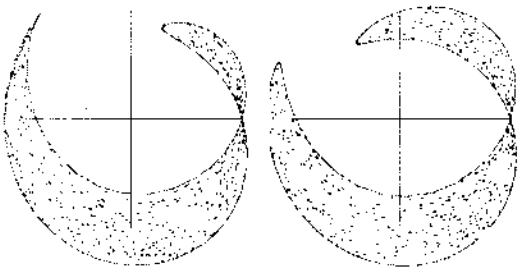


Figure 15:Debris at t=28,29τ of explosion 5X.

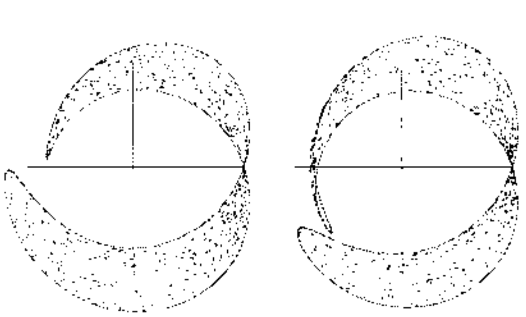


Figure 16:Debris at t=30,31τ of explosion 5X.

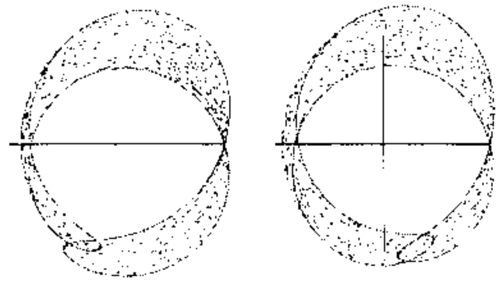


Figure 17:Debris at t=32,33τ of explos. 5X.

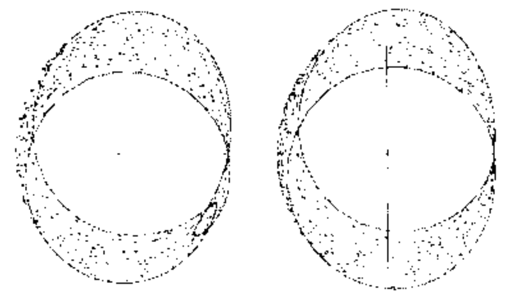


Figure 18:Debris at t=34,35 τ of explos. 5X.

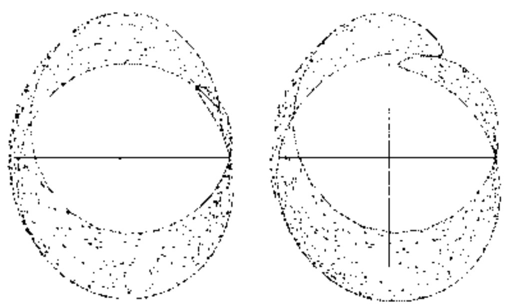


Figure 19:Debris at t=36, 37τ of explos. 5X.

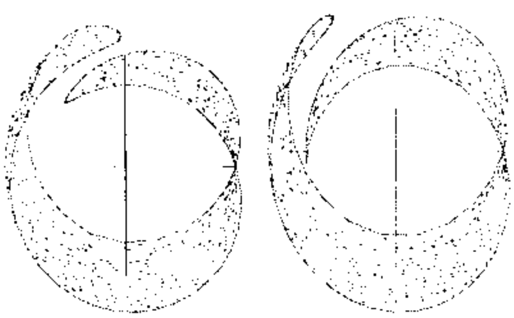


Figure 20:Debris at t=38,39τ of explos. 5X.

In the simulations the parent body describes a circular orbit with radius R=1 (dimensionless) and period T=2π. The parent body is a small circle, 10^{-6} diameter. It is rotating at the same orbital angular speed. This represents quite well an Earth-pointed 2 meters satellite on a low Earth orbit. It could, however, represent anything on a circular orbit. Moreover, the discussion below concerns most to the qualitative aspects of the motion of the cloud of debris and actual numbers are of little, if any, interest.

The Simulation Program 1X simulates de motion of an ensemble of N particles departing from a parent body with velocities resulting from the satellite translation and rotation velocities, and of a radial explosion. It uses a circular reference orbit with radius r, period T, for an homogeneous rotating disk with radius R such that, after a convenient change of variables, they become: r = 1 length unit, T = 2π time units, and $R \cong 1 \times 10^{-6}$ r. The initial conditions are: initial boundary = disk circumference, initial radial velocity of the satellite bound = 10^{-2} or 1% of the orbital linear speed. Program 1X propagates the motion of the initial envelope and plots it with period T at each τ=0.1T.

The Simulation Program 5X simulates de motion of the same ensemble, departing from the same parent body, with the same initial conditions, but we put 25 times more dynamite on the spacecraft, to get a bound radial speed 5 times higher than the previous case. This does not matches a usual case but enhances the effects on the very first orbit. In Figures 1-20 we simulated two orbits after the explosion of a satellite in circular orbit for explosion 5X. The continuous line are the points initially on the surface of the satellite. The points/traces indicate points in the interior (position and velocity). Again, no ellipses!

OBSERVATION AND INTERPRETATION OF THE BASIC PROPERTIES OF SUCH PROCESS

The observation and interpretation of Figures 1-20 suggest the basic properties of such process, namely:

P1-A 1st. property that can be observed in all simulations is that the debris distribution evolves into crescent non elipsoidal (bananoid, boomerang, amoebba,twisted spiral) forms, limiting covariance analysis quickly.

P2-A 2nd. property that can be observed in all simulations is that initially interior points can go out of initially boundary points in the configuration space.

P3-A 3rd. property that can be observed in all simulations is that all fragments eventually return to the blast point. This is(after a little thinking) quite obvious, since thę fragments are describing closed Keplerian, thus periodic, motion and the blast point is common to every orbit. Usual covariance analysis is not able to spot this.

P4-A 4th. property but less obvious is that the debris occupies a 2-dimensional manifold embedded in the 4-dimensional phase space (two coordinates plus two linear momenta) but projected in a 2-dimensional configuration space. Just before the blast the parent body was a rigid body and for rigid bodies the velocity field is determined uniquely by the position and rotation vector. Thus the satellite points occupy a 2-dimensional manifold of the phase space at the starting point. After the blast the motion of the debris is a continuous function, thus preserving topology. Dimension is a topological property, so it is preserved. The 2-dimensional nature of the cloud can be best appreciated on Figures 15-16 when it looks like a potato chip. Remember that these images are epuræ or a 4-dimensional object!

P5-A 5^{th}. property on the deformed sets resulting from the motion (with the shape of bananas/potatoes/boomerangs) is that they remain 2-dimensional even if we consider the Earth oblatness and other gravitational effects. And this happens because the preservation of the topology is a property of any continuous system. The manifold structure will break down only for a dissipative system, e.g., with the colision of particles or in the presence of external random accelerations. Again, covariance propagation is unable to predict this behavior.

P6-A 6^{th}. property is that this is generalizable to a 3-dimensional manifold embedded in the 6-dimensional phase space (three coordinates plus three linear momenta) but projected in a 3-dimensional configuration space.

TIME EVOLUTION OF THE CENTER OF MASS – CM

A preliminary study on the evolution of the center of mass of the space debris, considering-the numerical method and later, the analytical method, revealed another property:

P7-A 7^{th}. property that can be observed in all simulations, specially in Figure 21, is that the center of mass of the space debris distribution spirals monotonically and asymptotically towards the center of gravitational attraction.

This can be verified in Figure 21, where the symbol '+' refers to the center of mass using the numerical method, and the symbol 'x' refers to the center of mass using the analytical method presented ahead.

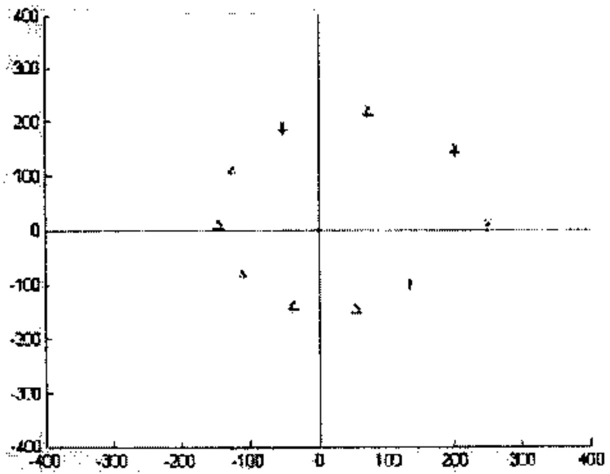


Figure 21: Evolution of the CM's via the: – numerical (+) and analytical (x) methods.

THEORETICAL DISCUSSIONS

To overcome the limitations of the traditional approach we turn now to the differential equations of the motion (1)-(4) and its topological and measure properties in the phase space (X_i , Y_i , U_i , V_i) where $U_i(t) = m_i(t).\dot{X}_i(t)$, $V_i(t) = m_i(t).\dot{Y}_i(t)$ are generalized momenta associated to the generalized coordinates X_i, Y_i . The partial diferential equation that describes the time history of the debris density function (ddf) in the phase space $\rho = \rho($ $X_i(t)$, $Y_i(t)$, $U_i(t)$, $V_i(t)$, t) is of the same mathematical nature of the Lagrangian or material derivative (D/Dt) of the fluid mechanics, as shown in eqs.5-11:

$$\frac{\partial\rho}{\partial t} + \vec{\nabla} \circ (\rho \cdot \vec{M}) = 0 \quad (5)$$

$$\frac{\partial\rho}{\partial t} + (\rho.\vec{\nabla}) \circ \vec{M} + \rho(\vec{\nabla} \circ \vec{M}) = 0 \quad (6)$$

$$\frac{D\rho}{Dt} \equiv \frac{\partial\rho}{\partial t} + (\rho.\vec{\nabla}) \circ \vec{M} = -\rho(\vec{\nabla} \circ \vec{M}) \quad (7)$$

$$\frac{D\rho}{Dt} \equiv \frac{\partial\rho}{\partial t} + \frac{\partial\rho}{\partial Xi} \cdot \dot{X}i + \frac{\partial\rho}{\partial Yi} \cdot \dot{Y}i + \frac{\partial\rho}{\partial Ui} \cdot \dot{U}i + \frac{\partial\rho}{\partial Vi} \cdot \dot{V}i = -\rho \cdot (\frac{\partial\dot{X}i}{\partial Xi} + \frac{\partial\dot{Y}i}{\partial Yi} + \frac{\partial\dot{U}i}{\partial Ui} + \frac{\partial\dot{V}i}{\partial Vi}) \quad (8)$$

$$\rho = \rho(Xi(t), Yi(t), Ui(t), Vi(t), t) \quad (9)$$

$$\vec{M} = \dot{X}i(t) \cdot \hat{X}i + \dot{Y}i(t) \cdot \hat{Y}i + \dot{U}i(t) \cdot \hat{U}i + \dot{V}i(t) \cdot \hat{V}i \quad (10)$$

$$\vec{\nabla} = \frac{\partial}{\partial Xi} \cdot \hat{X}i + \frac{\partial}{\partial Yi} \cdot \hat{Y}i + \frac{\partial}{\partial Ui} \cdot \hat{U}i + \frac{\partial}{\partial Vi} \cdot \hat{V}i \quad (11)$$

Liouville's theorem says that, for Hamiltonian systems, the 2^{nd}. member of Eqs.7 or 8 are identically zero. So, the Liouville equation in such phase space is ρ = constant. If we add random forces the motion becomes a diffusion process. Its density function is thus governed by the well known Kolmogorov-Fokker-Planck-KFP partial differential equation. Covariance matrix propagation is indeed an approximate solution to the KFP equation. Direct integration of KFP is not practical for real problems so some indirects methods may be pursued. We propose that more elaborate approximations be used, either including higher moments or moving to a new set of natural base functions.

A 1^{st}. method is to stay on covariance propagation on a different set of coordinates. The space of initial conditions may be a good choice. In this space the problem is quite regular, the debris does not scatter too much, indeed it does not scatter at all in the deterministic case. To obtain the actual distribution of debris on the **real** space it is sufficient to change the coordinates via a proper canonic transformation. The problem is that the function that makes the magic is transcendental, for its evaluation depends on the solution of Kepler's equation. To make things a little worse the Jacobian determinant of the tranformation has to be computed as well. In principle any set of first integrals of the Keplerian motion could be used as the set "where covariance propagation works". Keplerian elements and Rodrigues' parameters are good candidates.

A 2^{nd}. method is to reduce the order of the problem and then attack the KFP equation directly. Recall our simulations and the fact that the debris is on a 2-dimensional. We can then solve the partial differential equation by finite elements or other standard technique on the reduced dimension manifold. In the 3-dimensional world we do not have to integrate the KFP in 6 dimensions, but only on a reduced 3-D manifold. In theoretical terms this may becomes very complicated since we are talking of conditional probability or of measure of quotient spaces. On the other hand, to start from initial conditions where the coordinates of the phase space are independent may be very interesting to treat the case with noise/perturbations due to the separation of variables.

A 3rd. method is to use better approximations by: including higher moments, using a more appropriate function, or moving to a new set of natural base functions suggested by the dynamics of the problem.

INITIAL TESTS OF A SIMPLE ANALYTICAL MODEL AND COMPARISONS OF THEIR PREVISIONS WITH NUMERICAL ONES

After the numerical work just described, and to avoid the analytical difficulties of the 1st and 2nd methods, we tried the simplest version of the 3rd.method, namely using a simple but appropriate function that could fit the propagation longer and better than the regular ellipsoid propagated by covariance analysis. The first candidate to be this function is an ellipsoid deformed by: turning it around a center of rotation with position and rate to be adjusted, bending its tangent axis to form a central angle to be found, and increasing its tangent and normal axes by rates to be found. These parameters characterize a geometrical/ kinematic description of the motion of the debris. To do that we wrote a C program with the help of the MS Visual C++6.0 of the MS Visual Studio 6.0 environment, capable of reading the coordinates produced by the program KK and parameterizing the propagation of such debris by adjustment. We assumed the same initial distribution of angular velocities. After that, we plotted the debris cloud in the same instants using the same MATLAB routines.

Figure 22 shows a new example for such comparison at t = 0.6 T. In Figures 22.a and 22.b, the explosion was done considering the satellite as a disk containing 100 points in the border and 500 points at the interior. The example continues in Figures 23a and 23b at time t=0.8T, where T is the period. These and other Figures suggest the next property:

P8-A 8th. property that can be observed in Figures 21- 23 is that the analytical parameteri zation used, despite simple, is already capable of reproducing some of the properties of the numerical propagation (P1, P2, P3, P7, etc). So, it is promising and deserves more study.

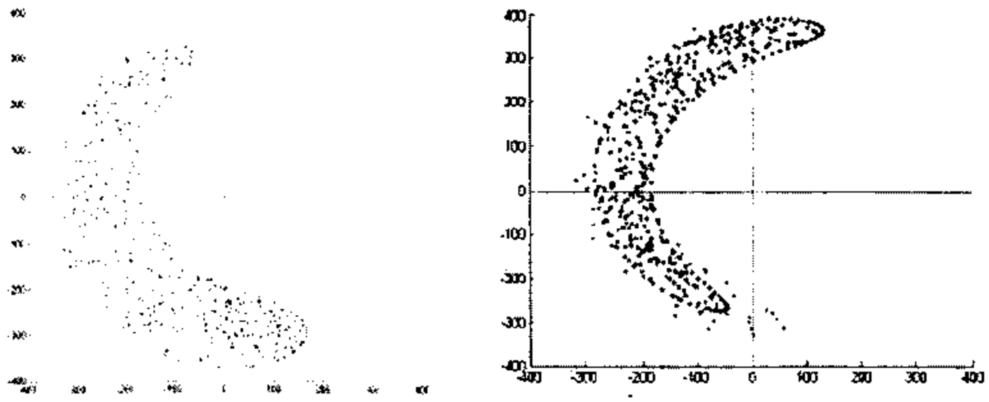
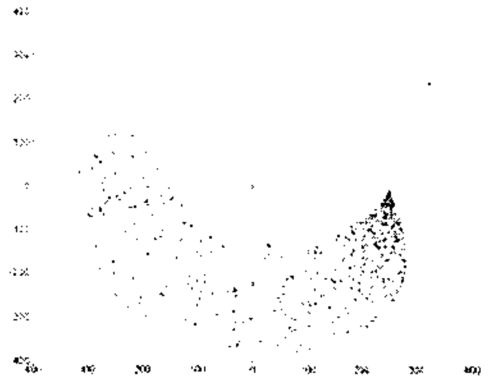


Fig.22.a – Analytical method (t=0.6T)   Fig.22.b – Numerical method (t=0.6T)

Figura 22 – Comparison between the analytical and the numerical methods (t=0.6T)



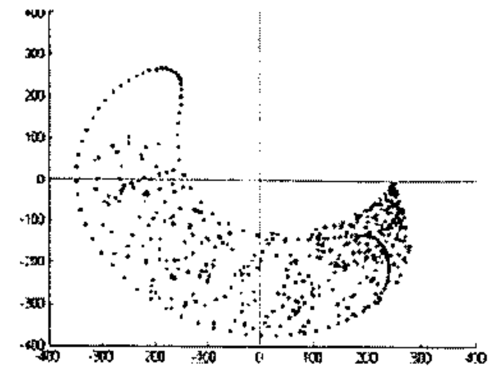


Fig.23a – Analytical method (t=0.8T) Fig.23b – Numerical Method (t=0.8T)

Figure 23 - Comparison between the analytical and the numerical methods (t=0.8T)

The next steps could be: 1) to refine the parameterization of a distribution at each instant; and 2) to characterize the evolution of the parameters with time. To do that, we might use: 1) the method of least squares; 2) the method of curve fitting; respectively, and then, 3) the method of identical substitution of the parameterization in equations 5-11.

CONCLUSIONS

This work showed how traditional covariance matrix propagation is not fit for the purpose of forecasting either the distribution of space debris or (which turns out to be equivalent) the probability of finding a body drifting on a gravitational field with incomplete knowledge of its initial conditions, for important topological properties of the cloud are not modelled by a Gaussian distribution. These and other properties were shown and explained. The general results suggest a progressive deformation of the points distribution along the nominal orbit. Given enough time, the chaotic nature of the motion under gravity stretches and bends the initial debris distribution into distorted and growing shapes (called by us **"bananoids"**) that the two lowest moments can no longer model it. This suggests 3 methods of solution for the equations of motion, beginning with the search of a more appropriate function or basis of functions to solve the KFP equation approximately. We are working on this search and we are testing a parameterized solution for it. Based on these data, we intend later, to study the advanced properties of such process to calculate the probability of collision avoidance of such space debris distribution with some object of importance and size. So, it will be possible to analyze better the problems of collisions and/or interferences with other objects found in space as satellites, constellations, the Space Shuttle or the International Space Station.

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