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Reconstruction of a Spatial Dependent Scattering Albedo in a Radiative Transfer Problem Using a Hybrid Ant Colony System Implementation and a Pre-Regularization Scheme

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Abstract
This work presents an inverse radiative transfer problem that is written as an optimization problem and solved by an Ant Colony System (ACS) method. The ACS employs a metaheuristic based on the collective behaviour of ants choosing a path between the nest and the food source. Each ant marks its path with an amount of pheromone, and the most marked paths are further employed by other ants as reference. In the generation of each ant of a population, the ACS must randomly assign values to the unknowns. These values compose a candidate solution and can be viewed as a path associated to that particular ant. Each ant must be evaluated. The ACS was formerly proposed for the Traveling Salesman Problem (TSP) and other graph like problems that use visibility information in the choice of each unknown. Inverse problems associated to the reconstruction of smooth profiles may include such information in the generation of ants in order to perform a pre-selection of ants. This scheme can be viewed as a kind of pre-regularization. This work shows the use of a pure ACS and a hybridization of the ACS with the LM method in an inverse radiative transfer problem with a spatial dependent albedo. The ACS based pre-regularization scheme is applied for the albedo in reconstructions using noiseless and noisy data. The direct radiative transfer problem is modeled by the linear version of the Boltzmann equation using a discrete ordinates method combined with the finite difference method.
Keywords: Inverse problems, Radiative transfer, Hybrid optimization, Ant Colony System and Levenberg-Marquardt.

1. Introduction

Among several other relevant applications, inverse radiative transfer problems have been used in computerized tomography [1], reconstruction optical spectroscopy [2], radiative properties estimation [3-5], climate modeling [6], hydrologic optics [7], and space science [8].

When formulated implicitly inverse problems are usually written as optimization problems. Several heuristics that mimic natural behaviors have been proposed for the solution of optimization problems.

In particular some of the most recent algorithms, classified within the field of swarm intelligence, are based on the observation of social insects like bees, ants, etc.

In the late nineties the Ant Colony System (ACS) was applied successfully in the solution of combinatorial optimization problems, and more recently it has been proposed for the solution of some specific inverse problems associated with the estimation of real parameters [9,10], being Ref. [11] related to an inverse radiative transfer problem.

The ACS is a method that employs a metaheuristic based on the collective behavior of ants choosing a path between the nest and the food source. Each ant marks its path with an amount of pheromone, and the most marked paths are further employed by other ants as a reference. In the generation of each ant in a population the ACS must randomly assign values to the unknowns. These values compose a candidate solution and can be viewed as a path associated with a particular ant.

The ACS was formerly proposed for the Traveling Salesman Problem (TSP) and other graph like problems that use visibility information in the choice of each unknown. Inverse problems associated with the reconstruction of smooth properties may include such *a priori* information in the generation of ants in other to perform a pre-selection of ants. This scheme may be viewed as a pre-regularization and was proposed in Ref. [9].

Hybrid approaches coupling stochastic and deterministic methods are becoming very popular. In particular, inverse radiative transfer problems have been solved for the estimation of radiative properties using the coupling of Simulated Annealing, Genetic Algorithms and Artificial Neural Networks with the Levenberg-Marquardt method [12-14]. These works are all related to the estimation of radiative properties such as the optical thickness, single-scattering albedo, diffuse reflectivities and anisotropic scattering phase function of one-dimensional homogeneous plane-parallel medium.

In the present work we investigate the use of a pure ACS and a hybridization of the ACS with the Levenberg-Marquardt method (LM) for the estimation of a space-dependent albedo. The direct problem was first considered by Garcia and Siewert [15] and Cengel et al. [16].

Here the ACS based pre-regularization is applied for the albedo reconstruction using noiseless and noisy data. The mathematical formulations of the direct and inverse problems are presented as well as the results for a few test cases.

2. Mathematical Formulation and Solution of the Direct Problem

Consider a one-dimensional, gray, heterogeneous, isotropically scattering participating medium of optical thickness τ_0 and transparent boundaries. The left boundary of the medium at $\tau = 0$ is subject to the incidence of isotropic radiation originated at an external source while there is no radiation coming into the medium through the boundary at $\tau = \tau_0$.

The mathematical formulation for the radiative transfer in the medium is given by the linear Boltzmann equation [17], which for the case of azimuthally symmetry and a space-dependent albedo is written in the dimensionless form as

$$\mu \frac{\partial I(\tau,\mu)}{\partial \tau} + I(\tau,\mu) = \frac{\omega(\tau)}{2}\int_{-1}^{1} I(\tau,\mu')d\mu' \text{ in } 0<\tau<\tau_0, -1\le\mu\le 1 \quad (1a)$$

$$I(0,\mu) = A_1,\ \mu>0 \quad (1b)$$

$$I(\tau_0,\mu) = 0,\ \mu<0 \quad (1c)$$

where I is the radiation intensity, τ is the optical variable, μ is the cosine of the polar angle, A_1 is the intensity of the external isotropic radiation source, and the space dependent albedo, $\omega(\tau)$, is expressed in the following polynomial form

$$\omega(\tau) = \sum_{k=0}^{K} D_k \tau^k \quad (2)$$

When the radiative properties and the boundary conditions are known, problem (1) can be solved in order to determine the intensity of the radiation in the spatial and angular domains, i.e. $0\le\tau\le\tau_0$ and $-1\le\mu\le 1$, respectively. This is the so called direct problem.

In order to solve the direct radiative transfer problem we use in the present work the Chandrasekhar's discrete ordinates method [18] in which the integral term in the right hand side of Eq. (1a) is replaced by a Gaussian quadrature. Here we used a finite difference approximation for the terms in the left hand side of this equation.

3. Mathematical Formulation and Solution of the Inverse Problem

In the direct problem, we consider that the space dependent albedo represented in the polynomial form given in Eq. (2) is unknown. The inverse problem world then consist on the estimation of the coefficients D_k, $k = 0,1,..., K$. However, in the inverse formulation considered in this work the unknown space dependent albedo is determined using a function estimation approach. This approach was adopted in order to take advantage of a recently developed pre-regularization scheme. The albedo is thus estimated as a sampled function with a total of N_u discrete values and, therefore, we can write the vector of unknowns as

$$\vec{Z} = \{\omega_1, \omega_2, ..., \omega_{N_u}\} \quad (3)$$

It is considered also that experimental data on the exit radiation intensity measured at both boundaries, $\tau = 0$ and $\tau = \tau_0$, at different polar angles is available, i.e. $Y_i, i = 1,2,..., N_d$, where N_d is the total number of experimental data. From this experimental data we then try to solve the inverse radiative transfer problem of estimating the space dependent albedo.

As the number of experimental data, N_d, is assumed to be higher than the number of unknowns to be determined, N_u, the inverse problem is formulated as a finite dimensional optimization problem in which we seek to minimize the squared residues cost function

$$Q(\vec{Z}) = \sum_{i=1}^{N_d} [I_i(\omega_1, \omega_2, ..., \omega_{N_u}) - Y_i]^2 = \vec{R}^T \vec{R} \quad (4)$$

where I_i and Y_i are the calculated and measured values of the radiation intensity obtained at the same boundary, $\tau = 0$ or $\tau = \tau_0$, and at the same polar angle represented by μ_i, with $i = 1,2,..., N_d$. The elements of the vector of residues $\vec{R}$ are given by

$$R_i = I_i(\omega_1, \omega_2, ..., \omega_{N_u}) - Y_i,\ i = 1,2,..., N_d \quad (5)$$

As real experimental data was not available, we generated synthetic experimental data by adding a random noise to the calculated values of the radiation intensity I_{exact_i} obtained from the solution of problem (1) using the exact values for the unknowns we want to determine, $\vec{Z}_{exact}$, i.e.

$$Y_i = I_{exact_i}(\vec{Z}_{exact}) + \sigma\, e_i,\ i = 1,2,..., N_d \quad (6)$$

where e_i is a computer generated pseudo-random number in the range [-1,1] and σ emulates the standard deviation of the measurement errors.

In order to minimize the cost function given by Eq.(4) we use in the present work a gradient based deterministic method, the Levenberg-Marquardt method (LM), the stochastic Ant Colony System (ACS), and a hybridization ACS-LM in which the stochastic method is used to provide an initial guess for the deterministic method.

In the ACS implementation we use also an idea originally developed in Ref.[9] in which a priori information on the smoothness of the curve which represents the space dependent albedo is used as a pre-regularization scheme. These methods are briefly described in the next sections.

We must stress that the inverse problem discussed here is somewhat artificial because the optical thickness is considered known. As the scattering and absorption coefficients are unknown, and therefore the scattering albedo, the optical thickness should also be considered unknown. This subject will be investigated in future works.

3.1. Deterministic Gradient Based Method: Levenberg-Marquardt (LM)

In order to minimize the cost function $Q(\vec{Z})$ given by Eq.(4) we write the critical point equation yielding a system of nonlinear equations. We then use a Taylor's expansion for the vector of residues and then we are able to write the following system of linear equations

$$[(J^T)^n J^n + \lambda^n J]\Delta\vec{Z}^n = -(J^T)^n \vec{R}(\vec{Z}^n) \tag{7}$$

where the elements of the Jacobian matrix are given by

$$J_{ij} = \frac{\partial \mathrm{I}_i}{\partial Z_{ij}}, i = 1,2,..., N_d, \ j = 1,2,..., N_u \tag{8}$$

J is the identity matrix, n is the iteration index, λ^n is the damping factor introduced by Levenberg-Marquardt [19] in order to improve the convergence of the method and

$$\Delta\vec{Z}^n = \vec{Z}^{n+1} - \vec{Z}^n \tag{9}$$

such that

$$\vec{Z}^{n+1} = \vec{Z}^n + \Delta\vec{Z}^n, n = 0,1,2,... \tag{10}$$

Starting with an initial guess $\vec{Z}^0$ new estimates for the vector of unknowns are obtained with Eq. (10) being the vector of corrections $\Delta\vec{Z}^n$ calculated from the solution of the linear system of equations (7).

The iterative procedure is interrupted when a stopping criterion such as

$$\left|\frac{\Delta Z_j^n}{Z_j^n}\right| < \varepsilon, \ j = 1,2,..., N_u \tag{11}$$

is satisfied, where ε is a small tolerance, say 10^{-5}.

3.2. Stochastic Method: Ant Colony System (ACS)

The Ant System (ACS) is a method that employs a metaheuristic based on the collective behavior of ants choosing a path between the nest and the food source [20]. Each ant marks its path with an amount of pheromone, and the marked path is further employed by other ants as a reference. As an example, when an obstacle is put in the middle of the original path, blocking the flow of the ants between the nest and the food source, two new paths are then possible, either going to the left of the obstacle or to the right. The shortest path causes a greater amount of pheromone to be deposited by the preceding ants, and then more and more ants choose this path.

This behavior of the ants is then used for the formulation and solution of an optimization problem. In the ACS optimization method, several generations of ants are produced. For each generation, a fixed amount of ants (*na*) is evaluated. Each ant is associated to a feasible path that represents a candidate solution, being composed of a particular set of edges of the graph that contains all possible solutions. Figure 1 represents the discretization of the feasible range of each unknown. Here we consider $ns = N_u$ unknowns. Each unknown ω_i has a range represented by the lower and upper bounds, $\underline{\omega_i}$ and $\overline{\omega_i}$ respectively, with $i = 1,2,\ldots, N_u$ as in Eq. (3). This range is discretized in *np* values in order to deal with real valued unknowns. Each ant consists on a set of possible values for the corresponding unknowns. Choosing the values of the unknowns on a probabilistic basis generates each ant.

This approach was successfully used for the Travelling Salesman Problem (TSP) and other graph like problems [21]. The best ant of each generation is then chosen and it is allowed to mark its path with pheromone, i.e. to increase the amount of pheromone associated to its set of estimated values for the unknowns. This will influence the creation of ants in further generations, since it increases the probability of new ants being generated using these particular values for the unknowns. In addition, the pheromone put by the ants decays according to an evaporation rate denoted by ϕ_{decay}. At the end of all generations, the best solution is assumed to be achieved. A possible solution (ant) can be depicted by graphically linking the ns nodes by $(ns-1)$ edges. In our inverse radiative transfer problem ns corresponds to the total number of unknowns, i.e. $ns = N_u$ and, for each unknown, np discrete values can be chosen, subjected to its lower and upper bounds, $\underline{\omega_i} \leq \omega_i \leq \overline{\omega_i}$, as shown in Fig. 1.

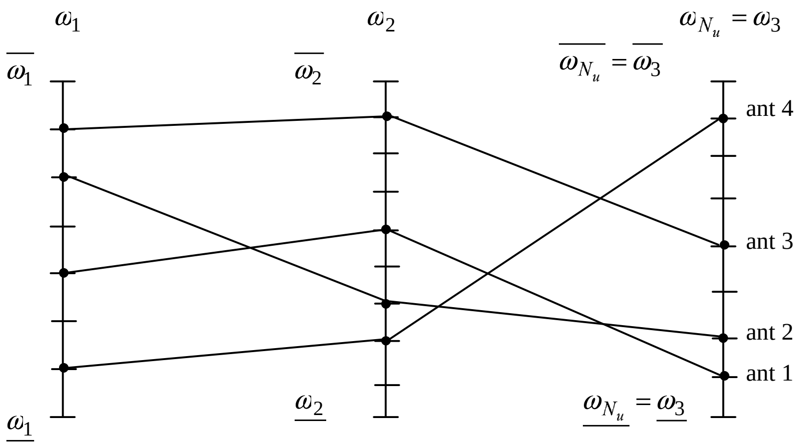


Figure 1. Schematical representation of the random generation of four ants. In this example we have considered $N_u = 3$.

It is defined an array ϕ of pheromone of dimension $np \times ns$. A given element ϕ *[i,j]* represents the amount of pheromone associated to the discrete value *i (i= 1, 2, ..., np)* of the unknown *j (j=1,2,....,ns).* At the beginning of the algorithm, generation $k = 0$, all elements of this array are assigned with the concentration of pheromone $\phi_{ij}^{k=0} = \phi_0$. The amount ϕ_0 is calculated with a greedy heuristics, as suggested in Ref. [22], using an evaluation of the objective function $Q(\vec{Z})$ given by Eq. (4)

$$\phi_0 = \frac{1}{ns \times Q(\overrightarrow{Z^*})} \tag{12}$$

Since in inverse problems we are not able to determine *a priori* some greedy heuristics, we decided to arbitrarily choose $\overrightarrow{Z^*} = \{1,1,...,1\}^T$ in order to evaluate $Q(\overrightarrow{Z^*})$ to be used in Eq.(12).

The best ant in a given generation is allowed to mark its path, i.e. to increase its corresponding values of ϕ *[i,j]* by the amount ϕ_0 of pheromone, and this will have an influence in further ants. For the next generations, $k = 1,2,...,$ the amount of pheromone is given by

$$\phi_{ij}^k = (1 - \phi_{decay})\phi_{ij}^{k-1} + \delta_{ij,best}^{k-1}\phi_0 \tag{13}$$

where $\delta_{ij,best}^{k-1}$ is the Krönecker delta associated with the best ant in generation *(k-1)*, i.e. the one who yields the lowest value for the objective function at the preceding generation $(k-1)$.

A probability *P[i.j]* is associated to each discrete value *j* of each unknown *i* at generation *k*, given by [22]

$$P_{ij}^k = \frac{[\phi_{ij}^k]^\alpha [\eta_{ij}]^\beta}{\sum_{i=1}^{np} \{[\phi_{ij}^k]^\alpha [\eta_{ij}]^\beta\}}, \qquad i = 1,2,..., np, \qquad j = 1,2,..., ns \tag{14}$$

where η_{ij} is the visibility/cost associated to the choice of each unknown, a concept that arises from TSP, where the cost is the inverse of the distance of a particular edge of the TSP graph.

In Eq.(14) we assume that all edges are possible for any ant, but this is not the case for the TSP. The parameters α and β are weights used to establish a tradeoff between the influence of the pheromone and the visibility in the probability associated to the choice of a particular discrete value of an unknown. In order to generate a new ant, that corresponds to a new candidate solution, discrete values must be obtained for the *ns* unknowns. For each unknown, one of the *np* discrete values must be chosen. For the *j*-th unknown, a random number is generated and the elements of the *j*-th column of matrix *P* are gradually summed up. The summation stops when it gets larger the previously this random number. This determines that the *i*-th discrete value is chosen for that unknown.

There is an additional scheme for the choice of the value for an unknown. According to a roulette, a random number in the range [0, 1] is generated and it is compared with a parameter q_0 previously chosen for the problem. If the random number is greater than this parameter, the *j*-th unknown is taken according to the above scheme. If not, the discrete value that corresponds to the highest value *P[i,j]* of the *j*-th column is assigned.

3.3. Pre-regularization Scheme for the ACS

In this work, the ACS based inverse solver was coupled to a recently developed intrinsic regularization scheme [9] and employed for noiseless and noisy data. Since there is an *a priori* information about the smothness of the solution profile, such knowledge is included in the generation of the candidate solutions. A larger number of ants is randomly generated, but only a subset of these ants is selected according to a smoothness criteria. Only the ants of this subset are evaluated by the ACS, requiring the calculation of the objective function by solving the direct problem.

This pre-selection scheme can be viewed as a pre-regularization. It was conceived for problems that do not present a visibility criteria for the ACS. It can be shown that the ACS has poor performance compared to other stochastic optimization algorithms when no visibility information can be defined. In the reconstruction of curves known to be smooth, the smoothness can be interpreted as a visibility information for the ACS. The smoothness criterion adopted in the present work corresponds to the 2^{nd} order Tikhonov regularization term [23].

There is also a performance gain since the pre-selection reduces the number of ants that must be evaluated by solving the direct problem.

In the case of noisy data, the standard explicit regularization is implemented by adding a regularization function weighted by a regularization parameter to the objective function. There are some criteria for the choice of the regularization parameter, but an optimal value can be difficult to adjust, as it requires a choice criteria (Morozov discrepancy principle, L-curve, etc.) that may demand many executions of the inverse solver. A value too small may yield a profile with fluctuations, while the opposite makes the profile flat. The pre-regularization scheme coupled to the ACS does not require the regularization parameter.

3.4. Hybrid Method ACS-LM

By probing the project space (range of the unknowns) in a random way, a stochastic method, such as the ACS, may lead to the vicinity of the global minimum, usually at the expense of a high computational cost. Deterministic gradient based methods when converge are usually much faster than the stochastic methods, but they lead to the closer local minimum.

In order to try to use the best feature of the both methods we use here the same strategy adopted in [12, 14]. The stochastic method ACS is used with a small number of ants and generations. The best estimate obtained with this fast run of the stochastic method, $\hat{\vec{Z}}_{ACS}$, is then used as the initial guess for the deterministic method, i.e.

$$\vec{Z}_{LM}^{0} = \hat{\vec{Z}}_{ACS} \tag{15}$$

4. Results and Discussion

As real experimental data was not available, we generated synthetic experimental data by adding noise to the values calculated for the exit radiation intensities using the exact values of the radiative properties. In all test cases we have considered noiseless data as well as data with noise in the order of, or smaller than, 2% and 5%.

In order to evaluate the performance of the ACS minimizer we chose a relatively difficult test case with a space-dependent albedo given asa polynomial with the following coefficients D_0=0.2, D_1=0.2 and D_2=0.6, according to Eq. (2). The incident radiation was taken as given in Eqs.(1b) and (1c), with $A_1 = 1.0$.

The inverse solver estimates the space dependent albedo as a set of 10 points, i. e. $ns = Nu = 10$,

$$\vec{Z}_{exact} = \{\omega_1, \omega_2, \cdots, \omega_9, \omega_{10}\}^T \tag{16}$$

For each set of experimental data (noiseless data, and 2% and 5% error noisy data) the ACS was executed using 10 different seeds for the random generation of the ants. The results are presented always considering the average of the reconstructions for the 10 different seeds. An important aspect is that it cannot be observed an appreciable variation between the results for different seeds.

As in Ref. [11], two schemes were tested: pure ACS minimization and the hybridization of the ACS with the Levenberg – Marquardt method (ACS-LM). However, since this problem is related to a space dependent albedo, in both strategies the pre-regularization scheme was adopted. In Section 4.1 are presented the results obtained with 120 ants and 500 generations (ACS 120/500) and with the same parameters, but with the pre-regularization reducing the number of ants to 10 (ACS 10/500). Section 4.2 shows the results for the hybridization. The initial guess for the Levenberg – Marquardt method (LM) was given by the ACS using 10 ants, 40 iterations (ACS 10/40) and the pre-regularization.

The quality of the results for the three schemes, ACS 120/500, ACS 10/500 and ACS 10/40 + LM is shown in Table 1. The residue is the final value of the objective function $Q(\vec{Z})$. The error is given by the quadratic difference between the exact and the estimated values for the albedo,

$$d^2 = \sum_{i=1}^{ns} \left(\vec{Z}_{i_{exact}} - \vec{Z}_{i_{estimated}} \right)^2 \qquad (17)$$

In Table 1, the first line presents the number of evaluations of the objective function, and the next line, the execution times. In the case of the LM, the number of calls is different for noiseless (19), 2% noisy (15) and 5% noisy (13) data. These values are discussed in the next sections.

Table 1. Comparison of the performance of the three optimization schemes.

Noise level		ACS 120/500	ACS 10/500	ACS 10/40	LM
	No. evaluations	60000	5000	400	19, 15, 13
	Time (sec.)	877.00	110.00	8.20	< 20
0%	Residue	7.22E-06	5.55E-06	2.24E-04	4.39E-10
	Error	9.88E-03	5.19E-04	6.37E-03	7.94E-04
2%	Residue	–	8.60E-06	2.35E-04	4.54E-10
	Error	–	7.69E-04	7.66E-03	1.26E-03
5%	Residue	–	6.77E-06	8.28E-05	1.80E-11
	Error	–	6.63E-04	2.28E-03	1.31E-03

4.1. Pure ACS minimization

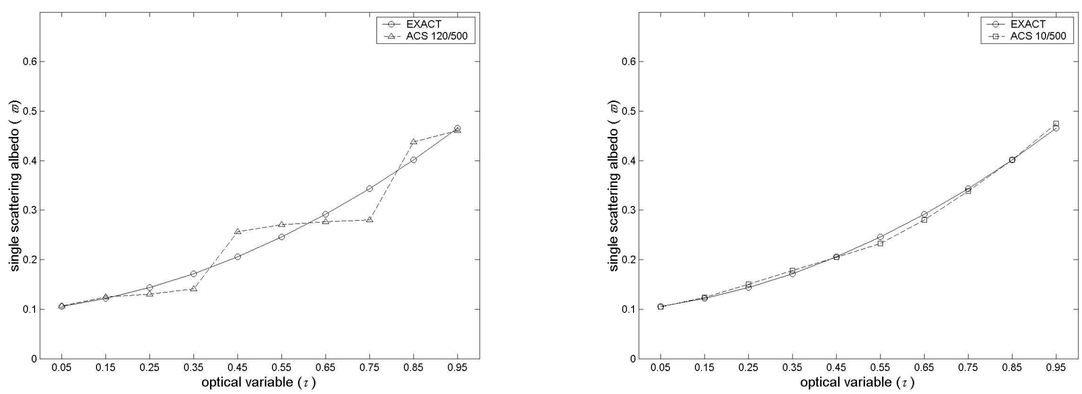
As in most of stochastic optimization algorithms (and deterministic also), the quality of the solution obtained is related to the proper choice and fine tuning of the control parameters. The present section include the test cases ACS 120/500 (pure ACS minimization) and ACS 10/500 (pure ACS minimization with pre-regularization). In the former, each generation is composed of 120 ants, and 500 iterations are performed. In the latter, for each generation 120 ants are generated, but only 10 are selected by the pre-regularization scheme. This selection is based on the smoothness of the albedo curve that corresponds to each ant using the 2th order Tikhonov criterion. For every dozen of ants that is generated the smoother one is selected and so on. The ACS is actually executed with 10 ants for 500 generations.

In both cases we have considered: $\phi_{decay} = 0.03$ for the pheromone decay rate and $q_0 = 0.0$ for the parameter related to the choice of a new edge. This value implies that edges are chosen according to Eq. (14). In this equation, since visibility was not taken into account in it, the control parameters in were Eq. (14) taken as $\alpha = 1$ and $\beta = 0$.

We are interested in the estimation of 10 discrete values of the single scattering albedo as described in Eq. (16). The range for each of these unknowns ($ns = 10$), already shown in the schematical representation given in Fig. 1, is discretized in 3000 values ($np = 3000$).

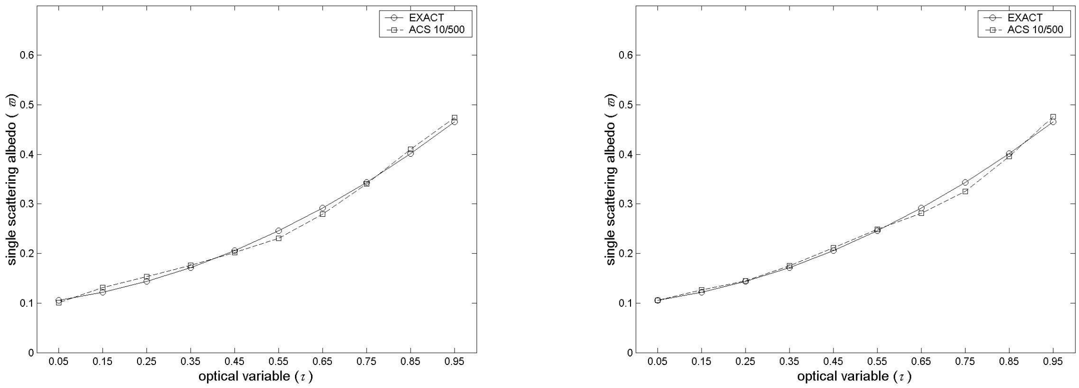
In Fig. 2 is shown a comparison of the results for the ACS 120/500 and ACS 10/500 schemes using noiseless data. It can be seen that the latter s (with pre-regularization) yielded a better result. This can be confirmed by the values of the residue and error in Table 1. The ACS 10/500 required less processing time (about 1/8th) since it evaluates only a fraction of the generated ants.

Figure 3 shows the results only for the ACS 10/500 scheme (with pre-regularization) using 2% and 5 % data. The estimated albedo is also very close to the exact one in both cases. Table 1 shows that residues and errors are slightly worst than for the case using noiseless data.



(a) (b)

Figure 2. Comparison of the exact and estimated albedo for (a) ACS 120/500 (without pre-regularization) and (b) ACS 10/500 (with pre-regularization) schemes using noiseless data.



(a) (b)

Figure 3. Comparison of the exact and estimated albedo for ACS 10/500 scheme (with pre-regularization) using (a) 2% noisy data, and (b) 5% noisy data.

4.2. Hybridization ACS-LM

Recently**,** hybrid approaches coupling stochastic methods and the Levenberg-Marquardt method have been used successfully for the solution of inverse heat transfer problems of parameter estimation [12, 13]: SA-LM (Simulated Annealing and Levenberg-Marquardt) and GA-LM (Genetic Algorithms and Levenberg-Marquardt). Other hybrid strategies combining stochastic and deterministic methods have also been implemented.

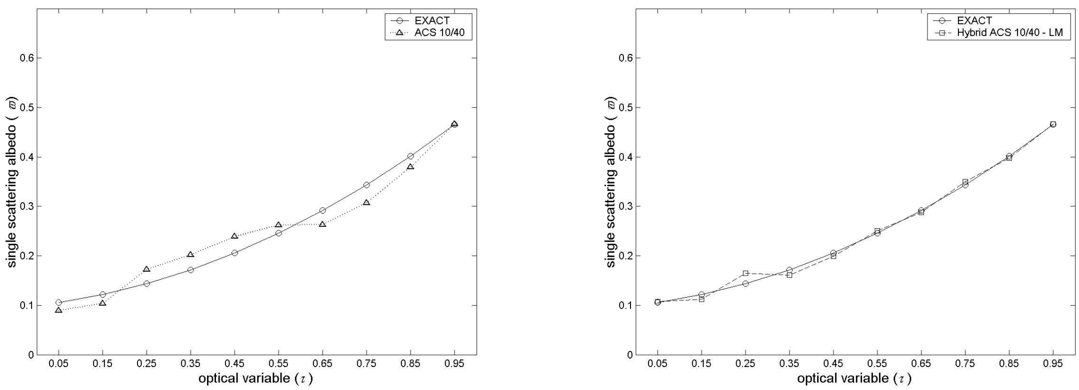
In such hybrid approach the stochastic method (SA or GA) is run for a small number of individuals and generations (or cycles), requiring therefore a much smaller number of function evaluations. The solution obtained with the stochastic method is then used as the initial guess for the gradient based method. If necessary this approach may be iterated.

Artificial Neural Networks (ANN) have also been used for the same strategy of generating a good initial guess for the gradient based method: ANN-LM [14].

In the hybridization ACS-LM, the ACS 10/40 scheme is used to generate an initial guess for the Levenberg-Marquardt method. In this section, we considered the ACS 10/40 scheme described in Section 4.1, with only 40 generations, 10 ants per generation (pre-selected out of 120). The remaining parameters of the ACS are the same as those for the ACS 120/500 or the ACS 10/500, except for the pheromone decay rate that is ϕ_{decay}=0.30.

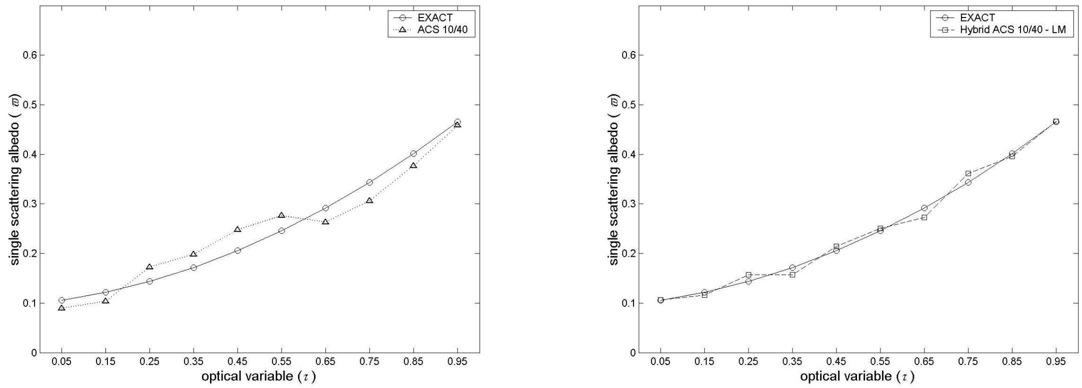
Test results are presented for noiseless, 2% and 5% noisy data. In each case, the initial guess generated by the ACS 10/40 scheme is also shown. Figures 4, 5 and 6 show the ACS initial guess and the final LM results for noiseless, 2% and 5% noisy data, respectively.

These results show that the hybridization ACS-LM proved to be robust yielding reasonable solutions for the space-dependent single scattering albedo, even when noisy data is used. Residues and errors were better than those for the ACS pure minimization (ACS 10/500 scheme) as shown in Table 1. The number of evaluations of the objective function was lower than that for the pure minimization (419, 415 and 413 for noiseless, 2% and 5% noisy data, respectively). Processing times were also lower, as expected, since the LM processing times were always lower than 20 seconds. Therefore, the hybridization required about 1/3rd of the processing time of the pure ACS minimization (ACS 10/500 scheme).



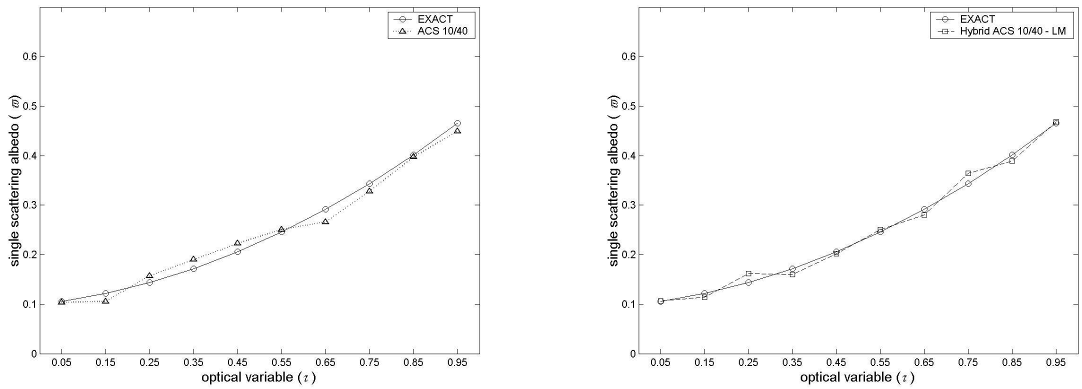
(a) (b)

Figure 4. Comparison of the exact and estimated albedo for the initial guess obtained by the (a) ACS 10/40 scheme and for the final solution yielded by the (b) hybridization ACS-LM using noiseless data.



(a) (b)

Figure 5. Comparison of the exact and estimated albedo for the initial guess obtained by the (a) ACS 10/40 scheme and for the final solution yielded by the (b) hybridization ACS-LM using 2% noisy data.



(a) (b)

Figure 6. Comparison of the exact and estimated albedo for the initial guess obtained by the (a) ACS 10/40 scheme and for the final solution yielded by the (b) hybridization ACS-LM using 5% noisy data.

5. Conclusions

In the present work both the Ant Colony System algorithm (ACS) and the hybridization of such method with the gradient based Levenberg-Marquardt method (LM), i.e. ACS-LM, yielded good estimates for the space dependent albedo using the measured data of the intensity of the radiation acquired only by external detectors. Test cases were performed using noiseless and noisy data. An implicit regularization scheme that pre-selects ants was successfully employed to improve the performance of the ACS. The ACS-LM hybridization yielded a better reconstruction at a lower processing time. In future works we intend to include other parameters in the reconstruction such as the optical thickness of the medium.

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