# EXPERIMENTS ON REAL TIME POSITIONING OF A GPS RECEIVER USING DIFFERENTIAL GPS

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*Abstract. The purpose of this paper is to estimate in real time, the position coordinates of a fixed GPS receiver ("user") based on another receiver located on a landmark with known position ("base"), through the differential GPS technique (DGPS). The "user" receiver is purposely placed at a known distance, which makes possible the verification of accuracy of the method. The differential GPS technique consists of two well defined steps: 1) generating corrections in the reference "base"; 2) applying the corrections in the "user" receiver. The reference base has its position coordinates precisely known, so that it can calculate either the correction on position coordinates with respect to the landmark, or the correction on the raw pseudorange measurements for each GPS satellite. Due to the base and user proximity, the differential GPS principle assumes that environmental effects (troposphere and ionosphere mainly) are the same, for close baselines. Thus, complex modeling effects are prevented and the technique gives origin to a high accuracy relative positioning. The data were collected by dual frequency GPS receivers, geodetic quality, Ashtec Z12, in a campaign of about one hour with 1 Hz of sampling rate. They were processed using both techniques, namely DGPS on position, using a least squares method, and DGPS on measurements, through a Kalman filter, and the results are compared with the known distance between receivers. The results obtained with DGPS techniques show an improvement with respect to stand-alone GPS, with an accuracy better than 3 m for the test cases. They allow to establish the basis for the development of a real time estimation method of the receiver position and to verify the accuracy achieved with the DGPS techniques. This is an important step in developing a differential GPS system to be further implemented in dynamically mobile platforms, like aircraft, ships, and cars.*

*Keywords. GPS, navigation solution, differential positioning.*

## 1. Introduction

The purpose of this work is to obtain, in real time, the position coordinates of the stationary "user" GPS receiver applying corrections given by the differential GPS technique (DGPS), as well as to compare different methods of DGPS implementations. The differential GPS technique consists of two major steps: 1) generating corrections on a reference "base"; 2) applying the corrections on the "user" receiver. The reference base used has its position coordinates precisely known being a landmark surveyed by IBGE (Brazilian Institute of Geography and Statistics). Therefore it is rather simple to calculate corrections to the results obtained using the GPS measurements for each satellite. The basic idea behind DGPS assumes that if two receivers are relatively close to each other (about up to 150 km), the GPS radio signals will travel in very close paths, thus the atmospheric (ionosphere and troposphere) effects will be practically the same. These effects are spatially correlated, i. e., the magnitude error is dependent of the baseline length. The application of the corrections to both receivers makes therefore the final result more accurate (Strang and Borre, 1997). In this DGPS technique care of using only the simultaneously visible satellites for both receivers should be taken. In this work, the "user" GPS receiver is located at a known distance to the base, which is used to verify the accuracy of the procedures proposed. The data were collected by two dual frequency GPS receivers, geodetic quality, Ashtec Z12, in a campaign of about one hour with 1 Hz of sampling rate. They were processed using two techniques, namely DGPS on position and DGPS on measurements, and the outcomes are compared with the known distance between receivers. The results obtained show the feasibility of developing simple real time estimation methods with fair accuracy through the DGPS techniques. This investigation paves the basis to develop a real time differential GPS system to be further implemented in dynamically mobile platforms, like aircraft, ships, and cars.

## 2. DGPS on position

Essentially this technique applies corrections directly in position coordinates. Two steps are required: first of all, using the raw CA pseudorange measurements, each receiver (base and user) computes their positions by the same algorithm, e.g. Bancroft (1985) or a conventional Least Squares method. Then the position correction on base receiver is applied to the user receiver. Follows the details of the technique.

## 2.1 Base position correction

Let the base receiver be placed at a known landmark, with coordinates $r_{ref} = (x_{ref}, \ y_{ref}, \ z_{ref})$ given in WGS84 coordinates, and let $r_b = (x_b, \ y_b, \ z_b)$ be the position coordinates computed by using the raw pseudorange measurements, through the Least Squares method. Then the correction to the base position are inferred directly from comparison:

$$\delta r = r_{ref} - r_b \tag{1}$$

## 2.2. Differential position determination

Assuming that both receivers are close to each other, it can be assumed that the position, in analogy to pseudorange measurement model, can be modeled by:

$$r = r_g + b - B + I + T + \varepsilon \tag{2}$$

where $r_g$ is the geometrical position, $b$ are errors due to receiver clock offset, $B$ are errors due to GPS satellite clock, $I$ are due to ionosphere, $T$ are due to troposphere, and $\varepsilon$ are other unmodelled errors (multipath, internal delays, etc.) assumed white noise. It is clear that if both receivers are near, the atmospheric effects $I$ and $T$ are almost the same, $B$ as well if the set of GPS satellites are the same, and $b$ is known or computed to each receiver. Therefore the correction $\delta r$ computed for the base receiver can be applied straightforwardly to the user receiver:

$$\hat{r}_u = r_u + \delta r \tag{3}$$

where $\hat{r}_u$ is the differentially corrected position estimate of the user receiver, and $r_u$ is the position computed using raw pseudorange measurements of the user receiver.

## 3. DGPS on pseudorange measurements

This method applies corrections directly in the raw GPS pseudorange measurements. If the coordinates of a reference base are known, then the actual pseudorange measurement for each GPS satellite can be calculated. For a given GPS satellite, the difference between the actual and the raw pseudorange measurement is applied to the raw pseudorange measurement of the user receiver. Then user position calculation can be achieved by using such corrected pseudorange measurements.

## 3.1 Base pseudorange corrections

Let the pseudorange measurement corresponding to the i-th GPS satellite be modeled by (Parkinson and Spilker, 1996):

$$\rho_i = \left| R_i - r \right| + b - B_i + c(T_i + I_i) + \varepsilon_i \tag{4}$$

where $\rho_i$ is the pseudorange measurement, $R_i$ is the position vector of the i-th GPS satellite at the transmission time, $r$ is the position vector of the receiver at the reception time, $c$ is the light speed, $b$ is the receiver clock offset, $B_i$ is the satellite clock offset, $T_i$ is the delay due to troposphere, $I_i$ is the delay due to ionosphere (group delay), and $\varepsilon_i$ represents noise due to multi-path, errors between receiver channels, and other errors. The magnitude of the geometric distance is represented by $\left| R_i - r \right|$.

The GPS satellite clock offset $B_i$ is broadcasted together with the GPS signal and is promptly available. The receiver clock offset $b$ is known or computable. Therefore, the correction $\delta\rho_i$ generated on the base receiver is simply the difference between the actual pseudorange calculated with the reference base receiver coordinates and the measured pseudorange. It is easy to verify that this correction would mostly cover the atmosphere effects, due to proximity of the receivers. It is simply computed by:

$$\delta\rho_i = \rho_i - \rho_{bi} \tag{5}$$

where $\rho_i$ is the measured pseudorange, and $\rho_{bi}$ is the "actual" pseudorange.

### 3.2. User position estimation

The $\delta\rho_i$ of the base receiver is used to correct the pseudorange measurement of the corresponding GPS satellite:

$$\hat{\rho}_{ui} = \rho_{ui} + \delta\rho_i \tag{6}$$

where $\hat{\rho}_{ui}$ is the corrected pseudorange measurement, and $\rho_{ui}$ is the pseudorange measurement. Then this set of corrected measurements must be processed to finally obtain the user position. This is achieved through two Kalman filters: the first one estimates the user receiver clock bias and the second one is used to finally obtain position estimates. They work in cascade, the first filter estimates the "user" receiver clock bias from the best position estimate available. This bias estimate is used in the second filter for estimating the user position. The two filters are described in the following.

### 3.3. Kalman filter for receiver clock bias

Given the pseudorange measurement model:

$$\rho_i = \left| R_i - r \right| + b - B_i + \varepsilon_i \tag{7}$$

where $\rho_i$ is the corrected pseudorange measurement. Notice the absence of the atmosphere terms which are supposed to be corrected by the correction term $\delta\rho_i$. Here, if $r$ is known, than the only unknown is b, the receiver clock bias. This bias is modeled as a second order polynomial with respect to the time interval $\delta t$ between measurements:

$$b = H\, x_{bias} = \begin{bmatrix} 1 & \delta t & \delta t^2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{8}$$

where $x_{bias} = \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix}^T$ is the vector of state parameters to be estimated time wise by the Kalman filter. This model gives a more accurate clock bias estimation than trying a linear model, as shown in Marques Filho et all (2003). Therefore the measurement equation for applying the Kalman filter is:

$$\rho_i - \left| R_i - r \right| + B_i = \begin{bmatrix} 1 & \delta t & \delta t^2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} + \varepsilon_i \tag{9}$$

Assuming a parameter estimation via Kalman filter, then the equations for state and covariance update from time $k$ to time $k + 1$ are (Brown et all, 1997):

$$\begin{aligned} \overline{x}_{biask+1} &= \hat{x}_{biask} \\ \overline{P}_{biask+1} &= \hat{P}_{biask} + Q_{bias} \end{aligned} \tag{10}$$

where $P_{bias}$ is the covariance matrix and $Q_{bias}$ is the noise covariance matrix. The upper bar indicates time updated states and covariance, and the upper hat indicates measurement updated states and covariance. The measurement updated state and covariance for this filter are obtained by the classical Kalman form:

$$\begin{aligned} K_k &= \overline{P}_{biask} H^T [H\ \overline{P}_{biask} H^T + R]^{-1} \\ \hat{x}_{biask} &= \overline{x}_{biask} + K_k \cdot (\rho_{ci} - \rho_i) \\ \hat{P}_{biask} &= (I - K_k H)\overline{P}_{biask} \end{aligned} \tag{11}$$

where $\rho_{ci}$ is the measured corrected pseudorange vector from Eq. (6), $\rho_i$ is the predicted pseudorange from Eq. (9), and $R$ is the pseudorange measurement error covariance matrix.

### 3.4. Kalman filter for receiver position

In this filter, the state $x_u = \begin{bmatrix} x & y & z & b \end{bmatrix}^T$ contains the three position coordinates and the clock bias $b$. The initial value of $b$ is the value estimated by the previous filter (section 3.3).

The receiver is known to be in a stationary position, so its model is very simple. Thus, the state and covariance update from time $k$ to time $k + 1$ is (Brown et all, 1997):

$$\overline{x}_{uk+1} = \hat{x}_{uk}$$
$$\overline{P}_{uk+1} = \hat{P}_{uk} + Q_u \tag{12}$$

where $P_u$ is the covariance matrix and $Q_u$ is the noise covariance matrix. The upper bar is for time updated states and covariance, and the upper hat is for measurement updated states and covariance. Eq. (7) for pseudorange can be explicitly given as (Parkinson et all, 1996):

$$\rho_i = \left[ (X_i - x)^2 + (Y_i - y)^2 + (Z_i - z)^2 \right]^{1/2} + b - B_i + \varepsilon_i \tag{13}$$

where $R_i = \begin{bmatrix} X_i & Y_i & Z_i \end{bmatrix}^T$ is the position vector of the i-th GPS satellite. The measurement residual, which is the difference between the predicted and actual measurement, can be modeled as linearly related to the error in the state estimate, $\Delta x_u = \hat{x}_u - x_u$, by performing a Taylor expansion about the current state estimate. The linearized result is given by the following:

$$\Delta\rho = \hat{\rho}_i - \rho_i = G_k \Delta x_u + \Delta\varepsilon_i = \begin{bmatrix} -\dfrac{X_i - \hat{x}}{\rho_i} & -\dfrac{Y_i - \hat{y}}{\rho_i} & -\dfrac{Z_i - \hat{z}}{\rho_i} & 1 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta b \end{bmatrix} + \Delta\varepsilon_i \tag{14}$$

Therefore, the measurement update cycle of this filter, for each satellite measurement, is:

$$K_k = \overline{P}_{uk} G_k^T [G_k \overline{P}_{uk} G_h^T + R]^{-1}$$
$$\hat{x}_{uk} = \overline{x}_{uk} + K_k \cdot (\rho_{ci} - \hat{\rho}_i) \tag{15}$$
$$\hat{P}_{uk} = (I - K_k G_k)\overline{P}_{uk}$$

where $R$ is the pseudorange measurement error covariance matrix.

## 4. Results

The data set was collected by two Ashtech Z12 real time GPS receivers with dual frequency capability and geodetic quality, during a campaign of about one hour with data being recorded at 1 Hz of sampling rate. One of the receivers, named "base", was placed on top of a surveyed landmark, with ECEF position coordinates in WGS-84 system given by $x_{ref}$ = 4084765.0762m, $y_{ref}$ = -4209370.0341m, $z_{ref}$ = -2498478.221m. The other receiver, named "user", was placed at 5.20m of distance. The navigation and observation data from both receivers were downloaded in RINEX-2 standard files. Both observation files were edited so that the interval and the set of satellites processed were the same.

### 4.1. Results for DGPS on position

In this test, data of both receivers were processed independently to produce a least squares solution for the position and receiver clock biases. Then the procedure described in section 2 was applied to get the position correction of the "user" receiver. Figure (1) shows the error in distance of the user with respect to the 5.20m baseline length. One notices that the error remained most of the time less than 2m, with some ripples reaching 5-7m, probably due to bad clock offset estimation. However this result is within the level of accuracy expected when applying DGPS techniques, as stated in Parkinson and Spilker (1996): horizontal 1-sigma error 2.2m, and vertical 1-sigma error 2.8m.
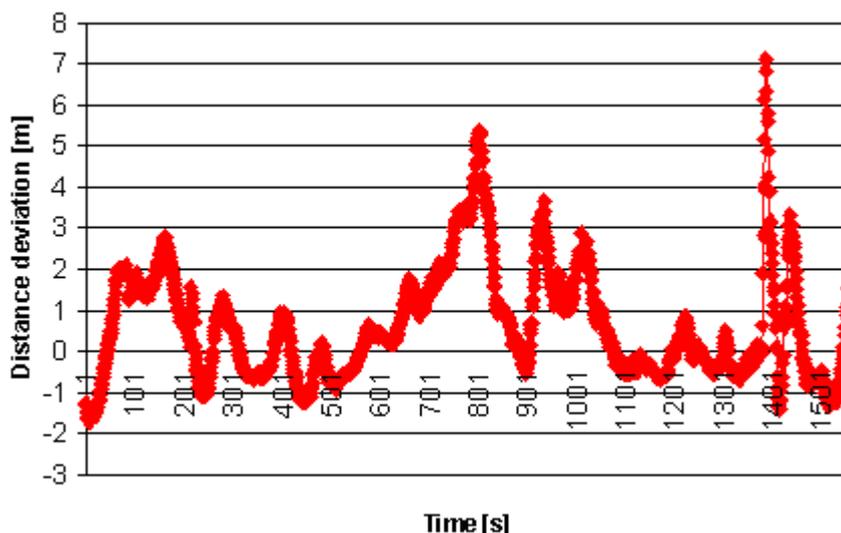
Figure 1. Distance deviation for DGPS on position.

## 4.2. Results for DGPS on pseudorange measurements

In this test, data were processed as described in section 3, first computing the pseudorange measurements corrections through the "base" receiver, and then applying the corrections to the corresponding measurements of the "user" receiver. Both Kalman filters were tuned and then applied to process the data. The final results are shown here. Figure (2) shows the behavior of the pseudorange residuals along the filtering time. After initial higher values they decrease to the level of less than 1m.
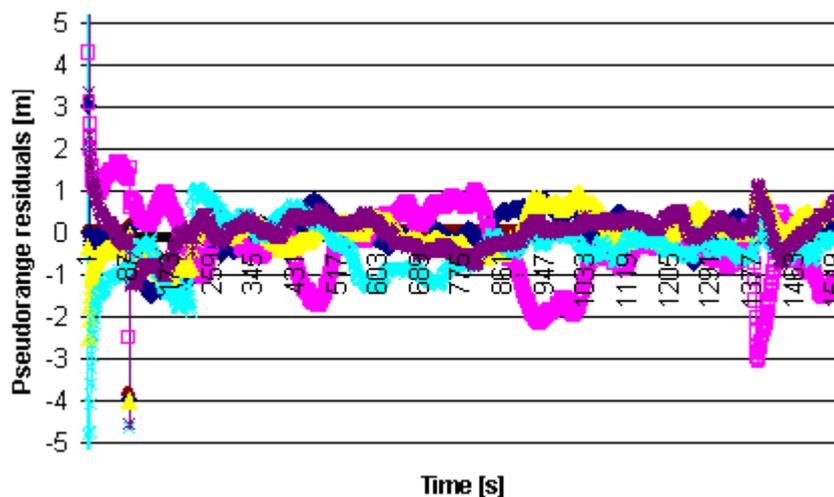


Figure 2. Pseudorange residuals.

Figure (3) shows the distance deviation from 5.20m, together with ±1sigma error estimation (smooth curves) given by the trace of covariance matrix. At the beginning the filter was initialized with the "base" coordinates, which explain the initial 4m error. After some seconds the filter converges and remains within the 0.5m deviation range. It is a remarkable improvement when compared to the expected 10-12m for a single receiver. Also it seems more stable than the results of DGPS on position, Fig. (1), which presented higher errors and presented not so smooth behavior.
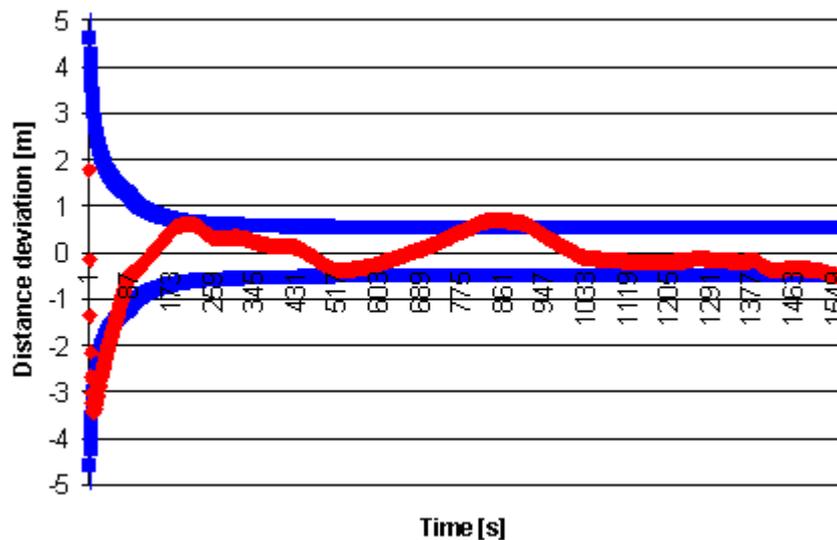
Figure 3. Distance deviation for DGPS on pseudorange measurements.

## 5. Conclusion

The results obtained prove the usefulness of differential GPS techniques to improve accuracy of static positioning. The use of a short baseline, 5.20 m, can practically eliminate the atmospheric error components, because they are spatially correlated. The remaining errors may be due to unmodelled errors, like multipath, internal receiver errors etc. It also shows the feasibility of developing simple Kalman filter real time estimation method to deliver fair accuracy at the prescribed DGPS levels. This investigation builds-up the basis for development of a real time differential GPS system with the aim of future implementation in dynamically mobile platforms, like aircraft, ships, and cars.

## 6. Acknowledgment

## 7. References

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