BI-IMPULSIVE ORBITAL TRANSFERS BETWEEN COPLANAR ORBITS WITH MINIMUM TIME FOR A PRESCRIBED FUEL CONSUMPTION

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In this work we consider the problem of bi-impulsive orbital transfers between coplanar circular or elliptical orbits with minimum time for a prescribed fuel consumption. We used the equations presented by Eckel and Vinh (1984), that provides the transfer orbit between non-coplanar elliptical orbits with minimum fuel and fixed time of transfer; or minimum time of transfer for a prescribed fuel consumption. But in this work we consider only the problem with minimum time of transfer for a prescribed fuel consumption. The case of minimum fuel and fixed time of transfer was already studied in Rocco (1997) and Rocco et al. (1999; 2002). The case of orbital transfer between non-coplanar orbits with minimum time for a prescribed fuel consumption was already studied in Rocco et al. (2000). Then, we modified the equations presented by Eckel and Vinh (1984) to consider the problem of coplanar orbital transfer and develop a software for orbital maneuvers. This software is available to be used in the next missions developed by INPE. The original method, developed by Eckel and Vinh, was presented without numerical results in that paper. Thus, the modifications considering the coplanar case, the implementation and the solutions using this method are contributions of this work. The software was tested, simulating real maneuvers with success.

Keywords *Astrodynamics, Orbital Transfer, Impulsive Maneuvers, Maneuvers Optimization, Minimum Time.*

1. Introduction

The majority of the spacecrafts that have been placed in orbit around the Earth utilizes the basic concept of orbital transfers. During the launch, the spacecraft is placed in a parking orbit distinct from the final orbit for which the spacecraft was designed. Therefore, to reach the desired final orbit the spacecraft must perform orbital transfers. Besides that, the orbit of the spacecraft must be corrected periodically because there are perturbations acting on the spacecraft. Both maneuvers are usually calculated with minimum fuel consumption but without a time constraint. This time constraint imposes a new characteristic to the problem that rules out the majority of the transfer methods available in the literature: Hohmann (1925), Hoelker et al. (1959), Gobetz (1969), Prado (1989), etc. Therefore, the transfer methods must be adapted to this new constraint: Wang (1963), Lion et al. (1968), Gross et al. (1974), Prussing (1969), Prussing (1970), Prussing et al. (1986), Ivashin et al. (1981), Eckel (1982), Eckel et al. (1984), Lawden (1993) and Taur et al. (1995). In Brazil, we have important applications with the launch of the Remote Sensing Satellites RSS1 and RSS2 that belongs to the Complete Brazilian Space Mission and with the launch of the China Brazil Earth Resources Satellites CBERS1 and CBERS2.

In this work we consider the problem of bi-impulse orbital transfers between coplanar circular and elliptical orbits with minimum time for a prescribed fuel consumption. This problem is very important because most of the spacecrafts utilize a propulsion system only capable of providing a fixed value of velocity increment, and the velocity increment is direct related to the fuel consumption. On the other hand, in many missions it is important to perform the maneuvers in the minimum time, as for instance in the case of remote sensing satellites because during the maneuver the collected data are of low quality and therefore they can not be used. Thus, we used the equations presented by Eckel et al. (1984), add some new equations to consider cases with different geometries, and solved those equations to develop a software for orbital maneuvers. This software is available to be used in the next missions developed by INPE.

2. Definition of the Problem

The orbital transfer of a spacecraft from an initial orbit to a desired final orbit consists (Marec, 1979) in a change of state (position, velocity and mass) of the spacecraft, from initial conditions $\vec{r}_0$, $\vec{v}_0$ and m_0 at time t_0 to final conditions $\vec{r}_f$, $\vec{v}_f$ and m_f at time t_f ($t_f \geq t_0$) as shown in the Fig. 1.

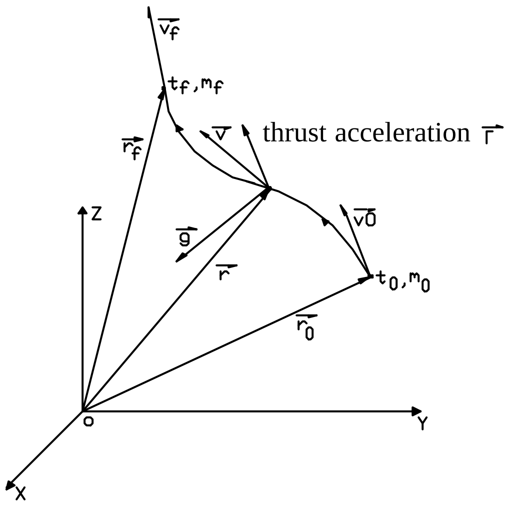

Figure 1. Orbital Transfer Maneuver, cf. Marec (1979).

The maneuvers can be classified in: maneuvers partially free, when one or more parameter is free (for example, the time spent with the maneuver); or maneuvers completely constrained, when all parameters are constrained. In this case the spacecraft perform an orbital transfer maneuver from a specific point in the initial orbit to another specific point in the final orbit (for example, rendezvous maneuvers). In this work we consider the orbital transfer maneuvers partially free, and that the spacecraft propulsion system is able to apply an impulsive thrust. Therefore, we have the instantaneous variation of the spacecraft velocity.

3. Presentation of the Method

The bases for this method are the equations presented by Eckel et al. (1984). These equations provide the transfer orbit between non-coplanar elliptical orbits with minimum fuel and fixed time transfer (MFFT), or the transfer orbit with minimum time transfer for a prescribed fuel consumption (MTPF). But in this work we study the MTPF problem considering coplanar maneuvers between circular or elliptical orbits. The MFFT problem has already been studied by Rocco (1997) and by Rocco et al.(1999, 2002). The non-coplanar MTPF problem was already studied in Rocco et al. (2000).

The equations were presented in the literature but the method was not implemented neither tested in Eckel et al. (1984), and, in the non-coplanar case, they used the plane of the transfer orbit as the reference plane but we decide to use the equatorial plane as the reference plane because in this way it is easier to obtain and to apply the results in real applications. Using the transfer orbit as the reference plane almost all the results obtained belong to the same specific geometry, so we change the reference system, adding the equations 1 to 6, to make possible the implementation of a software. For the coplanar and circular case, we should substitute some equations, eliminating the singularities that appear when the eccentricity and the difference between the orbits inclinations tend to zero.

Therefore, the method was implemented to develop a software for orbital maneuvers. Thus, the modification, the implementation and the solutions using this method are contributions of this work. By varying the total velocity increment necessary to the maneuver, the software developed furnishes a set of results which are the solution of the problem of bi-impulsive optimal orbital transfer with minimum time for a prescribed fuel consumption.

Initially considering the non-coplanar case, because the solution for the coplanar case can be obtained from the non-coplanar case eliminating the singularities, we have: given two terminal orbits we desire to obtain a transfer orbit which performs an orbital maneuver from the initial orbit to the final orbit with minimum time and fixed total velocity increment. The orbits are specified by the elements shown in the Table 1 (subscript 1: initial orbit; subscript 2: final orbit; no subscript: transfer orbit):

Table 1 – Elements Characterizing the Maneuvers.

a	Semi-major axis
e	Eccentricity
p	Semi-latus rectum
w	Longitude of the periapsis
i	Inclination
W	Longitude of the ascending node
M	Mean anomaly
E	Eccentric anomaly

l	Angle between the planes of the initial and final orbits
b_1	True anomaly of the point N obtained in the plane of the initial orbit
b_2	True anomaly of the point N obtained in the plane of the final orbit
I_1	Location of the first impulse
I_2	Location of the second impulse
D	Transfer angle obtained in the plane of the transfer orbit
V_1	Velocity increment generated by the first impulse
V_2	Velocity increment generated by the second impulse
V	Total velocity increment
T	Time spent in the maneuver
a_1	True anomaly of the point I_1 obtained in the plane of the initial orbit
a_2	True anomaly of the point I_2 obtained in the plane of the final orbit
r_1	Distance from point I_1
r_2	Distance from point I_2
f_1	True anomaly of the point I_1 obtained in the plane of the transfer orbit
f_2	True anomaly of the point I_2 obtained in the plane of the transfer orbit
x_1	Radial component of the first impulse
x_2	Radial component of the second impulse
y_1	Transverse component of the first impulse in the plane of the initial orbit
y_2	Transverse component of the second impulse in the plane of the transfer orbit
z_1	Component of the first impulse orthogonal to the initial orbit
z_2	Component of the second impulse orthogonal to the transfer orbit
h_i	Horizontal component of V_i

The geometry of the non-coplanar maneuver is shown in Fig. 2.

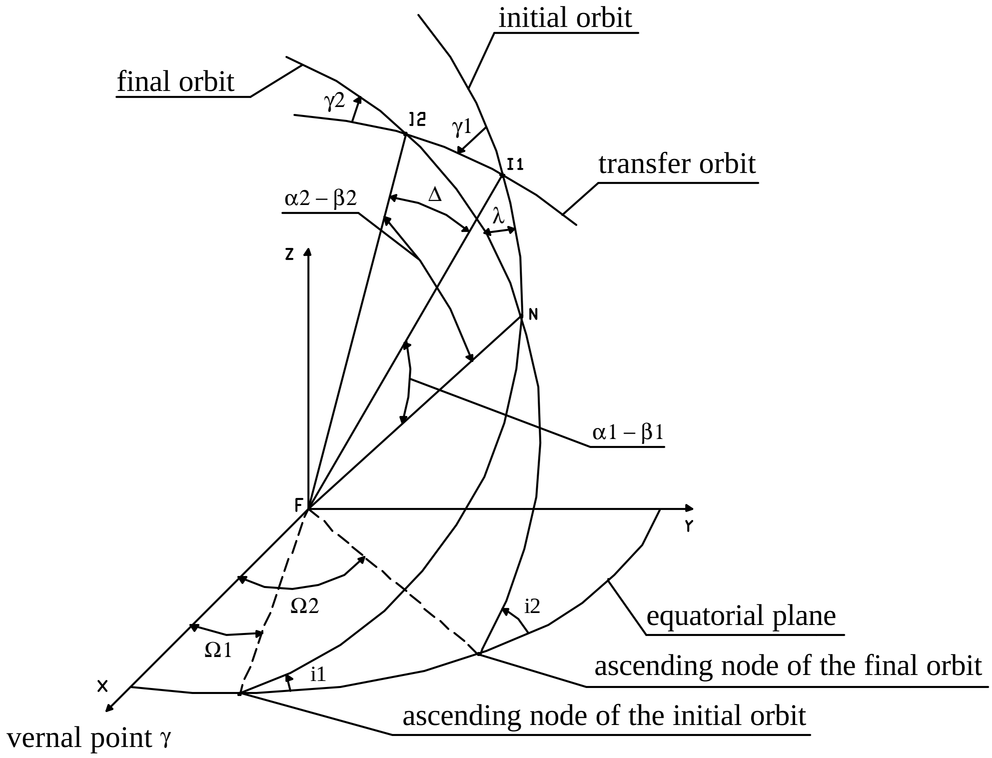

Figure 2. Geometry of the Maneuver.

From the geometry of the non-coplanar maneuver we obtain $\boldsymbol{b}_1$, $\boldsymbol{b}_2$, $\boldsymbol{l}$ and the transfer angle D:

$$\boldsymbol{b}_1 = \arctan\left[\frac{\sin(\boldsymbol{W}_2 - \boldsymbol{W}_1)\tan(180^\circ - i_2)}{\sin i_1 + \tan(180^\circ - i_2)\cos i_1 \cos(\boldsymbol{W}_2 - \boldsymbol{W}_1)}\right] - \boldsymbol{w}_1 \tag{1}$$

$$\boldsymbol{b}_2 = \arctan\left[\frac{sin(\boldsymbol{W}_2 - \boldsymbol{W}_1)\tan i_1}{sini_2 + \tan i_1 \cos(180^\circ - i_2)\cos(\boldsymbol{W}_2 - \boldsymbol{W}_1)}\right] - \boldsymbol{w}_2 \tag{2}$$

$$\boldsymbol{l} = \arcsin\left[\frac{sin(\boldsymbol{W}_2 - \boldsymbol{W}_1)sini_1}{sin(\boldsymbol{w}_2 + \boldsymbol{b}2)}\right] = \arcsin\left[\frac{sin(\boldsymbol{W}_2 - \boldsymbol{W}_1)sini_2}{sin(\boldsymbol{w}_1 + \boldsymbol{b}_1)}\right] \tag{3}$$

$$\cos D = \cos(b_1 - a_1)\cos(a_2 - b_2) + sin(b_1 - a_1)sin(a_2 - b_2)\cos(180^\circ - l) \tag{4}$$

$$sin\,\boldsymbol{D} = \frac{sin(\boldsymbol{a}_2 - \boldsymbol{b}_2)sin(180^\circ - \boldsymbol{l})}{sinB} \tag{5}$$

$$B = \arctan\left[\frac{sin(180^\circ - \boldsymbol{l})}{sin(\boldsymbol{b}_1 - \boldsymbol{a}_1)\cot(\boldsymbol{a}_2 - \boldsymbol{b}_2) - \cos(\boldsymbol{b}_1 - \boldsymbol{a}_1)\cos(180^\circ - \boldsymbol{l})}\right] \tag{6}$$

Considering that the spacecraft propulsion system is able to apply an impulsive thrust, and that the maneuver is bi-impulsive, the total velocity increment is:

$$V = V_1 + V_2 = F(\boldsymbol{C}) \tag{7}$$

The time of the transfer maneuver is:

$$\boldsymbol{T} = G(\boldsymbol{C}) \tag{8}$$

Therefore, the problem is the minimization of $\boldsymbol{T}$ for a prescribed V. If the total velocity increment is prescribed, being equal to a value V_0, we have the constrained relation:

$$V - V_0 = 0 \tag{9}$$

Thus, we have the performance index:

$$J = T + k(V - V_0) \tag{10}$$

From Eckel et al. (1984) we know that the solution of the problem depend on three variables: the semi-latus rectum p of the transfer orbit and the true anomaly $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ that define the position of the impulses in the initial and final orbits. Therefore, we have the necessary conditions:

$$\frac{\P V}{\P p} + k\frac{\P \boldsymbol{T}}{\P p} = 0 \quad ; \quad \frac{\P V}{\P \boldsymbol{a}_1} + k\frac{\P \boldsymbol{T}}{\P \boldsymbol{a}_1} = 0 \quad ; \quad \frac{\P V}{\P \boldsymbol{a}_2} + k\frac{\P \boldsymbol{T}}{\P \boldsymbol{a}_2} = 0 \tag{11}$$

By eliminating the Lagrange's multiplier k from equations 11 we have the set of two equations:

$$\frac{\partial V}{\partial p}\frac{\partial T}{\partial \boldsymbol{a}_1} - \frac{\partial V}{\partial \boldsymbol{a}_1}\frac{\partial T}{\partial p} = 0 \quad ; \quad \frac{\partial V}{\partial p}\frac{\partial T}{\partial \boldsymbol{a}_2} - \frac{\partial V}{\partial \boldsymbol{a}_2}\frac{\partial T}{\partial p} = 0 \tag{12}$$

Evaluating the partial derivatives in these equations and doing some simplifications we have the final optimal conditions:

$$(X_1 + YZes\inf_2)(S_1 q_1 - T_1 es\inf_1) + S_1 T_1 + W_1\left(\frac{W_1 - W_2}{sin\boldsymbol{D}}q_2 - W_1 \tan\frac{\boldsymbol{D}}{2}\right) - \frac{W_1 Z er_1 e_1 sin\,\boldsymbol{a}_1}{q_1 p_1 s\inf_1 sin\,\boldsymbol{g}_1} = 0 \tag{13}$$

$$\left(X_2 + Y Z es\ \inf_1\right)\left(S_2 q_2 - T_2 es\ \inf_2\right) + S_2 T_2 - W_2\left(\frac{W_2 - W_1}{sin\mathbf{D}} q_1 - W_2 \tan\frac{\mathbf{D}}{2}\right) + \frac{W_2 Z\, er_2 e_2 sin\, a_2}{q_2\, p_2 s\ \inf_2\, sin\mathbf{g}_2} = 0 \tag{14}$$

For the coplanar case we should substitute the two previous equations by the following equations:

$$\left(X_1 + Y Z e \sin f_2\right)\left(S_1 q_1 - T_1 e \sin f_1\right) + S_1 T_1 - \sqrt{\frac{m}{p_1}}\, \frac{Ze\ e_1 \sin \boldsymbol{a}_1}{q_1 V_1 \sin f_1} = 0 \tag{15}$$

$$\left(X_2 + Y Z e \sin f_1\right)\left(S_2 q_2 - T_2 e \sin f_2\right) + S_2 T_2 - \sqrt{\frac{m}{p_2}}\, \frac{Ze\ e_2 \sin \boldsymbol{a}_2}{q_2 V_2 \sin f_2} = 0 \tag{16}$$

which utilize the relations shown in the appendix A.

Thus, we have an equation system composed by Eqs. (9), (15) and (16). Solving this equation system by Newton Raphson Method (cf. Press et al., 1992) or by the Least Square Method (cf. Rocco, 2002), we obtain the transfer orbit which performs the maneuver between two coplanar terminal orbits spending a minimum time but with a specific fuel consumption. Initially we used the Newton Raphson Method, however, after that we decided to use the Least Square Method because, in this way, it was possible to obtain more accurate results. Other numeric methods can also be used, but the Least Square Method supplied satisfactory results.

4. Results

Figures (3) to (14) present some results obtained with the software developed. They not only show the tendency of the parameters, but they quantify the evolution of the variables studied. The graphs were obtained through the variation of the total velocity increment necessary to perform the maneuver. Thus, each point was obtained executing the software to the specific total velocity increment. The points were joined by a line that shows the behavior of that orbital element.

We utilized as a first example, shown in Figs. (3) to (8), the correction maneuver between two circular coplanar orbits where the initial orbit have the semi-major axis of 7000 km, and the final orbit have the semi-major axis of 7020 km. We utilized in this example the initial values l = 7011.94497514 km, $\boldsymbol{a}_1$ = 0.1 rad, and $\boldsymbol{a}_2$ = 0.2 rad. The graphs were obtained through the variation of the total necessary velocity increment from 0.1 to 14.0 km/s.

We utilized as a second example, shown in Figs (9) to (14), the correction maneuver between two elliptical coplanar orbits where the initial orbit have the semi-major axis of 7000 km, eccentricity 0.015 and longitude of the periapside 1.0 rad. The final orbit have the semi-major axis of 7020 km, eccentricity 0.015 and longitude of the periapside 1.0 rad. We utilized in this example the initial values l = 7011.94497514 km, $\boldsymbol{a}_1$ = 0.1 rad, and $\boldsymbol{a}_2$ = 0.15 rad. The graphs were obtained through the variation of the total necessary velocity increment from 0.1 to 11.0 km/s.

5. Conclusion

In Figs. (3) to (6) and (9) to (12) we can verify that when the total velocity increment increases the semi-major axis and the eccentricity of the transfer orbit also increase, however, the transfer angle and the time spent in the maneuver decrease. These behaviors occur because when the maneuver is performed with a high value of the velocity increment the transfer orbit approaches a parabolic orbit, so the eccentricity approaches one. Then, we have a high value of the semi-major axis and a small value of the transfer angle. In Figs. (7) and (13) we have the true anomaly of the positions of application of the impulses in the transfer orbit (points I_1 and I_2). We can verify that, when the necessary velocity increment increases, the true anomaly of the points I_1 and I_2 approaches to the same value. This is expected because for a high value of the velocity increment the maneuver can be performed with a small transfer angle. In Figs. (8) and (14) we can see that when the maneuver spends more time, the velocity increment is smaller than when the maneuver spends less time. This is expected because when the maneuver spends more time the impulse directions approaches the movement directions. In these graphs we can see that it was possible to obtain results when we fixed a small value of the velocity increment, but there is a lower limit which occur when we reach the solution for time free. Besides that, we should advise that the developed program can not supply the solution for all combinations of the input parameters. For certain values of the total velocity increment it can be impossible to obtain one solution, because for a very small or very large values of the total velocity increment the solution can not exist, or the numerical algorithms used in the program do not converge for the solution, because the initial values used can be too far from the solution. So, it is recommended a physical analysis of the problem, that takes into account the geometry of the maneuver, to find the range of values for the total velocity increment so that it is possible to accomplish the maneuver. Whether the

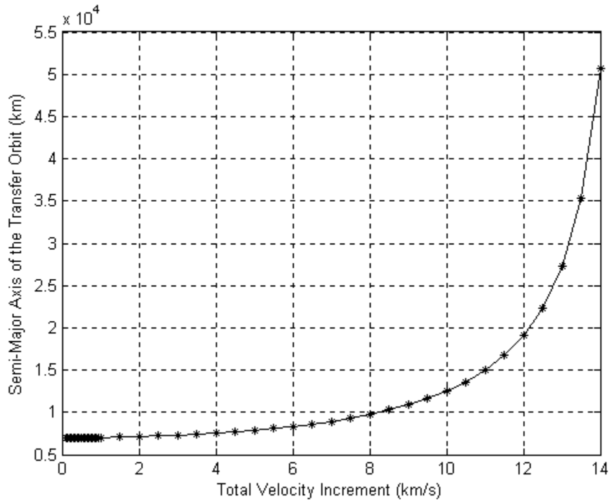

Fig. 3: Semi-Major Axis vs. Total Velocity Increment.

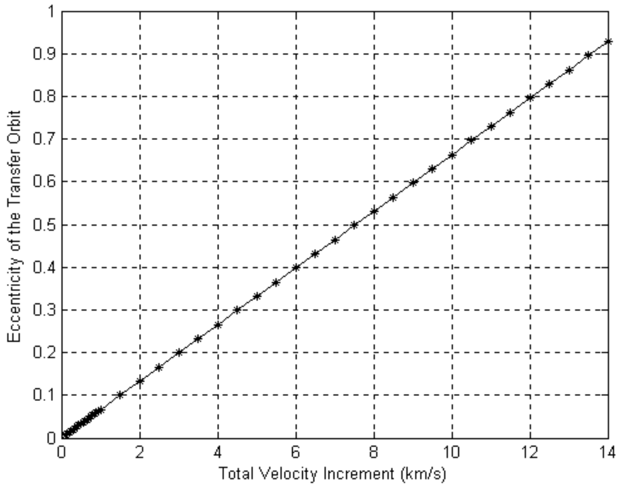

Fig.4: Eccentricity vs. Total Velocity Increment.

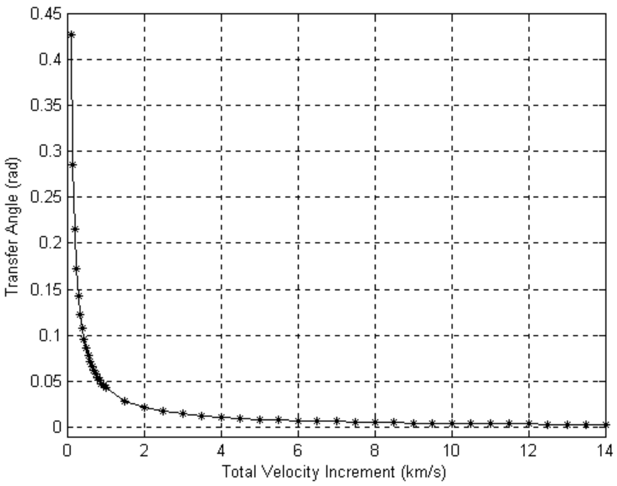

Fig. 5: Transfer Angle vs. Total Velocity Increment.

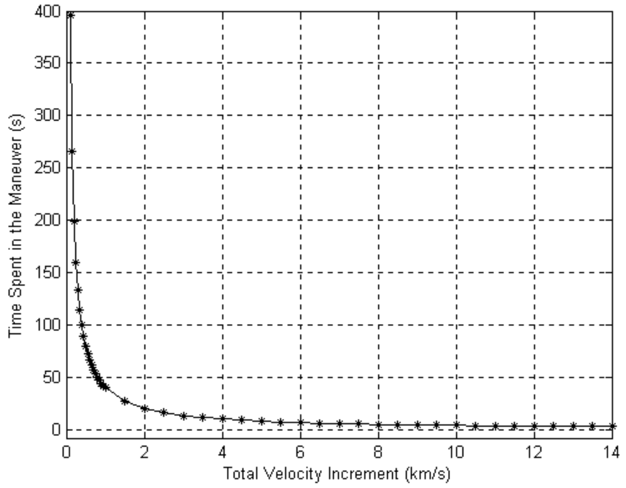

Fig. 6: Time Spent in Maneuver vs. Total Velocity Increment.

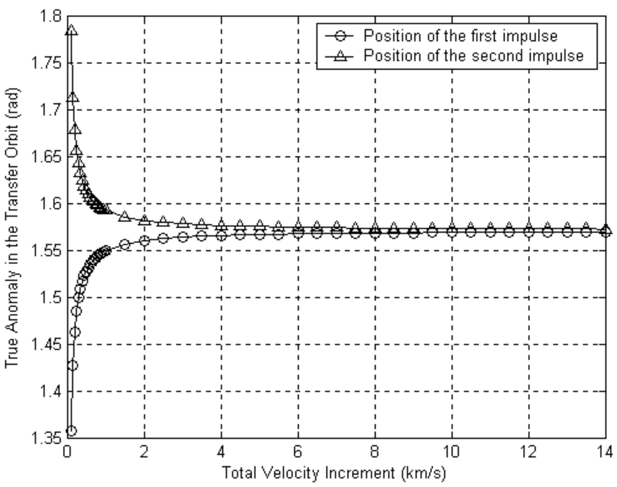

Fig. 7: True Anomaly vs. Total Velocity Increment.

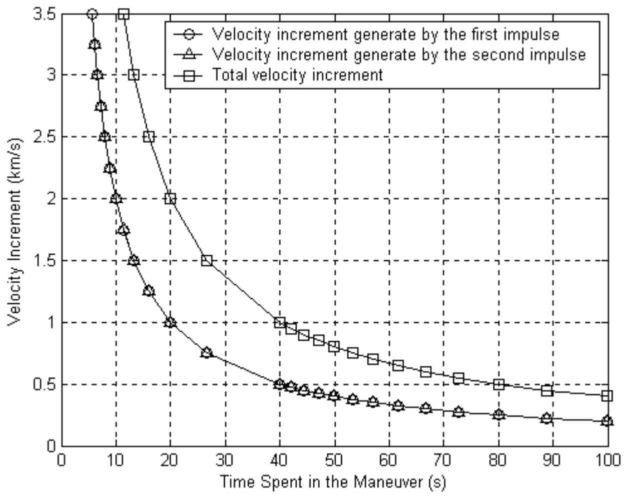

Fig. 8: Velocity Increment vs. Time Spent in Maneuver.

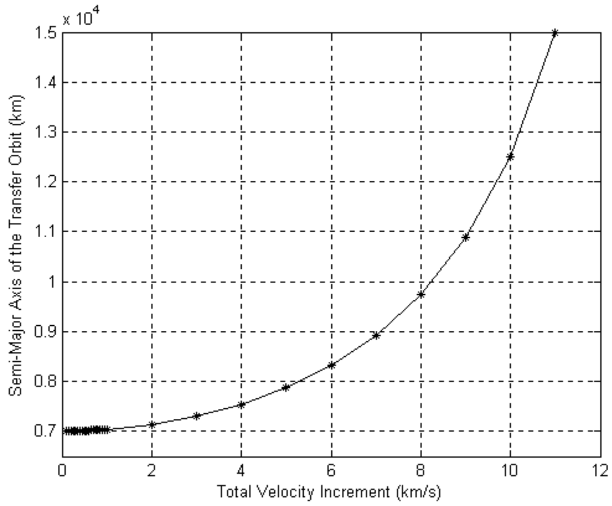

Fig. 9: Semi-Major Axis vs. Total Velocity Increment.

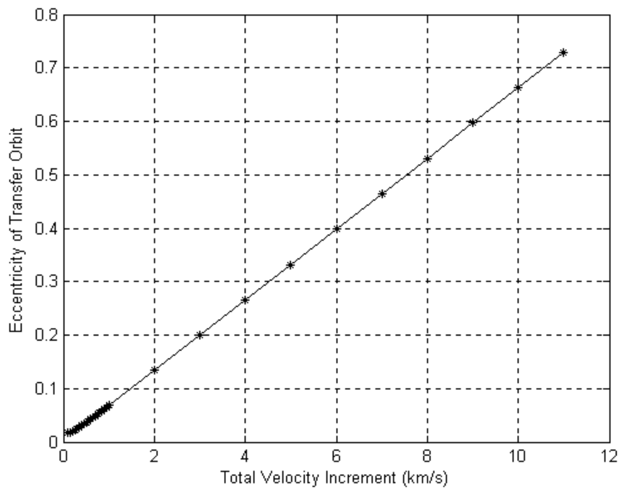

Fig.10: Eccentricity vs. Total Velocity Increment.

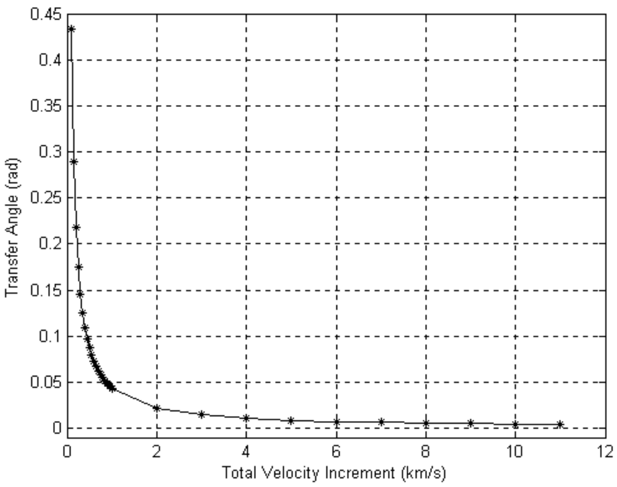

Fig. 11: Transfer Angle vs. Total Velocity Increment.

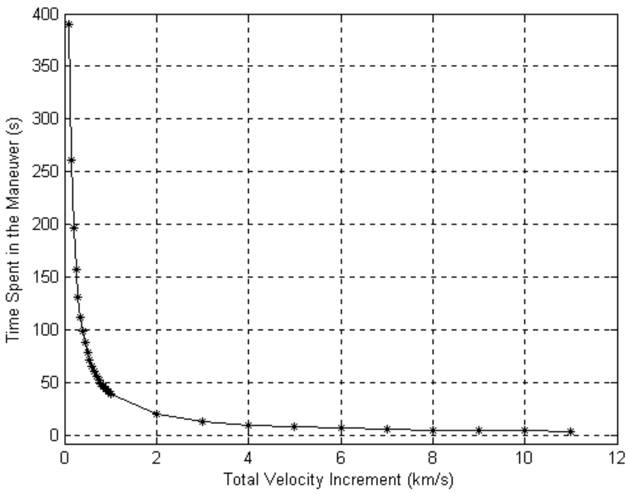

Fig. 12: Time Spent in Maneuver vs. Total Velocity Increment.

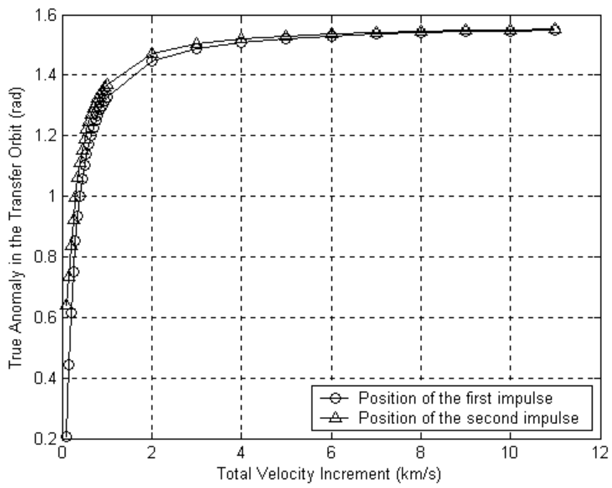

Fig. 13: True Anomaly in the Transfer Orbit vs. Total Velocity Increment.

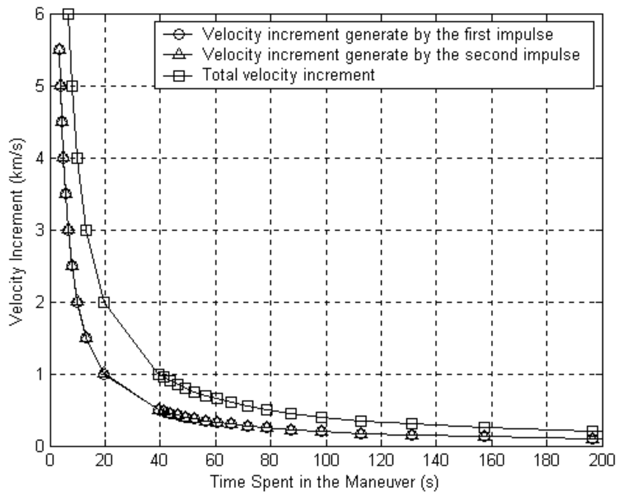

Fig. 14: Velocity Increment vs. Time Spent in Maneuver.

solution is a local or global minimum is another question to be solved. As far as we verified, the solution obtained seems to be a global minimum because, for the same input parameters, but using different initial values, it was not possible until the moment, to obtain better results. However, the equations presented by Eckel et al. (1984) are the necessary conditions for a local minimum. Thus, we cannot affirm that the obtained solutions are global minimum. It is important to notice that the software tests automatically all the results, verifying if the maneuver obtained is just a mathematical solution or if it can really be implemented. When we use numerical methods there are some solutions, which satisfy the equations, however, in practice, they are impossible. Concluding, we can verify that these results are very similar to the results obtained by Rocco (1997) for the non-coplanar MFFT case and by Rocco et al. (2000) for the non-coplanar MTPF case. Thus it is clear that the MTPF case is almost the converse of the MFFT case. Therefore, both cases, considering circular and elliptical orbits, coplanar and non-coplanar transfer maneuvers, were studied, implemented and tested with success. The simulations showed that the software developed can be used in real applications and it is capable to generate reliable results.

6. Acknowledgements

The authors express their thanks to the National Council for Scientific and Technological Development in Brazil (CNPq) and to the Foundation for Supporting Research in São Paulo State (FAPESP), for supporting this research.

7. Appendix A

$$r_i = \frac{p_i}{1 + e_i \cos \boldsymbol{a}_i} \tag{A.1}$$

$$f_1 = \arctan\left[\cot \boldsymbol{D} - \frac{r_1(p - r_2)}{r_2(p - r_1)\sin \boldsymbol{D}}\right]$$

(A.2)

$$f_2 = \arctan\left[\frac{r_2(p - r_1)}{r_1(p - r_2)\sin \boldsymbol{D}} - \operatorname{cotg} \boldsymbol{D}\right]$$

$$p = \frac{r_1 r_2(\cos f_1 - \cos f_2)}{r_1 \cos f_1 - r_2 \cos f_2} \tag{A.3}$$

$$e = \frac{r_2 - r_1}{r_1 \cos f_1 - r_2 \cos f_2} \tag{A.4}$$

$$a = \frac{p}{1 - e^2} \tag{A.5}$$

$$\boldsymbol{g}_1 = \arcsin\left[-\frac{\sin(\boldsymbol{b}_2 - \boldsymbol{a}_2)}{\sin \boldsymbol{D}} \sin \boldsymbol{f}\right]$$

(A.6)

$$\boldsymbol{g}_2 = \arcsin\left[-\frac{\sin(\boldsymbol{b}_1 - \boldsymbol{a}_1)}{\sin \boldsymbol{D}} \sin \boldsymbol{f}\right]$$

$$x_1 = \sqrt{\boldsymbol{m}}\left(\frac{e}{\sqrt{p}} \sin f_1 - \frac{e_1}{\sqrt{p_1}} \sin \boldsymbol{a}_1\right)$$

(A.7)

$$x_2 = \sqrt{\boldsymbol{m}}\left(\frac{e_2}{\sqrt{p_2}} \sin \boldsymbol{a}_2 - \frac{e}{\sqrt{p}} \sin f_2\right)$$

$$
\begin{aligned}
y_1 &= \frac{\sqrt{\mathbf{m}}}{r_1}\left(\sqrt{p} - \sqrt{p_1}\cos \mathbf{g}_1\right) \\
y_2 &= \frac{\sqrt{\mathbf{m}}}{r_2}\left(\sqrt{p_2}\cos \mathbf{g}_2 - \sqrt{p}\right)
\end{aligned}
\tag{A.8}
$$

$$
z_i = \frac{\sqrt{\mathbf{m}\, p_i}}{r_i}\sin \mathbf{g}_i \tag{A.9}
$$

$$
h_i = \left(y_i^{\,2} + z_i^{\,2}\right)^{1/2} \tag{A.10}
$$

$$
V_i = \left(x_i^{\,2} + h_i^{\,2}\right)^{1/2} \tag{A.11}
$$

$$
S_i = \frac{x_i}{V_i} \tag{A.13}
$$

$$
T_i = \frac{y_i}{V_i} \tag{A.14}
$$

$$
W_i = \frac{z_i}{V_i} \tag{A.15}
$$

$$
q_i = \frac{p}{r_i} \tag{A.16}
$$

$$
E_i = \operatorname{arc}\cos\left(\frac{e + \cos f_i}{1 + e\cos f_i}\right) \tag{A.17}
$$

$$
\sin E_i = \frac{\sqrt{1 - e^2}\,\sin f_i}{1 + e\cos f_i} \tag{A.18}
$$

$$
M_i = E_i - e\sin E_i \tag{A.19}
$$

$$
\mathbf{T} = \sqrt{\frac{a^3}{\mathbf{m}}}\left(M_{I_2} - M_{I_1} + 2\mathbf{p}N\right) \tag{A.20}
$$

$$
\begin{aligned}
X_1 &= \frac{S_1\cos \mathbf{D} - S_2}{\operatorname{sen} \mathbf{D}} + T_1 \\
X_2 &= \frac{S_1 - S_2\cos \mathbf{D}}{\operatorname{sen} \mathbf{D}} + T_2
\end{aligned}
\tag{A.21}
$$

$$
Y = \frac{1}{\left(1 - e^2\right)\operatorname{sen}\mathbf{D}}\left[3e^2\mathbf{T}\sqrt{\frac{\mathbf{m}}{p^3}} - 2e\left(\frac{1}{q_2\operatorname{sen} f_2} - \frac{1}{q_1\operatorname{sen} f_1}\right) + \operatorname{cotg} f_2 - \operatorname{cotg} f_1\right] \tag{A.22}
$$

$$
Z = \frac{q_2X_2 - q_1X_1 + \left(S_1 + S_2\right)\operatorname{tg}\mathbf{D}/2}{\operatorname{cotg} f_1 - \operatorname{cotg} f_2 + Y\left[\left(1 + e^2\right)\operatorname{sen}\mathbf{D} + 2e\left(\operatorname{sen} f_2 - \operatorname{sen} f_1\right)\right]} \tag{A.23}
$$

8. References

Eckel, K.G., 1982, “Optimal Impulsive Transfer with Time Constraint” *Acta Astronautica*, Vol. 9, No. 3, pp. 139-146.

Eckel, K.G. e Vinh, N.X., 1984, “Optimal Swtitching Conditions for Minimum Fuel Fixed Time Transfer Between non Coplanar Elliptical Orbits” *Acta Astronautica*, Vol. 11, No. 10/11, pp. 621-631.

Gobetz, F.W.; Doll, J.R., 1969, “A Survey of Impulsive Trajectories. AIAA Journal, Vol. 7, No. 5: 801-834.

Gross, L.R.; Prussing, J.E., 1974, Optimal Multiple-Impulse Direct Ascent Fixed-Time Rendezvous” *AIAA Journal*, Vol. 12, No. 7, 1969, pp. 885-889.

Hoelker, R.F.; Silber, R., 1959, “The Bi-Elliptical Transfer Between Circular Co-Planar Orbits”, *Army Ballistic Missiles Agency*, Redstone Arsenal, Alabama, EUA.

Hohmann, W., 1925, “Die Erreichbarkeit Der Himmelskoprper” Oldenbourg, Munique.

Ivashkin, V.V.; Skorokhodov, A.P., 1981, “Definition and Analysis of Properties of Optimal Three-Impulse Point-to-Orbit Transfers Under Time Constraint” *Acta Astronautica,* Vol. 8, No. 1, pp. 11-23.

Lawden, D.F., 1993, “Time-Closed Optimal Transfer by Two Impulses Between Coplanar Elliptical Orbits” *Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 3.

Lion, P.M.; Handelsnman, M., 1968, “Primer Vector on Fixed-Time Impulsive Trajectories”, *AIAA Journal*, Vol. 6 No. 1, pp. 127-132.

Marec, J.P., 1979, “Optimal Space Trajectories”, New York, NY, Elsevier.

Prado, A.F.B.A., 1989, “Análise, Seleção e Implementação de Procedimentos que Visem Manobras Ótimas de Satélites Artificiais”. Master Thesis, (INPE-5003-TDL/397).

Press, W.H.; Teukolsky, S.A.; Vetterling, W.T.; Flannery, B.P., 1992, “Numerical Recipes in FORTRAN (The Art of Scientific Computing)”, 2nd Ed. Cambridge University Press.

Prussing, J.E., 1969, “Optimal Four-Impulse Fixed-Time Rendezvous in the Vicinity of a Circular Orbit”, *AIAA Journal*, Vol. 7, No. 5, pp. 928-935.

Prussing, J.E., 1970, “Optimal Two and Three Impulse Fixed-Time Rendezvous in the Vicinity of a Circular Orbit”, *AIAA Journal*, Vol. 8, No. 7, pp. 1221-1228.

Prussing, J.E. e Chiu, J.H., 1986, “Optimal Multiple-Impulse Time-Fixed Rendezvous Between Circular Orbits”, *Journal of Guidance, Control, and Dynamics*, Vol. 9, No. 1, pp. 17-22.

Rocco, E.M., 1997, “Transferências Orbitais Bi-Impulsivas com Limite de Tempo” Master Thesis, (INPE-6676-TDI/626).

Rocco, E.M.; Prado, A.F.B.A.; Souza, M.L.O., 1999, “Bi-Impulsive Orbital Transfers Between Non-Coplanar Orbits with Time Limit”, *Sixth Pan American Congress of Applied Mechanics PACAM VI / 8th International Conference on Dynamic Problems in Mechanics DINAME*. Rio de Janeiro – RJ.

Rocco, E.M.; Prado, A.F.B.A.; Souza, M.L.O., 2000, “Orbital Trasfers Between Non-Coplanar Orbits Using Bi-Impulsive Maneuvers with Minimum Time for a Prescribed Fuel Consumption”. 15th International Symposium of Spaceflight Dynamics. Biarritz – France, June 26 – 30.

Rocco, E.M.; Souza, M.L.O.; Prado, A.F.B.A., 2002, “Comparison Between Two Methods of Optimal Coplanar Orbital Transfers With Time Limit”. XI Colóquio Brasileiro de Dinâmica Orbital. Viçosa - MG, November 4 - 8.

Rocco, E.M., 2002, “Manutenção Orbital de Constelações Simétricas de Satélites Utilizando Manobras Impulsivas Ótimas com Vínculo de Tempo” Doctoral Thesis.

Taur, D.R.; Carroll, V.C.; Prussing, J.E., 1995, “Optimal Impulsive Time-Fixed Orbital Rendezvous and Interception with Path Constraints”, *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 1, pp. 54-60.

Wang, K., 1963, “Minimum Time Transfer Between Coplanar Circular Orbits by Two Impulses and Propulsion Requirements”, *Astronautica Acta*, Vol. 9 No. 1, pp. 12-19.