ANALYTICAL ATTITUDE PROPAGATION OF SPIN STABILIZED EARTH ARTIFICIAL SATELLITES

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ABSTRACT *- An analytical approach for spin-stabilized satellites attitude propagation is presented, considering the influence of the residual magnetic torque and eddy currents torque. It is assumed the inclined dipole model for the Earth´s magnetic field and the method of averaging such torques, over each orbital period, is applied to obtain the components of the torques in the satellite body frame reference system. The inclusion of these torques on the rotational motion differential equations of spin stabilized satellites yields the conditions to derive an analytical solution. The solution shows that the eddy currents torques causes an exponential decay of the angular velocity magnitude and the coupled effect of both torques produces a precession on the spin axis. Numerical simulations performed with data of the Brazilian satellites (SCD1 and SCD2) show the agreement between the analytical solution and the actual satellite behaviour.*

KEYWORDS: artificial satellite attitude, spin-stabilized satellite, residual magnetic torque, eddy current torque, angular velocity magnitude, right ascension of spin axis, declination of spin axis**.**

INTRODUCTION

This paper aims at analyzing the rotational motion dynamics of spin stabilized Earth artificial satellites, through derivation of an analytical attitude prediction. Emphasis is placed on modeling the torques steaming from residual magnetic and eddy currents perturbations, as well as their influences on the satellite angular velocity and space orientation. A spherical coordinates system fixed in the satellite is used to located the spin axis of the satellite in relation to the terrestrial equatorial system. The direction of the spin axis are specified by the right ascension (α) and the declination (δ) as represented in the Fig. 1. The magnetic residual torque occurs due to the interaction between the Earth magnetic field and the residual magnetic moment along the spin axis of the satellite. In spin stabilized satellites, equipped with nutation dumpers, such effect is usually the most important perturbing torque. The eddy currents torque appears due to the interaction of such currents circulating along the satellite structure chassis and the Earth magnetic field.

The torque analysis is performed through the modeling of the inclined Earth magnetic dipole, which orientation depends on the magnetic colatitude and on the longitude of ascending node of the magnetic plane. Essentially an analytical averaging method is applied to determine the mean torque over an orbital period.

To compute the average components of both the residual magnetic and eddy current torques in the satellite body frame reference system (satellite system), an average time in the fast varying orbit element, the mean anomaly, is utilized. This approach involves several rotation matrices, which are dependent on the orbit elements, right ascension and declination of the satellite spin axis, the magnetic colatitude and on the longitude of ascending node of the magnetic plane. Afterwards, the inclusion of such torques on the rotational motion differential equations of spin stabilized satellites yields the conditions to derive an analytical solution. The theory is developed accounting also for orbit elements time variation, not restricted to circular orbits, giving rise to some hundreds of curvature integrals solved analytically.
Numerical simulations performed with data of the spin stabilized SCD1 and SCD2 Brazilian satellites show the agreement between the analytical solution an the actual satellite behaviour.

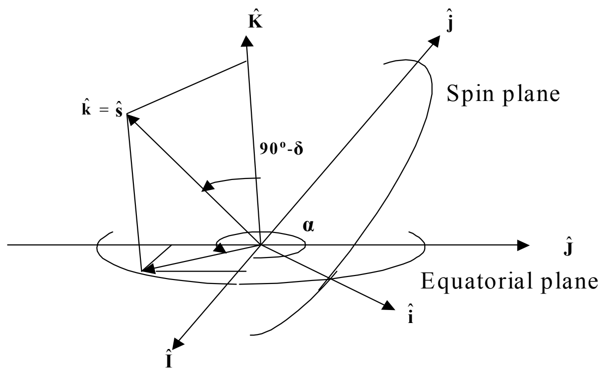

Figure 1 - Orientation of the spin axis ($\hat{s}$): Equatorial System ($\hat{I}, \hat{J}, \hat{K}$), satellite body frame reference system ($\hat{i}$, $\hat{j}, \hat{k}$), right ascension (α) and declination (δ) of the spin axis.

GEOMAGNETIC FIELD

An inclined Earth magnetic dipole model is assumed in this paper. Its orientation depends on the magnetic colatitude (β) and on the longitude of ascending node of the magnetic plane (η). The magnetic reference system, which axis z_m is along the dipole vector, β and η are represented in the Fig.2 .

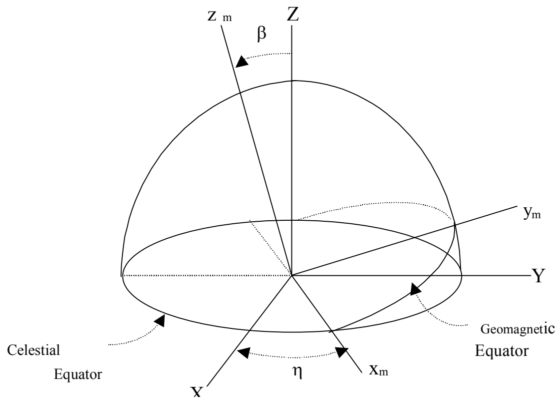

Figure 2- Magnetic System (O\`x_m $y_m z_m$) and Equatorial System (O´XYZ).

It is well known that the Earth magnetic dipole model (Thomas and Capellari, 1964; Wertz,1978) may be expressed by:

$$\vec{B} = \frac{\ell}{4\pi\mu_o r^3}\left[\hat{k}_m - 3\left(\hat{i}_s \cdot \hat{k}_m\right)\hat{i}_s\right], \tag{1}$$

where ℓ is the magnetic moment of Earth's field magnitude, μ_0 the permeability of free space, r the radius vector magnitude of the satellite, $\hat{k}_m$ the unit vector along the dipole vector and $\hat{i}_s$ the unit vector along the radius vector of the satellite ($\vec{r}$).

Thus, the unit vectors $\hat{k}_m$ and $\hat{i}_s$ can be expressed in the satellite system through rotation matrices dependent on the orbit elements, right ascension and declination of the satellite spin axis and the angles β and η.

RESIDUAL AND EDDY CURRENTS TORQUES

Magnetic residual torques result from the interaction between the spacecraft's residual magnetic field and the Earth´s magnetic fields. If $\vec{m}$ is the magnetic moment of the spacecraft and $\vec{B}$ is the geomagnetic field, the residual magnetic torques is given by (Wertz,1978):

$$\vec{N}_r = \vec{m} \times \vec{B}. \tag{2}$$

For the spin stabilized satellite, with appropriate nutation dampers, the magnetic moment is mostly aligned along the spin axis and the residual torque can be expressed by (Kuga et al., 1987):

$$\vec{N}_r = M_s \hat{k} \text{ x } \vec{B}, \tag{3}$$

where M_s is the satellite magnetic moment along its spin axis and $\hat{k}$ is the unit vector along the spin axis of the satellite. On the other hand, the torque induced by eddy currents is caused by the spacecraft spinning motion. If $\vec{W}$ is the spacecraft's angular velocity vector and p is the Foucault parameter representing the geometry and material of the satellite chassis (Wertz, 1978), this torque may be modeled by:

$$\vec{N}_i = p \ \vec{B} \text{ x } (\vec{B} \text{ x } \vec{W}). \tag{4}$$

For a spin stabilized satellite the spacecraft's angular velocity vector and the satellite magnetic moment are along the z-axis and induced eddy currents torque can the expressed by (Kuga et al., 1987):

$$\vec{N}_i = p W \ \vec{B} \text{ x } (\vec{B} \text{ x } \hat{k}). \tag{5}$$

MEAN RESIDUAL AND EDDY CURRENTS TORQUES

In order to obtain the mean residual and eddy currents torques, it is necessary to integrate the instantaneous torque $\vec{N}_r$ and $\vec{N}_i$ over one orbital period T:

$$\vec{N}_{r_m} = \frac{1}{T}\int_{t_i}^{t_i+T} \vec{N}_r \, dt \qquad \text{and} \qquad \vec{N}_{i_m} = \frac{1}{T}\int_{t_i}^{t_i+T} \vec{N}_i \, dt, \tag{6}$$

where: t is the time, t_i the initial time and T the orbital period. Changing the independent variable to the fast varying true anomaly, the mean residual and eddy currents torque can be obtained by (Quirelli, 2002):

$$\vec{N}_{r_m} = \frac{1}{T}\int_{\upsilon_i}^{\upsilon_i+2\pi} \vec{N}_r \frac{r^2}{h} d\upsilon \qquad \text{and} \qquad \vec{N}_{i_m} = \frac{1}{T}\int_{\upsilon_i}^{\upsilon_i+2\pi} \vec{N}_i \frac{r^2}{h} d\upsilon, \tag{7}$$

where υ_i is the true anomaly at instant t_i and h is the specific angular moment of orbit. Since the instantaneous torques are given by (3) and (5) and

$$r = \frac{a\left(1-e^2\right)}{1+e\,\mathrm{Cos}\,\upsilon}, \qquad h = \frac{2\pi a^2 \left(1-e^2\right)^{1/2}}{T}, \tag{8}$$

where a is the semi-major axis and e the eccentricity of orbit, the mean residual and eddy currents torque (7) becomes

$$\vec{N}_{r_m} = \Re \ \hat{k} \text{ x } \int_{\upsilon_i}^{\upsilon_i+2\pi} \left[\hat{k}_m - 3\left(\hat{i}_s \cdot \hat{k}_m\right)\hat{i}_s\right]\left(1+e\,\mathrm{Cos}\,\upsilon\right)d\upsilon, \tag{9}$$

$$\vec{N}_{i_m} = \Re_i \int_{\upsilon_i}^{\upsilon_i+2\pi} W \left[\hat{k}_m - 3\left(\hat{i}_s\cdot\hat{k}_m\right)\hat{i}_s\right]\times\left\{\left[\hat{k}_m - 3\left(\hat{i}_s\cdot\hat{k}_m\right)\hat{i}_s\right]\times\hat{k}\right\}(1+e\,Cos\,\upsilon)^4\, d\upsilon, \tag{10}$$

with x being the vector product and with

$$\Re = \frac{M_s\,\ell}{8\pi^2\,\mu_o\,a^3\left(1-e^2\right)^{3/2}} \quad \text{and} \quad \Re_i = \frac{p\,\ell^2}{32\pi^3\,\mu_o{}^2\,a^6\left(1+e^2\right)^{9/2}}. \tag{11}$$

To evaluate the integrals of (9) and (10) we will use the elliptic expansions of the true anomaly in terms of the mean anomaly M (Brouwer & Clemence, 1961), including terms up to first order in the eccentricity (e). Then the present development can be applicable for elliptical orbits without loss of precision. For simplification of the integrals we will consider the initial time for integration equal to the instant that the satellite passes through the perigee, without loss of generality.

The components of the unit vector $\hat{k}_m$ in the satellite system depends on the magnetic colatitude (β) and ascending node of the magnetic plane (η) and right ascension (α) and declination (δ) of the spin axis (Quirelli, 2002; Thomas & Capellari, 1964). In this paper, we will consider (Thomas and Capellari ,1964):

$$\eta = \eta_o + bM \quad \text{and} \quad b = \frac{\omega_e\,T}{2\pi} \tag{12}$$

where M is the mean anomaly, η_o is the initial position of the ascending node of the geomagnetic equator at the instant the satellite is at the perigee and ω_e is the angular velocity of the Earth.

The components of the unit vector $\hat{i}_S$ in the satellite system depend on ascending node orbit (Ω), orbital inclination (i), the true anomaly (ν) and right ascension (α) and declination (δ) of the spin axis (Quirelli, 2002). For one orbital period averaging the angles Ω , i, α, δ and β are fixed constant.

Thus, using trigonometric properties and after exhausting but simple algebraic developments, which involves some hundred of curvature integrals, the mean residual and mean eddy currents torques can be expressed by (Quirelli, 2002):

$$\vec{N}_{rm} = \Re\left\{N_{rx}\,\hat{i} + N_{ry}\,\hat{j}\right\} \quad \text{and} \quad \vec{N}_{im} = \Re_i\; W\left(N_{ix}\,\hat{i} + N_{iy}\,\hat{j} + N_{iz}\,\hat{k}\right), \tag{13}$$

where:

$$\begin{aligned} N_{rx} = {} & Sen\beta\{A\,Cos\alpha\,Sen\delta - B\,Sen\alpha\,Sen\delta + 3[Sen\delta\,Cos(\Omega-\alpha)(D_1\,Sen\Omega + \\ & + D_3\,Cos\Omega\,Cos\,i - C_1\,Cos\Omega + C_3\,Sen\Omega\,Cos\,i) + \left(Sen\,i\,Cos\delta + Cos\,i\,Sen\delta\,Sen(\Omega-\alpha)\right) \\ & \left[D_2\,Cos\Omega\,Cos\,i + D_3\,Sen\Omega + C_2\,Sen\Omega\,Cos\,i - C_3\,Cos\Omega\right]]\} \\ & + \pi Cos\beta\{-2Cos\delta + 3Sen\,i\,Cos\,i\,Sen\delta\,Sen(\Omega-\alpha) + 3Sen^2 i\,Cos\delta\}, \end{aligned} \tag{14}$$

$$\begin{aligned} N_{ry} = {} & Sen\beta\;\{A\,Sen\alpha - B\,Cos\alpha + 3\,[\,Sen(\Omega-\alpha)(D_1\,Sen\Omega + \\ & + D_3\,Cos\Omega\,Cos\,i - C_1\,Cos\Omega + C_3\,Sen\Omega\,Cos\,i) + \left(Cos\,i\,Cos(\Omega-\alpha)\right) \\ & \left[\,D_2\,Cos\Omega\,Cos\,i + D_3\,Sen\Omega + C_2\,Sen\Omega\,Cos\,i - C_3\,Cos\Omega\,\right]]\} + \\ & + \pi\cos\beta\left[3Sen\,i\,Cos\,i\,Cos(\Omega-\alpha)\right]\}, \end{aligned} \tag{15}$$

N_{ix}, N_{iy}, N_{iz} and the coefficients A, B, C_i, D_i, i = 1,2,3,4, are described explicitly in Quirelli(2002); and N_{ix}, N_{iy}, N_{iz} depend on orbital elements (Ω, ω, i), the attitude angles (δ, α) and the magnetic colatitude β as arguments of trigonometric functions.

THE ROTATIONAL MOTION EQUATIONS

The variations of the angular velocity, the declination and the ascension right of the spin axis for spin stabilized artificial satellites are given by the Euler equations in spherical coordinates (Kuga et al, 1987) :

$$\dot{W} = \frac{1}{I_z} N_z \tag{16}$$

$$\dot{\delta} = \frac{1}{I_z\,W} N_y \tag{17}$$

$$\dot{\alpha} = \frac{1}{I_z \; W \; \mathrm{Cos}\,\delta} \; N_x \tag{18}$$

where I_z is moment of inertia along the spin axis, N_x, N_y, N_z are the components of the external torques in the satellite body frame reference system. By substituting N_{rm} and N_{im}, given in (13), in equations (16), (17) and (18), we get:

$$\frac{d\,W}{d\,t} = \frac{1}{I_z} \; N_{iz} \; \Re_i \; W \;, \tag{19}$$

$$\frac{d\,\delta}{d\,t} = \frac{1}{I_z \; W} \left(N_{ry} \; \Re + N_{iy} \; \Re_i \; W \right), \tag{20}$$

$$\frac{d\,\alpha}{d\,t} = \frac{1}{I_z \; W \; \mathrm{Cos}\,\delta} \left(N_{rx} \; \Re + N_{ix} \; \Re_i \; W \right). \tag{21}$$

Then it is possible to observe that the eddy currents torque affects the angular velocity magnitude and the spin axis, while the residual torque does not affect the angular velocity magnitude. The equations (19), (20) and (21) will be integrated assuming that the orbital elements, attitude angles (δ, α) and the magnetic colatitude (β) are held constant over one orbital period.

Analysis of the Angular Velocity Magnitude

The variation of the angular velocity magnitude (19) can the expressed as:

$$\frac{d\,W}{d\,t} = k \; d\,t \quad \text{with} \quad k = \frac{N_{iz} \; \Re_i}{I_z} \;. \tag{22}$$

If the parameter k is considered constant for one orbital period, the analytical solution of eq. (22) is:

$$W = W_0 \; e^{k\,t} \quad , \tag{23}$$

where W_0 is the initial angular velocity. Then when $k < 0$ the angular velocity magnitude decays with an exponential profile.

Analysis of the Declination of Spin Axis

By substituting the solution of the angular velocity magnitude (23) in the equation (20) and performing an integration over one orbit period, the variation of the spin axis declination is given by:

$$\delta = \delta_0 - \frac{k_1}{k} \; (\, e^{-kt} \; - \; 1) \; + \; k_2 t \;, \tag{24}$$

where:

$$k_1 = \frac{N_{ry} \; \Re}{I_z \; W_0} \qquad \text{and} \qquad k_2 = \frac{N_{iy} \; \Re_i}{I_z} . \tag{25}$$

Therefore the spin axis declination has a secular variation modulated by k_2 (associated to eddy currents torque), an exponential variation modulated by k_1/k (associated with residual and eddy currents torques) and an average term k_1/k for orbital period. The effect is a slow drift of the satellite spin axis**.**

Analysis of the Right Ascension of Spin Axis

By substituting the solution of the angular velocity magnitude (23) in the equation (21) and by integration over one orbit period, the variation of the right ascension of the spin axis is expressed by:

$$\alpha = \alpha_0 \; + \; \frac{k_3}{k} \; (1 - \; e^{-kt} \;) + \; k_4 t \;, \tag{26}$$

where:

$$k_3 = \frac{N_{rx} \; \Re}{I_z \; W_0 \; \mathrm{Cos}\,\delta} \qquad \text{and} \qquad k_4 = \frac{N_{ix} \; \Re_i}{I_z \; \mathrm{Cos}\,\delta} \tag{27}$$

Therefore the eddy currents torque causes a small precession in the spin axis and short periodic variations would be caused by the residual and eddy currents torque.

The solutions presented in the equations (22), (24) and (26), for the angular velocity magnitude, declination and right ascension of the spin axis respectively, are valid for one orbital period. Thus, for every orbital period, the orbital data must be updated, taking into account at least the main influences of the Earth oblateness. In a similar way, the initial values of the magnetic residual moment, the parameter of Foucault, right ascension and declination of the spin must be updated. With this approach, the long term analytical solution will be close to the actual behavior of the satellite.

APPLICATIONS

Here, applications of the developed theory will be presented, for the spin stabilized Brazilian satellites (SCD1 and SCD2), which are quite appropriated for verification and comparison of the theory with the data generated and processed by the satellite control center of INPE.

Results for SCD 1 satellite

The initial conditions of attitude had been taken for date of 24 of July, 1993 to the 00:00:00 GMT, supplied by the INPE's Satellite Control Center (SCC). Also, the orbital elements supplied by the SCC was updated daily by an orbit propagation program. A table of results with the data from SCC and computed values by the present analytical theory is presented in the Appendix A.

The behaviour of the SCD1 attitude along time is shown in Fig. 3.

In accordance with the results, for the period of the test it is possible to note that the mean error deviation in right ascension was –1.07°, 0.003° in declination, and in the angular velocity magnitude it was 0.10 rpm.

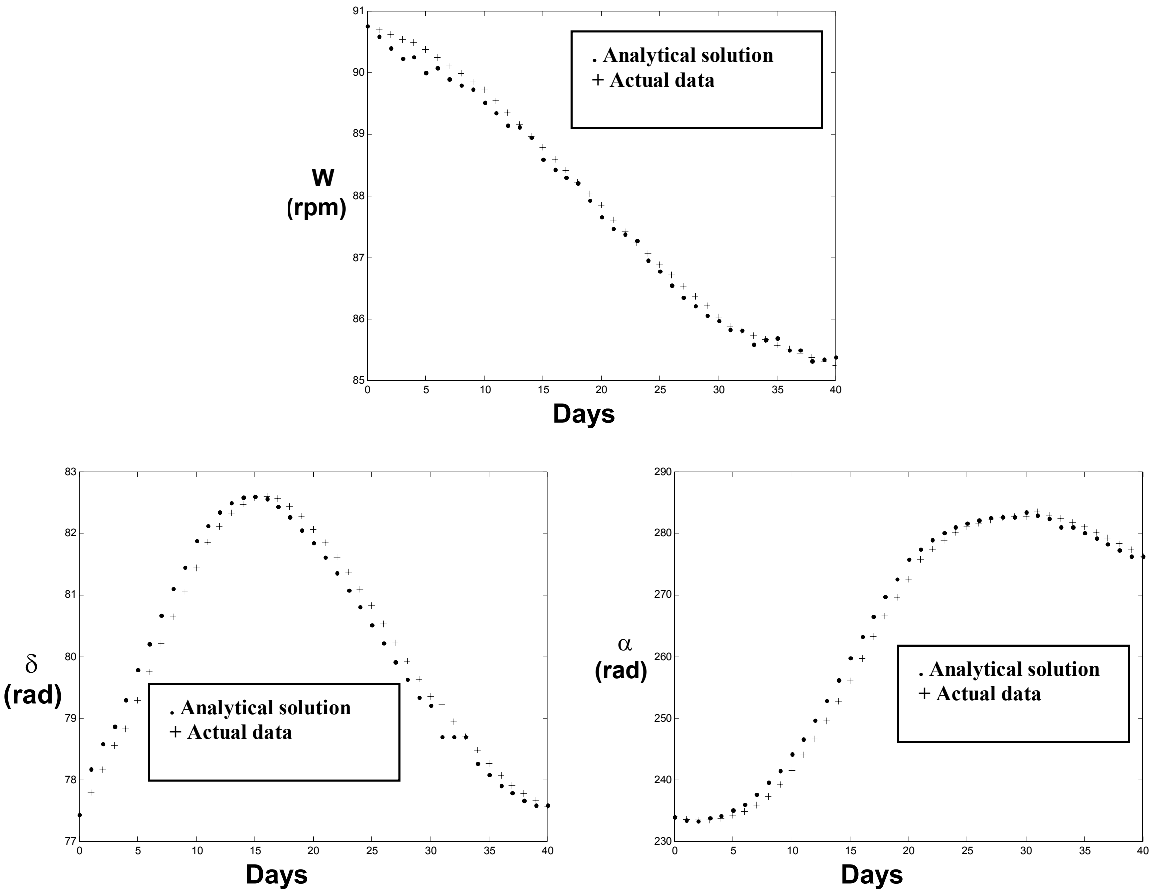

Figure 3 - Evolution of the angular velocity magnitude (W) , declination (δ) and right ascension (α) of satellite spin axis for SCD1

Results for SCD2 satellite

Here, the initial conditions of attitude had been taken for 01 February, 2002 at 00:00:00 GMT. Similarly to SCD1 case, the orbital elements supplied by the SCC was updated daily by an orbit propagation program. A table of results with the data from SCC and computed values by our analytical theory is presented in the Appendix B.

The behavior of the SCD2 attitude is shown in Fig. 4. The discontinuities observed in Fig. 4, for the right ascension and declination of the spin axis respectively, occur due to the attitude control corrections effected by the SCC during the period of test.

In accordance with the results, for the period of the test it is possible to note that the mean error deviation in right ascension was 0.357^o , 0.166^o in declination, and in the angular velocity magnitude it was 0.0366 rpm.

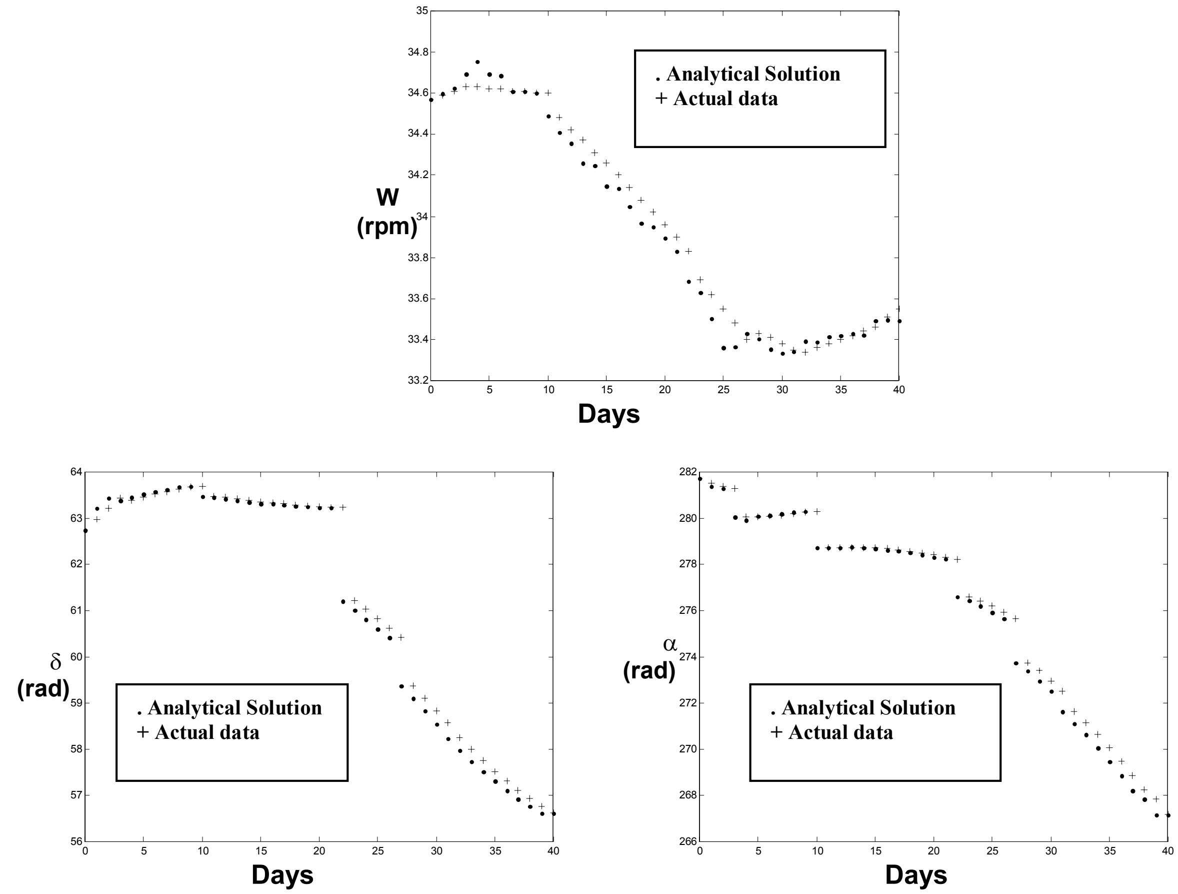

Figure 4 - Evolution of the angular velocity magnitude (W) , declination (δ) and right ascension (α) of satellite spin axis for SCD2

Mean pointing deviation

For the tests it is important to observe the deviation between the actual SCC supplied and the analytically computed attitude, for each satellite. It can be computed by :

$$\theta = \mathrm{Cos}^{-1}\left(\hat{i}\ \hat{i}_c + \hat{j}\ \hat{j}_c + \hat{k}\ \hat{k}_c\right)$$

where ($\hat{i}, \hat{j}, \hat{k}$) indicates the unity vectors computed by SCC and ($\hat{i}_c, \hat{j}_c, \hat{k}_c$) indicates the unity vector computed by the presented theory.

Fig. 5 and 6 present the pointing deviation for the test period. The mean pointing deviation was 0.788^o for the SCD1 and 0.481^o for SCD2, which is within the dispersion range of the attitude determination system performance of INPE's control center.

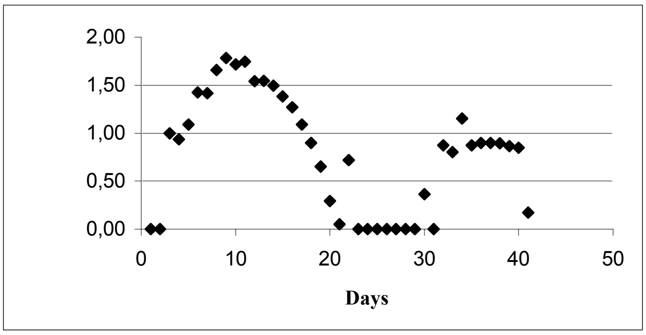

Figure 5 – Pointing Deviation Evolution (in degrees) for SCD1.

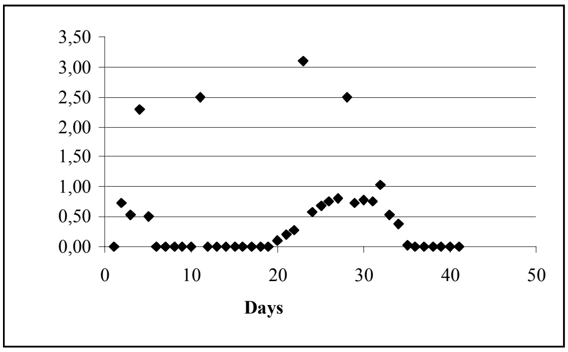

Figure 6 – Pointing Deviation Evolution (in degrees) for SCD2.

SUMMARY

In this work an analytical approach for the spin-stabilized satellite rotational motion was presented. It was assumed the influence of the residual and eddy currents torques. The models for the residual and eddy currents torques was discussed, considering the inclined Earth magnetic dipole.

The analytical solution shows that the eddy currents torque causes an exponential decay of the angular velocity magnitude and the coupled effect of both torques produce a precession on the spin axis. The theory was developed for non circular orbits and it can be applicable for elliptical orbits with precision.

The theory was coded in a PC micro-computer. Then the program was executed using the data of SCD1 and SCD2 Brazilian satellites. Results have shown the agreement between the analytical solution and the actual satellite behaviour.

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APPENDIX A

Table 1. W, α and δ supplied by the SCC, calculated values (index c) and deviations for SCD1 satellite.

Day	W (rpm)	W_c (rpm)	$W-W_c$ (rpm)	α (°)	α_c (°)	$\alpha - \alpha_c$ (°)	δ(°)	δ_c (°)	$\delta - \delta_c$ (°)
24/07/93	90.76	90.76	0	233.94	233.94	0	77.43	77.43	0
25/07/93	90.69	90.58	0.11	233.61	233.42	0.19	77.79	78.17	-0.38
26/07/93	90.61	90.40	0.21	233.44	233.81	-0.37	78.17	78.58	-0.41
27/07/93	90.54	90.23	0.31	233.43	233.80	-0.37	78.56	78.87	-0.31
28/07/93	90.49	90.26	0.23	233.75	234.24	-0.49	78.83	79.30	-0.47
29/07/93	90.37	90.00	0.37	234.18	235.17	-0.99	79.29	79.78	-0.49
30/07/93	90.24	90.07	0.17	234.91	236.00	-1.09	79.75	80.21	-0.46
31/07/93	90.11	89.90	0.21	235.94	237.65	-1.71	80.21	80.67	-0.46
01/08/93	89.99	89.79	0.20	237.35	239.60	-2.25	80.65	81.10	-0.45
02/08/93	89.85	89.73	0.12	239.20	241.57	-2.37	81.06	81.45	-0.39
03/08/93	89.72	89.52	0.20	241.45	244.24	-2.79	81.44	81.88	-0.44
04/08/93	89.54	89.34	0.20	244.04	246.64	-2.60	81.86	82.12	-0.26
05/08/93	89.35	89.14	0.21	246.62	249.65	-3.03	82.12	82.34	-0.22
06/08/93	89.16	89.12	0.04	249.53	252.86	-3.33	82.33	82.50	-0.17
07/08/93	88.97	88.94	0.03	252.74	256.18	-3.44	82.48	82.58	-0.10
08/08/93	88.79	88.59	0.20	256.15	259.77	-3.62	82.58	82.60	-0.02
09/08/93	88.59	88.42	0.17	259.70	263.21	-3.51	82.60	82.56	0.04
10/08/93	88.41	88.30	0.11	263.20	266.61	-3.41	82.56	82.43	0.13
11/08/93	88.22	88.21	0.01	266.55	269.80	-3.25	82.44	82.27	0.17
12/08/93	88.03	87.93	0.10	269.70	272.58	-2.88	82.28	82.05	0.23
13/08/93	87.85	87.66	0.19	272.54	275.77	-3.23	82.06	81.84	0.22
14/08/93	87.61	87.46	0.15	275.75	277.46	-1.71	81.85	81.61	0.24
15/08/93	87.42	87.37	0.05	277.45	278.93	-1.48	81.62	81.36	0.26
16/08/93	87.24	87.27	-0.03	278.90	280.13	-1.23	81.37	81.08	0.29
17/08/93	87.06	86.96	0.10	280.09	281.01	-0.92	81.10	80.81	0.29
18/08/93	86.88	86.77	0.11	281.01	281.74	-0.73	80.82	80.52	0.30
19/08/93	86.71	86.55	0.16	281.74	282.23	-0.49	80.53	80.22	0.31
20/08/93	86.54	86.35	0.19	282.24	282.55	-0.31	80.23	79.92	0.31
21/08/93	86.37	86.20	0.17	282.57	282.69	-0.12	79.93	79.63	0.30
22/08/93	86.21	86.06	0.15	282.70	282.65	0.05	79.64	79.34	0.30
23/08/93	86.04	85.96	0.08	282.67	283.47	-0.80	79.35	79.21	0.14
24/08/93	85.88	85.82	0.06	283.50	283.00	0.50	79.22	78.69	0.53
25/08/93	85.80	85.82	-0.02	283.01	282.40	0.61	78.95	78.69	0.26
26/08/93	85.73	85.58	0.15	282.43	281.00	1.43	78.70	78.69	0.01
27/08/93	85.66	85.66	0.00	281.76	280.99	0.77	78.48	78.27	0.21
28/08/93	85.58	85.68	-0.10	281.01	280.16	0.85	78.27	78.08	0.19
29/08/93	85.51	85.49	0.02	280.18	279.28	0.90	78.08	77.91	0.17
30/08/93	85.44	85.50	-0.06	279.29	278.32	0.97	77.91	77.79	0.12
31/08/93	85.37	85.32	0.05	278.34	277.35	0.99	77.78	77.67	0.11
01/09/93	85.31	85.35	-0.04	277.36	276.30	1.06	77.67	77.59	0.08
02/09/93	85.24	85.38	-0.14	276.34	276.29	0.05	77.58	77.59	-0.01

APPENDIX B

Table 2. W, α and δ supplied by the SCC, calculated values (index c) and deviations for SCD2.

Day	W (rpm)	W_c (rpm)	W-W_c (rpm)	α (°)	α_c (°)	α - α_c (°)	δ(°)	δ_c (°)	δ - δ_c (°)
01/02/02	34.57	34.57	0	281.72	281.72	0	62.74	62.74	0
02/02/02	34.59	34.59	0.00	281.53	281.37	0.16	62.98	63.20	-0.22
03/02/02	34.61	34.62	-0.01	281.38	281.27	0.11	63.21	63.42	-0.21
04/02/02	34.63	34.69	-0.06	281.28	280.04	1.24	63.43	63.38	0.05
05/02/02	34.63	34.75	-0.12	280.05	279.92	0.13	63.39	63.44	-0.05
06/02/02	34.62	34.69	-0.07	280.06	280.08	-0.02	63.46	63.52	-0.06
07/02/02	34.62	34.68	-0.06	280.09	280.12	-0.03	63.53	63.58	-0.05
08/02/02	34.61	34.61	0.00	280.13	280.17	-0.04	63.58	63.63	-0.05
09/02/02	34.61	34.60	0.01	280.18	280.24	-0.06	63.63	63.67	-0.04
10/02/02	34.60	34.60	0.00	280.25	280.30	-0.05	63.67	63.70	-0.03
11/02/02	34.60	34.48	0.12	280.31	278.70	1.61	63.70	63.47	0.23
12/02/02	34.48	34.40	0.08	278.71	278.72	-0.01	63.47	63.45	0.02
13/02/02	34.42	34.35	0.07	278.73	278.73	0.00	63.45	63.42	0.03
14/02/02	34.37	34.25	0.12	278.74	278.74	0.00	63.42	63.38	0.04
15/02/02	34.31	34.24	0.07	278.74	278.72	0.02	63.39	63.35	0.04
16/02/02	34.26	34.14	0.12	278.72	278.68	0.04	63.36	63.32	0.04
17/02/02	34.20	34.13	0.07	278.68	278.63	0.05	63.33	63.30	0.03
18/02/02	34.14	34.04	0.10	278.63	278.57	0.06	63.31	63.28	0.03
19/02/02	34.08	33.96	0.12	278.57	278.50	0.07	63.29	63.25	0.04
20/02/02	34.02	33.94	0.08	278.50	278.42	0.08	63.27	63.24	0.03
21/02/02	33.96	33.89	0.07	278.42	278.33	0.09	63.25	63.23	0.02
22/02/02	33.90	33.82	0.08	278.33	278.23	0.10	63.24	63.22	0.02
23/02/02	33.83	33.68	0.15	278.23	276.60	1.63	63.23	61.20	2.03
24/02/02	33.69	33.63	0.06	276.60	276.42	0.18	61.22	61.00	0.22
25/02/02	33.62	33.50	0.12	276.42	276.20	0.22	61.03	60.81	0.22
26/02/02	33.55	33.36	0.19	276.20	275.94	0.26	60.83	60.59	0.24
27/02/02	33.48	33.36	0.12	275.94	275.64	0.30	60.62	60.41	0.21
28/02/02	33.40	33.43	-0.03	275.64	273.74	1.90	60.42	59.38	1.04
01/03/02	33.43	33.40	0.03	273.75	273.38	0.37	59.38	59.10	0.28
02/03/02	33.41	33.35	0.06	273.39	272.96	0.43	59.11	58.82	0.29
03/03/02	33.38	33.33	0.05	272.97	272.50	0.47	58.84	58.54	0.30
04/03/02	33.35	33.34	0.01	272.52	271.62	0.90	58.57	58.24	0.33
05/03/02	33.34	33.39	-0.05	271.63	271.12	0.51	58.25	57.98	0.27
06/03/02	33.36	33.38	-0.02	271.14	270.62	0.52	58.00	57.74	0.26
07/03/02	33.38	33.41	-0.03	270.63	270.05	0.57	57.75	57.51	0.24
08/03/02	33.40	33.42	-0.02	270.07	269.48	0.59	57.52	57.30	0.22
09/03/02	33.42	33.42	0.00	269.49	268.85	0.64	57.31	57.10	0.21
10/03/02	33.44	33.42	0.02	268.87	268.21	0.66	57.11	56.92	0.19
11/03/02	33.46	33.49	-0.03	268.24	267.83	0.41	56.94	56.76	0.18
12/03/02	33.51	33.49	0.02	267.84	267.15	0.69	56.77	56.61	0.16
13/03/02	33.55	33.49	0.06	267.18	267.16	0.02	56.63	56.62	0.01