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ABSTRACT

In the present paper we present an analytical description of the powered swing-by maneuver in the three-dimensional space. Analytical equations are derived, based in the patched conics approximation and in the fact that the impulse applied is small compared to the velocity of the spacecraft, in order to calculate the variation in velocity, angular momentum, energy and inclination of the spacecraft during the maneuver. A study is also executed to investigate in which cases the impulse is more efficient when applied during or after the point of maximum approach. Finally, those same maneuvers are computed with the dynamics given by the restricted problem of three bodies and the results are compared with those obtained via the "patched-conics" approximation.

INTRODUCTION

The swing-by maneuver is a very popular technique used to decrease fuel expenditure in space missions. The literature shows many applications of the swing-by technique to different dynamical systems. The first dates from the sixties, when Flandro (1966) made the first studies for the (then) future Voyager mission, although it was only during the eighties that swing-by was acknowledged as a powerful tool for mission planning. In that decade, Farquhar & Dunham (1981) formulated a mission to study the Earth's geomagnetic tail, Byrnes and D'Amario (1982) designed a mission to flyby the comet Halley, while D'Amario *et al.* (1981, 1982), Marsh & Howell (1988) and Dunham & Davis (1985) studied multiple flyby for interplanetary missions. Additional studies during this time include mission planning for the ISEE-3/ICE, performed independently by Farquhar *et al.* (1985), Efron *et al.* (1985) and Muhonen *et al.* (1985).

During the next decade, swing-by was widely used in missions to the planets. As examples, Striepe & Braun (1991) used a swing-by in Venus to reach Mars, Swenson (1992) proposed a mission to Neptune using swing-by to gain energy due to close approaches to the inner planets, and Weinstein

(1992) made a similar study for a mission to Pluto. Also at this time, the mathematical construction of the swing-by mode was improved by including additional effects. In this sense, Prado & Broucke (1994) studied the effects of the atmosphere in a swing-by trajectory, Prado (1996) considered the possibility of applying an impulse during the passage by the periapsis and, more recently, Felipe & Prado (1999) studied numerically a swing-by in three dimensions. The most usual approach to study this problem is to divide the problem in three phases, each dominated by a two-body problem. This is the so-called "patched-conics" approximation. Other models used to study swing-by include the circular restricted three-body problem (used in Broucke (1988), Broucke & Prado (1993) and Prado (1993), and the elliptic restricted three-body problem (Prado, 1997).

The goal of this paper is to develop analytical equations for the variations of velocity, energy, angular momentum and inclination for a spacecraft that passes close to a celestial body and receives a small impulse when passing by the periapsis. This passage, called swing-by, is assumed to be performed around the secondary body of the system. Among the several sets of initial conditions that can be used to identify uniquely a swing-by trajectory, the following five variables are used in the present paper: V_p, the velocity of the spacecraft at periapsis of the orbit around the secondary body; two angles (α and β), that specify the direction of the periapsis of the trajectory of the spacecraft around M_2 in a three-dimensional space; r_p, the distance from the spacecraft to the center of M_2 in the moment of the closest approach to M_2 (periapsis distance); γ, the angle between the velocity vector at periapsis and the intersection between the horizontal plane that passes by the periapsis and the plane perpendicular to the periapsis that holds $\vec{V}_p$. Fig. 1 shows the sequence for this maneuver.

It is assumed that the system has three bodies: a primary (M_1) and a secondary (M_2) bodies with finite masses that are in circular orbits around their common center of mass and a third body with negligible mass (the spacecraft) that has its motion governed by the two other bodies. The spacecraft leaves the point A, passes by the point P (the periapsis of the trajectory of the spacecraft in its orbit around M_2) and goes to the point B. When passing by the point P the space vehicle receives a small impulse ($\delta\vec{V}$). The points A and B are chosen in such a way that the influence of M_2 at those two points can be neglected and, consequently, the energy can be assumed to remain constant after B and before A (the system follows the two-body problem). The initial conditions are clearly identified in the Fig.1. The distance r_p is not to scale, to make the figure easier to understand. The result of this maneuver is a change in velocity, energy, angular momentum and inclination in the Keplerian orbit of the spacecraft around the central body.

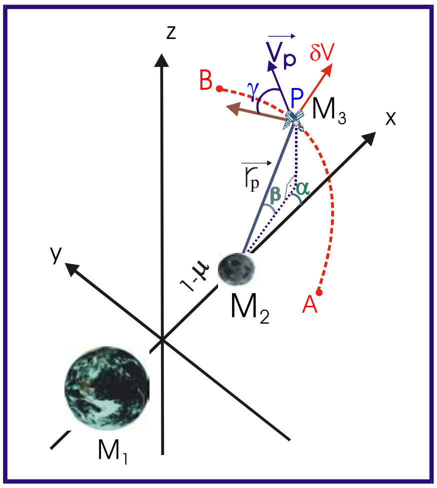

Fig. 1 – The powered Swing-By in Three Dimensions.

ANALYTICAL EQUATIONS FOR THE SWING-BY IN THREE DIMENSIONS

First, let us assume that the impulse has not been applied to obtain an expression for $\vec{V}_{\infty}^{-}$ and $\vec{V}_{\infty p}^{+}$, where $\vec{V}_{\infty p}^{+}$ denotes the pseudo velocity of the spacecraft with respect to M_2 after the swing-by, that represents the velocity that the spacecraft would have with respect to M_2 if the impulse was not applied. The position and velocity at periapsis, $\vec{V}_{\infty p}^{+}$ and $\vec{V}_{\infty}^{-}$ are (Prado, 2000):

$$x_i = r_p \cos\beta \cos\alpha \tag{1}$$

$$y_i = r_p \cos\beta \sin\alpha \tag{2}$$

$$z_i = r_p \sin\beta \tag{3}$$

$$V_{xiB} = -v_p \sin\gamma \sin\beta \cos\alpha \text{ - } v_p \cos\gamma \sin\alpha \tag{4}$$

$$V_{yiB} = -v_p \sin\gamma \sin\beta \sin\alpha + v_p \cos\gamma \sin\alpha \tag{5}$$

$$V_{ziB} = +v_p \cos\beta \sin\gamma \tag{6}$$

$$\begin{aligned}\vec{V}_{\infty}^{-} = {} & V_{\infty}\sin\delta\,(\cos\beta\cos\alpha, \cos\beta\sin\alpha, \sin\beta) + V_{\infty}\cos\delta\,(-\sin\gamma\sin\beta\cos\alpha - \cos\gamma\sin\alpha, \\ & \text{-}\sin\gamma\sin\beta\sin\alpha + \cos\gamma\cos\alpha, \cos\beta\sin\gamma)\end{aligned} \tag{7}$$

$$\begin{aligned}\vec{V}_{\infty p}^{+} = {} & -V_{\infty}\sin\delta\,(\cos\beta\cos\alpha, \cos\beta\sin\alpha, \sin\beta) + V_{\infty}\cos\delta\,(-\sin\gamma\sin\beta\cos\alpha - \cos\gamma\sin\alpha, \\ & \text{-}\sin\gamma\sin\beta\sin\alpha + \cos\gamma\cos\alpha, \cos\beta\sin\gamma)\end{aligned} \tag{8}$$

If we apply the impulse (Fig. 1), the position of the spacecraft remains unchanged, but the velocity components contain and additional term given by:

$$V_{xi} = -v_p \sin\gamma \sin\beta \cos\alpha \text{ - } v_p \cos\gamma \sin\alpha + \delta V_x \tag{9}$$

$$V_{yi} = -v_p \sin\gamma \sin\beta \sin\alpha + v_p \cos\gamma \sin\alpha + \delta V_y \tag{10}$$

$$V_{zi} = +v_p \cos\beta \sin\gamma + \delta V_z \tag{11}$$

For small impulses it is possible to assume that the vectors $\vec{r}_p$ and $\vec{v}_p$ will remain close to the position and velocity of the perigee. We also assume that the two-body problem is a valid approximation, and the whole maneuver takes place in the plane defined by the vectors $\vec{r}_p$ and $\vec{v}_p$. Thus, the vectors $\vec{V}_{\infty p}^{-}$ and $\vec{V}_{\infty}^{+}$ can be written as a linear combination of the versors associated with $\vec{r}_p$ and $\vec{v}_p$. Recall that $\vec{V}_{\infty p}^{-}$ is the velocity of the spacecraft when arriving at the sphere of influence of M_2, coming from a hyphotetical trajectory that leads to a velocity at periapsis given by Eq. (9)-(11), and $\vec{V}_{\infty}^{+}$ is the velocity vector after the swing-by, both with respect to M_2. Using $\vec{v}_{\infty}$ to represent both $\vec{V}_{\infty p}^{-}$ and $\vec{V}_{\infty}^{+}$, since the conditions are the same for both vectors and a double solution will give the values for $\vec{V}_{\infty p}^{-}$ and $\vec{V}_{\infty}^{+}$, we have:

$$\vec{V}_{\infty} = A\frac{\vec{r}_p}{r_p} + B\frac{\vec{V}_p}{V_p} \tag{12}$$

Explicitly, this can be written as:

$$\vec{V}_\infty = A(\cos\beta\cos\alpha, \cos\beta\sin\alpha, \sin\beta) + B(-\sin\gamma\sin\beta\cos\alpha - \cos\gamma\sin\alpha + \delta V_x, \sin\gamma\sin\beta\sin\alpha + \cos\gamma\cos\alpha + \delta V_y, \cos\beta\sin\gamma + \delta V_z) \quad (13)$$

where A, B are constants satisfying the relation $A^2 + B^2 = V_\infty^2$, and V_∞ can be obtained from the conservation of energy of the two-body problem: $V_\infty^2 = V_p^2 - \frac{2\mu}{r_p}$. Recall that $\mu = m_2/(m_1 + m_2)$, where m_1 and m_2 are the real masses of M_1 and M_2, respectively. A second requirement for $\vec{V}_\infty$ is that it makes an angle δ with $\vec{V}_p$, where δ is half of the total rotation angle described by the velocity vector during the maneuver (angle between $\vec{V}_\infty^-$ and $\vec{V}_\infty^+$) and it is assumed equal to the maneuver without impulse. This condition can be written as:

$$\vec{V}_\infty \bullet \vec{V}_p = V_\infty V_p \cos\delta \quad (14)$$

where the dot represents the scalar product between two vectors. From the two-body problem it is known that:

$$\sin\delta = \frac{1}{1 + \frac{r_p V_\infty^2}{\mu_2}} \quad (15)$$

where μ_2 is the gravitational parameter of M_2 (equal to 0.0121 in the case of the Moon). Using the equation for $\vec{V}_\infty$ as a function of $\vec{r}_p$ and $\vec{V}_p$, we also have:

$$\vec{V}_\infty \bullet \vec{V}_p = \left(A\frac{\vec{r}_p}{r_p} + B\frac{\vec{V}_p}{V_p} \right) \bullet \vec{V}_p = BV_p = V_\infty V_p \cos\delta \quad (16)$$

Thus, $B = V_\infty \cos\delta$, because $\vec{r}_p \bullet \vec{V}_p = 0$ (at the periapsis $\vec{r}_p$ and $\vec{V}_p$ are perpendicular) and $\vec{V}_p \bullet \vec{V}_p = V_p^2$. Consequently, since $A^2 + B^2 = V_\infty^2$ we have

$$A^2 = V_\infty^2 - B^2 = V_\infty^2 - V_\infty^2 \cos^2\delta = V_\infty^2 (1 - \cos^2\delta) = V_\infty^2 \sin^2\delta$$

which implies: $A = \pm V_\infty \sin\delta$

From those conditions, we have:

$$\vec{V}_{\infty p}^{-} = -\{(\cos\delta\,(\delta V_x - \cos\gamma\sin\alpha - \cos\alpha\sin\beta\sin\gamma) - \cos\alpha\cos\beta\sin\delta)V_\infty,$$
$$(\cos\delta\,(\delta V_y + \cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma) - \cos\beta\sin\alpha\sin\delta)V_\infty, \tag{17}$$
$$(\cos\delta(\delta V_z + \cos\beta\sin\gamma) - \sin\beta\sin\delta)V_\infty\}$$

$$\vec{V}_{\infty}^{+} = \{\cos\delta\,(\delta V_x - \cos\gamma\sin\alpha - \cos\alpha\sin\beta\sin\gamma) - \cos\alpha\cos\beta\sin\delta)V_\infty,$$
$$(\cos\delta\,(\delta V_y + \cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma) - \cos\beta\sin\alpha\sin\delta)V_\infty, \tag{18}$$
$$(\cos\delta(\delta V_z + \cos\beta\sin\gamma) - \sin\beta\sin\delta)V_\infty\}$$

For M_2, its velocity with respect to an inertial frame ($\vec{V}_2$) is assumed to be:

$$\vec{V}_2 = (0, V_2, 0) \tag{19}$$

By using vector addition:

$$\vec{V}_i = \vec{V}_\infty^{-} + \vec{V}_2 = \{-(\cos\delta\cos\gamma\sin\alpha + \cos\alpha\,(\cos\delta\sin\beta\sin\gamma - \cos\beta\sin\delta)),$$
$$\frac{V_2}{V_\infty^{-}} + (\cos\delta\cos\alpha\cos\gamma + \sin\alpha\,(-\cos\delta\sin\beta\sin\gamma + \cos\beta\sin\delta, \cos\beta\cos\delta\sin\gamma + \sin\beta\sin\delta\} \tag{20}$$

$$\vec{V}_0 = \vec{V}_\infty^{+} + \vec{V}_2 = \{\cos\delta\,(\delta V_x - \cos\gamma\sin\alpha - \cos\alpha\sin\beta\sin\gamma) - \cos\alpha\cos\beta\sin\delta,$$
$$\frac{V_2}{V_\infty^{-}} + \cos\delta\,(\delta V_y + \cos\alpha\cos\gamma - \sin\alpha\sin\beta\sin\gamma) - \cos\beta\sin\alpha\sin\delta, \cos\delta \tag{21}$$
$$(\delta V_z + \cos\beta\ \sin\gamma) - \sin\beta\sin\delta\}$$

where $\vec{V}_i$ and $\vec{V}_0$ are the velocity of the spacecraft with respect to the inertial frame before and after the swing-by, respectively.

From those equations, it is possible to obtain expressions for the variations in velocity, energy and angular momentum. They are:

$$\Delta\vec{V} = \vec{V}_0 - \vec{V}_i$$
$$\Delta E = \frac{1}{2}(\vec{V}_0^2 - \vec{V}_i^2) \tag{22}$$

which implies that:

$$\Delta V^2 = ((\delta V_x^2 + \delta V_y^2 + \delta V_z^2)\cos\delta^2 - 4\cos\delta\,(\delta V_x\ \cos\alpha\cos\beta + \delta V_y\cos\beta\sin\alpha +$$
$$\delta V_z\sin\beta)\sin\delta + 4\sin\delta^2)V_\infty^2 \tag{23}$$

$$\Delta E = \frac{1}{2} V_\infty (2V_2(\delta V_y \cos\delta - 2\cos\beta \sin\alpha \sin\delta) +$$
$$\cos\delta\,(\cos\delta\,(\delta V_x^2 + \delta V_y^2 + \delta V_z^2 + 2\delta V_y \cos\alpha \cos\gamma$$
$$- 2\delta V_x \cos\gamma \sin\alpha + 2\delta V_z \cos\beta \sin\gamma - 2\delta V_x \cos\alpha \sin\beta \sin\gamma - 2\delta V_y \sin\alpha \sin\beta \sin\gamma)$$
$$- 2(\delta V_x \cos\alpha \cos\beta + \delta V_y \cos\beta \sin\alpha + \delta V_z \sin\beta) \sin\delta) V_\infty) \tag{24}$$

Similarly, for the angular momentum ($\vec{C}$) the results are:

$$\vec{C}_i = \vec{R} \times \vec{V}_i = \{0, -d\,(\cos\beta \cos\delta \sin\gamma + \sin\beta \sin\delta) V_\infty,$$
$$d\,(V_2 + (\cos\alpha \cos\gamma \cos\delta\, V_\infty + \sin\alpha\,(-\cos\delta\ \sin\beta \sin\gamma\, V_\infty + \cos\beta \sin\delta))\, V_\infty)\} \tag{25}$$

$$\vec{C}_0 = \vec{R} \times \vec{V}_0 = \{0, -d\,(\cos\delta(\,\delta V_z + \cos\beta \sin\gamma) - \sin\beta \sin\delta) V_\infty,$$
$$d\,(V_2 + (\cos\delta\,(\delta V_y + \cos\alpha \cos\gamma - \sin\alpha \sin\beta \sin\gamma\,) - \cos\beta \sin\alpha \sin\delta) V_\infty)\} \tag{26}$$

where $\vec{R} = (d, 0, 0)$ is the position vector of M_2. Then:

$$\Delta\vec{C} = \vec{C}_0 - \vec{C}_i = \{0, \text{-}\,d\,\delta V_z \cos\delta\, V_\infty + 2\,d \sin\beta \sin\delta\, V_\infty, d\,\delta V_y \cos\delta\, V_\infty - 2\,d \cos\beta \sin\alpha \sin\delta\, V_\infty\} \tag{27}$$

So, its modulus can be expressed by

$$\Delta C = \left|\Delta\vec{C}\right| = \sqrt{(d\,\delta V_y \cos\delta V_\infty - 2\,d \cos\beta \sin\alpha \sin\delta\, V_\infty)^2 + (d\,\delta V_z \cos\delta\, V_\infty - 2\,d \sin\beta \sin\delta\, V_\infty)^2} \tag{28}$$

Using the definition of angular velocity $\omega = \frac{V_2}{d}$ for the inclination, the results are the following:

$$\vec{\omega} = \{0, 0, 1\}$$

$$\vec{C}_{iz} = \vec{C}_i \cdot \vec{\omega}_z = d\,(V_2 + (\cos\alpha \cos\gamma \cos\delta + \sin\alpha\,(\text{-}\cos\delta\ \sin\beta \sin\gamma + \cos\beta \sin\delta))\, V_\infty) \tag{29}$$

$$\vec{C}_{0z} = \vec{C}_0 \cdot \vec{\omega}_z = d\,(V_2 + (\cos\delta\,(\,\delta V_y + \cos\alpha \cos\gamma - \sin\alpha \sin\beta \sin\gamma\,) - \cos\beta \sin\alpha \sin\delta\,) V_\infty) \tag{30}$$

$$\Delta\vec{C}_z = \vec{C}_{0z} - \vec{C}_{iz} = d\, V_\infty (\delta V_y \cos\delta - 2 \cos\beta \sin\alpha \sin\delta) \tag{31}$$

$$\Delta C_i = \sqrt{\left|\Delta\vec{C}_i\right|} = \sqrt{\frac{(d \cos\beta \cos\delta \sin\gamma\, V_\infty + d \sin\beta \sin\delta\, V_\infty)^2 + d^2}{(V_2 + (\cos\alpha \cos\gamma \cos\delta + \sin\alpha\,(\text{-}\cos\delta \sin\beta \sin\gamma + \cos\beta \sin\delta)) V_\infty)^2}} \tag{32}$$

$$\Delta C_0 = \sqrt{\left|\Delta\vec{C}_0\right|} = \sqrt{\begin{array}{l} d^2((\cos\delta\,(\delta V_z + \cos\beta\,\sin\gamma) - \sin\beta\,\sin\delta)^2\; V_\infty^2 + \\ (V_2 + (\cos\delta\,(\delta V_y + \cos\alpha\,\cos\gamma - \sin\alpha\,\sin\beta\,\sin\gamma) - \cos\beta\,\sin\alpha\,\sin\delta)\; V_\infty^2) \end{array}} \qquad (33)$$

$$\mathrm{Cos}(i_i) = \frac{C_{iz}}{\left|\vec{C}_i\right|}$$

$$\mathrm{Cos}(i_i) = \frac{d\,(V_2 + \cos\alpha\,\cos\gamma\,\cos\delta + \mathrm{sen}\,\alpha\,(-\cos\delta\,\mathrm{sen}\beta\,\mathrm{sen}\gamma + \cos\beta\,\mathrm{sen}\delta))V_\infty))}{(d\cos\beta\,\cos\delta\,\mathrm{sen}\gamma\; V_\infty + d\,\mathrm{sen}\beta\,\mathrm{sen}\delta\; V_\infty)^2 + d^2(V_2 + (\cos\alpha\,\cos\gamma\,\cos\delta + \mathrm{sen}\alpha\,(-\cos\delta\,\mathrm{sen}\beta\,\mathrm{sen}\gamma + \cos\beta\,\mathrm{sen}\delta))V_\infty)^2} \qquad (34)$$

$$\mathrm{Cos}(i_0) = \frac{C_{0z}}{\left|\vec{C}_0\right|}$$

$$\mathrm{Cos}(i_0) = \frac{V_2 + (\cos\delta\,(\delta V_y + \cos\alpha\,\cos\gamma - \mathrm{sen}\alpha\,\mathrm{sen}\beta\,\mathrm{sen}\gamma) - \cos\beta\,\mathrm{sen}\alpha\,\mathrm{sen}\delta)V_\infty}{d\,((\cos\delta\,(\delta V_z + \cos\beta\,\mathrm{sen}\gamma) - \mathrm{sen}\beta\,\mathrm{sen}\delta)^2\; V_\infty^2 + (V_2 + (\cos\delta\,(\delta V_y + \cos\alpha\,\cos\gamma - \mathrm{sen}\alpha\,\mathrm{sen}\beta\,\mathrm{sen}\gamma) - \cos\beta\,\mathrm{sen}\alpha\,\mathrm{sen}\delta)V_\infty)^{)2}} \qquad (35)$$

The variation in inclination Δi can then be obtained from i_0-i_i

$$\Delta_i = \arccos\left(\frac{d\left(V_2 + (\cos\delta\,(\delta V_y + \cos\alpha\,\cos\gamma - \sin\alpha\,\sin\beta\,\sin\gamma) - \cos\beta\,\sin\alpha\,\sin\delta)V_\infty\right)}{\sqrt{d^2\,((\cos\delta\,(\delta V_z + \cos\beta\,\sin\gamma) - \sin\beta\,\sin\delta)^2\; V_\infty^2 + (V_2 + (\cos\delta\,(\delta V_y + \cos\alpha\,\cos\gamma - \sin\alpha\,\sin\beta\,\sin\gamma) - \cos\beta\,\sin\alpha\,\sin\delta)V_\infty)^2)}}\right) -$$
$$\arccos\left(\frac{d\,(V_2 + \cos\alpha\,\cos\gamma\,\cos\delta - \cos\delta\,\sin\alpha\sin\beta\,\sin\gamma + \cos\beta\sin\alpha\,\sin\delta)V_\infty))}{\sqrt{(d\cos\beta\,\cos\delta\,\sin\gamma\; V_\infty + d\,\sin\beta\,\sin\delta\; V_\infty)^2 + d^2(V_2 + (\cos\alpha\,\cos\gamma\,\cos\delta - \cos\delta\,\sin\alpha\,\sin\beta\,\sin\gamma + \cos\beta\,\sin\alpha\,\sin\delta))V_\infty)^2}}\right) \qquad (36)$$

where $\delta V_z = \sqrt{-\left(\delta V_x\right)^2 - \left(\delta V_y\right)^2 + \left(\left|\delta\vec{V}\right|\right)^2}$ and $\left|\delta\vec{V}\right|$ is the magnitude of the impulse applied in maneuver.

RESULTS

The following figures are ploted with different values and directions of angles to obtain ΔE, ΔC, ΔV as a function of the direction of an impulse with fixed magnitude. Figure 2 show the results obtained. The values attributed the variables are: $\mu = 0.0121$, $V_2 = 1.0$, $\left|\delta\vec{V}\right| = 0.01$, $\vec{V}_\infty = 2.0$, $d = 1.0$.

The figures show results with practical applications for missions, since it is possible to obtain the values of ΔE, ΔC, ΔV as a function of the direction of a impulse with fixed magnitude. Most of the extrema are located in the borders, although exceptions exist which indicate the existence of directions more efficient to apply the impulse.

Several similar graphs, built with other values of angles, are not shown here due to page limits. The comparisons in the variation of Energy between the maneuver standard swing-by and the propelled maneuver are show in Table 1.

CONCLUSIONS

We obtained analytic equations for the variation of velocity, energy, angular momentum and inclination for a propelled swing-by maneuver, for the particular case where the impulse is small. The propelled swing-by has values of the variation in Energy larger than the standard swing-by, what shows the pratical importance of this maneuver. The variations are closer to linear in the variables δV_x and δV_y. In most of the cases the more effecient maneuver is obtained when the impulse is aligned with “x”or “y”axis. So, the present research can show the best direction to apply the impulse for any desired criteria: maximum or minimum variation of energy or velocity or angular momentum or inclination.

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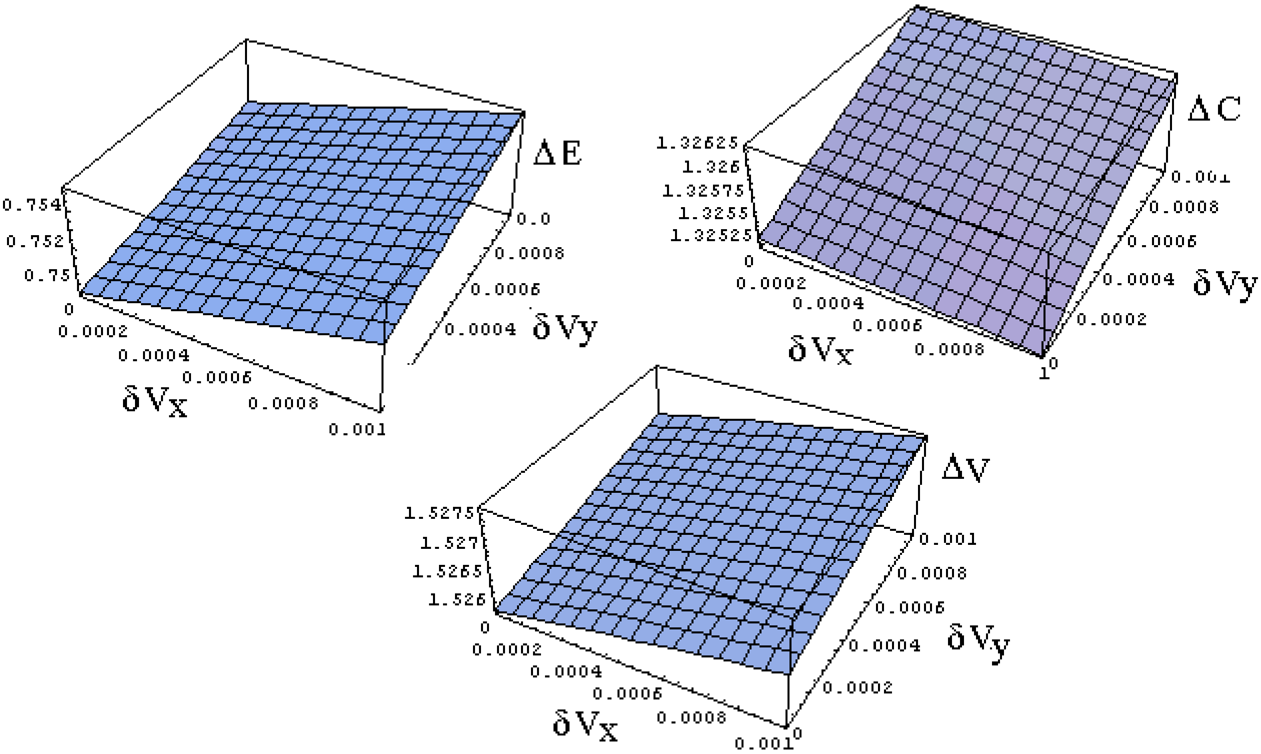
Table 1. Maxima values of the variation of Energy.

α = -135º, β = -45º, γ = -45º



angle	standard swing by ΔE max	propelled swing by ΔE max
α = -135º, β = -45º, γ = -45º	0.756453	0.75305
α = -90º, β = 45º, γ = -180º	1.06972	1.06288
α = 45º, β = -90º, γ = 180º	-0.0000409975	0.0164884
α = 0º, β = 0º, γ = 0º	0	0.00545071
α = 30º, β = 0º, γ = 0º	-0.756378	-0.752087
α = 60º, β = 0º, γ = 0º	-1.3101	-1.30757
α = 90º, β = 0º, γ = 0º	-1.51279	-1.51217
α = 120º, β = 0º, γ = 0º	-1.31015	-1.30998
α = 150º, β = 0º, γ = 0º	-0.756479	-0.756307
α = 180º, β = 0º, γ = 0º	-0.000115961	0.0014556
α = 210º, β = 0º, γ = 0º	0.756278	0.759376
α = 240º, β = 0º, γ = 0º	1.31004	1.31523
α = 270º, β = 0º, γ = 0º	1.51279	1.51964
α = 300º, β = 0º, γ = 0º	1.31021	1.31743
α = 330º, β = 0º, γ = 0º	0.756579	0.762772

$\alpha = -90^{o}$, $\beta = 45^{o}$, $\gamma = -180^{o}$

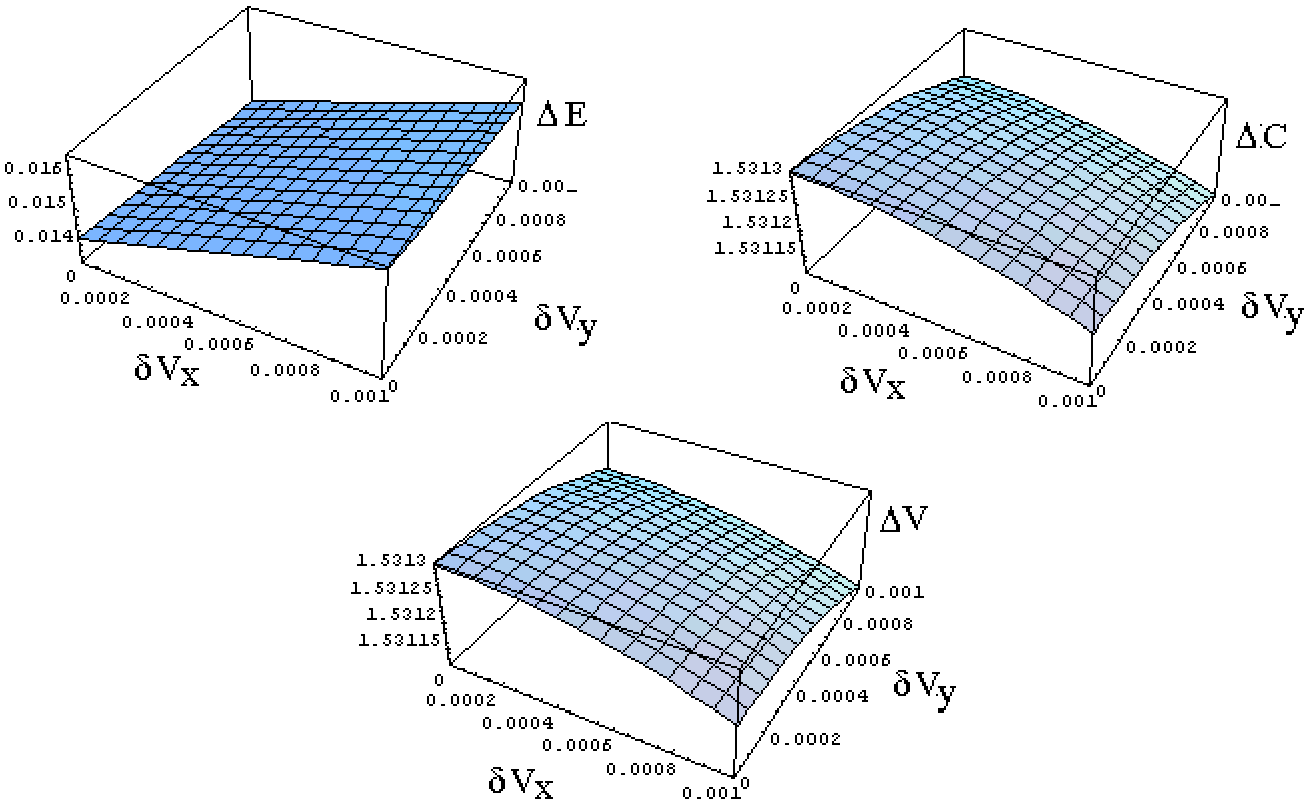


Fig.2 - Results of the numerical simulations.