MINISTÉRIO DA CIÊNCIA E TECNOLOGIA
INSTITUTO NACIONAL DE PESQUISAS ESPACIAIS

INPE-11293-PRE/6730

SPACECRAFT ATTITUDE PROPAGATIONWITH DIFFERENT REPRESENTATIONS

Domingos Sávio dos Santos Rodrigues
Maria Cecília Zanardi

ADVANCES IN SPACE DYNAMICS 4: CELESTIAL MECHANICS AND ASTRONAUTICS,
H. K. Kuga, Editor, 143-150 (2004).
Instituto Nacional de Pesquisas Espaciais – INPE, São José dos Campos, SP, Brazil.
ISBN 85-17-00012-9

INPE
São José dos Campos
2004

ADVANCES IN SPACE DYNAMICS 4: CELESTIAL MECHANICS AND ASTRONAUTICS,
H. K. Kuga, Editor, 143-150 (2004).
Instituto Nacional de Pesquisas Espaciais – INPE, São José dos Campos, SP, Brazil.
ISBN 85-17-00012-9

SPACECRAFT ATTITUDE PROPAGATION WITH DIFFERENT REPRESENTATIONS

Domingos Sávio dos Santos Rodrigues
Maria Cecília Zanardi
Group of Orbital Dynamics and Planetology
Department of Mathematics, FE/UNESP, Guaratinguetá - SP
cecilia@feg.unesp.br

ABSTRACT

The objective of this paper is to analyze the dynamic equations of the rotational motion of an artificial satellite in terms of different representations (Euler angles, Andoyer variables and quaternion). The external torques are not included in this study. Analytical solutions are presented for symmetrical satellites when the equation of motion are described in terms of the Andoyer variable or Euler angles and angular velocity. A semi-analytical solution is proposed for symmetrical satellite when the equations are described by quaternion. When compared, numerical and semi-analytical solutions have a good agreement during the time range considered. A numerical solution is presented for no symmetrical satellites when the motion equation are described by angular velocity and quaternion.

INTRODUCTION

The attitude of an artificial satellite represents its orientation in the space and it can be described by different forms. In this paper the dynamic equation of the rotational motion are described by Euler angles, angular velocity, Andoyer variables and quaternion. The objective of this paper is to analyze the spacecraft equations of motion in these three representations, without including external torques. Symmetrical and no symmetrical satellites are considered.

Analytical solutions are determined for symmetrical satellites (two principal moments of inertia are equal, $I_x = I_y$) and with the equation of motion described by Andoyer variables or Euler angles and angular velocity. The 4th order Runge Kutta method is used to get the numerical solution for the equations described by Euler angles and angular velocity. The dynamic equations described by quaternion are complex and a semi-analytical approach is presented for the symmetrical satellite. Comparisons between the analytical and numerical solutions are made and it is useful to check the developed numerical program

The equations of motion for no symmetrical satellites (satellite has different principal moments of inertia, $I_x \neq I_y \neq I_z$) are complex and in this paper only a numerical solution is presented.

The behavior of the angular velocity and the spin axis of the satellite is discussed and the precession and nutation of the spin axis are observed.

EQUATIONS OF THE FREE ROTATIONAL MOTION: ANGULAR VELOCITY AND EULER ANGLES

When external torques are not considered, the dynamic equations of the spacecraft rotational motion, described by the Euler equations and the kinematic equations,are given by (Moore & Pisacane, 1994) :

$$\dot{p} = q\ r\ (Iy - Iz)/Ix; \quad \dot{q} = p\ r\ (Iz - Ix)/Iy; \quad \dot{r} = p\ q\ (Ix - Iy)/Iz\ , \tag{1}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = (1/\sin\theta) \begin{bmatrix} \sin\psi & \cos\psi & 0 \\ \sin\theta\cos\psi & -\sin\theta\sin\psi & 0 \\ -\cos\theta\sin\psi & -\cos\theta\sin\psi & \sin\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}\ , \tag{2}$$

where:
I_x , I_y , I_z are the principal moments of inertia of the satellite ,
p, q , r are the components of the spin velocity in the system of principal moment of inertia (here it is called satellite system - Oxyz) and
θ, ϕ, ψ are sequence ZXZ of the Euler angles, which related the satellite system and inertial system (OXYZ).

For symmetrical satellite the moment of inertia I_x and I_y are equal, and the dynamic equation are simplified. In this case the Eq. (1) can be expressed by (Zanardi, 2000):

$$\dot{p} = k\,q \qquad \dot{q} = -k\,p \qquad \dot{r} = 0\ , \tag{3}$$

where $k = (Ix - Iz)\ r/Ix$.

The solution of the equations system (3) is given by (Moore & Pisacane, 1994):

$$p(t) = w_p \cos(k\,t + w_0) \qquad q(t) = -w_p \sin(k\,t + w_0) \qquad r(t) = r_0\ , \tag{4}$$

where w_0 and w_p are constants and they are computed by initial conditions at instant t_0. If p_0, q_0, r_0 are the initial components of the spacecraft spin velocity, w_0 and w_p (the projection of spin velocity on the plane xy) are given by (Zanardi, 2000):

$$w_0 = a\tan\left(\frac{-q_0}{p_0}\right) - k\,t_0 \qquad \text{and} \qquad w_p{}^2 = p_0^2 + q_0^2\,. \tag{5}$$

Therefor it is observed that the spin axis velocity describes a conical motion around the spacecraft symmetry axis. This motion is called *PRECESSION* of the spin axis. When the satellite is no symmetrical, the z-component of the angular velocity has a periodic variation, and it introduce a *NUTATION* motion of the spin axis.

In free rotational motion the angular momentum vector ($\vec{L}$) is constant and Z-axis of the inertial system can be put along it. Then the components of $\vec{L}$ in the satellite system are expressed like (Zanardi,2000):

$$L_1 = I_x\,p = L\sin\theta\sin\psi \quad L_2 = I_y\,q = L\sin\theta\cos \quad L_3 = I_z\,r = L\cos\theta\,. \tag{6}$$

If the satellite is symmetric, by Eq. (5) the z-component of the spin velocity is constant, then

$$\cos\theta = I_Z\,r/L = \text{cons}\tan t \tag{7}$$

and the kinematic equations (2) can be simplified ($\dot{\theta} = 0$):

$$p = \dot{\phi}\sin\theta\sin\psi \qquad q = \dot{\phi}\sin\theta\cos\psi \qquad r = \dot{\phi}\cos\theta + \dot{\psi}\,. \tag{8}$$

Substituting (6) and (7) in (8), after algebraic manipulations, the Euler angles variation rate are:

$$\dot{\phi} = h \qquad \dot{\psi} = k \qquad \dot{\theta} = 0\,, \tag{9}$$

where $k = \left(I_X - I_Z\right) r / I_X$ and $h = L / I_X$.

Integrating (9) it is possible to get the solution of the kinematic equations for symmetrical satellites:

$$\phi(t) = h\,t + \phi_0 \qquad \psi(t) = k\,t + \psi_0 \qquad \theta(t) = \theta_0\,, \tag{10}$$

where ϕ_0, ψ_0 e θ_0 are initial conditions at t_0.

EQUATIONS OF MOTION: QUATERNION

To avoid the singularities in the kinematic equations generated by Euler angles, the quaternion can be used. The quaternion q is defined by 4 x 1 matrix given by

$$q = [\vec{q} \quad q_4]^t \quad , \tag{11}$$

with

$$\vec{q} = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}^t = \sin(\Phi/2)\,\vec{n} \quad \text{and} \quad q_4 = \cos(\Phi/2), \tag{12}$$

where Φ is the rotational angle and $\vec{n}$ is the spin axis direction.

The kinematic equations that describe the variation rate of the attitude quaternion components, due to rotation of the satellite, are given by (Moore & Pisacane, 1994):

$$\begin{aligned} \dot{q}_1 &= \frac{1}{2}[p\, q_4 - q\, q_3 + r\, q_2] & \dot{q}_2 &= \frac{1}{2}[q\, q_4 - r\, q_1 + p\, q_3] \\ \dot{q}_3 &= \frac{1}{2}[r\, q_4 - p\, q_2 + q\, q_1] & \dot{q}_4 &= -\frac{1}{2}[p\, q_1 + q\, q_2 + r\, q_3] \end{aligned} \tag{13}$$

The Eqs. (13) depend on the component of spin velocity and a semi-analytical approach is applied to get the solution for the quaternion.

For the symmetrical satellites, this semi-analytical approach (Domingos, 2002) is developed , by computing the rotational angle (Φ) and the spin axis ($\vec{n}$) using the solution (8) for the component of the angular velocity. After that the components of the quaternion are computed by Eq. (12).

EQUATIONS OF MOTION: ANDOYER VARIABLES

The rotational motion can be also described by the Andoyer variables (Zanardi & Vilhena de Moraes, 1999). They are canonical variables and the dynamic equations can be solved using perturbation method. The Andoyer variables (G, L, H, g, l, h) are defined as: G is the magnitude of the rotational angular moment vector $\vec{G}$; L, the projection of $\vec{G}$ in satellite z-axis; H, the projection of $\vec{G}$ in inertial Z-axis; l, g, h are angles which relate the satellite system and inertial system (Zanardi & Lopes, 2000). The dynamic equations of the free rotational motion described by Andoyer variables are expressed by (Zanardi & Vilhena de Moraes, 1999):

$$\begin{aligned} &\dot{g} = \partial F/\partial G \quad && \dot{l} = \partial F/\partial L \quad && \dot{h} = \partial F/\partial H \\ &\dot{G} = -\partial F/\partial g \quad && \dot{L} = -\partial F/\partial l \quad && \dot{H} = -\partial F/\partial h \end{aligned} \tag{14}$$

where Hamiltonian F is

$$\begin{aligned} F = &\frac{1}{2}\left[1/I_z - 1/(2I_x) - 1/(2I_y)\right]L^2 + \frac{1}{4}\left(1/I_y + 1/I_x\right)G^2 + \\ &\frac{1}{4}\left(1/I_y - 1/I_x\right)\cdot(G^2 - L^2)\cdot\cos(2l) \end{aligned} \tag{15}$$

For symmetrical satellites ($I_x = I_y$), substituting (15) in (14), it can be observed that h, G, L, H are constant and:

$$g = n_g t + g_0 \qquad\qquad l = n_l t + l_0 \tag{16}$$

where $l_0, g_0, h_0, L_0, G_0, H_0$ are initial conditions and

$$n_g = G_0 / I_x \qquad\qquad n_l = L_0 \cdot (1/I_z - 1/I_x) \tag{17}$$

For no symmetrical satellites ($I_x \neq I_y \neq I_z$), the motion equations in terms of the variable of Andoyer are given by:

$$\begin{aligned}
\dot{g} &= (G/2)\left[1/I_x + 1/I_y + (1/I_y - 1/I_x)\cos(2l)\right] \\
\dot{l} &= L\left\{1/I_z - (1/2)\left[1/I_x + 1/I_y + (1/I_y - 1/I_x)\cos(2l)\right]\right\} \\
\dot{L} &= (1/2)\,(1/I_y - 1/I_x)(G^2 - L^2)\sin(2l) \\
\dot{h} &= 0 \qquad \dot{G} = 0 \qquad \dot{H} = 0
\end{aligned} \tag{18}$$

The analytical integration of the equations (18) is given in terms of elliptical integrals (Kinoshita, 1972) and it will be not discussed in this paper. However it is possible to observe that the metric variable L will be a periodic variation and the angles l and g will have a linear and periodic variations while h, G and H remain constant.

NUMERICAL SIMULATIONS

The numerical implementation of semi-analytical approach discussed for quaternion and symmetrical satellite is presented now. The software MATLAB and the 4^{th} Runge Kutta method are used to determine the numerical solution.
The result of the simulations for symmetrical satellites are presented in the Fig. (1)-(2) considering the following initial conditions:

Ix=3.9499 x 10^{-1}(kg.km^2); $I_y=I_x$; I_z=1.0307 x 10^{-1}(kg.km^2).
p_0 = 0.0246 (rad./s); q_0 = 0.01 (rad./s); r_0 = 0 (rad./s).
$\Phi_0 = 0$; $q_1 = 0$; $q_2 = 0$; $q_3 = 0$; $q_4 = 1$.

For no symmetrical satellites, the numerical results are presented in the Fig. (3)-(4), considering $I_y = 0.95\ I_x$ and the same previous initial conditions.

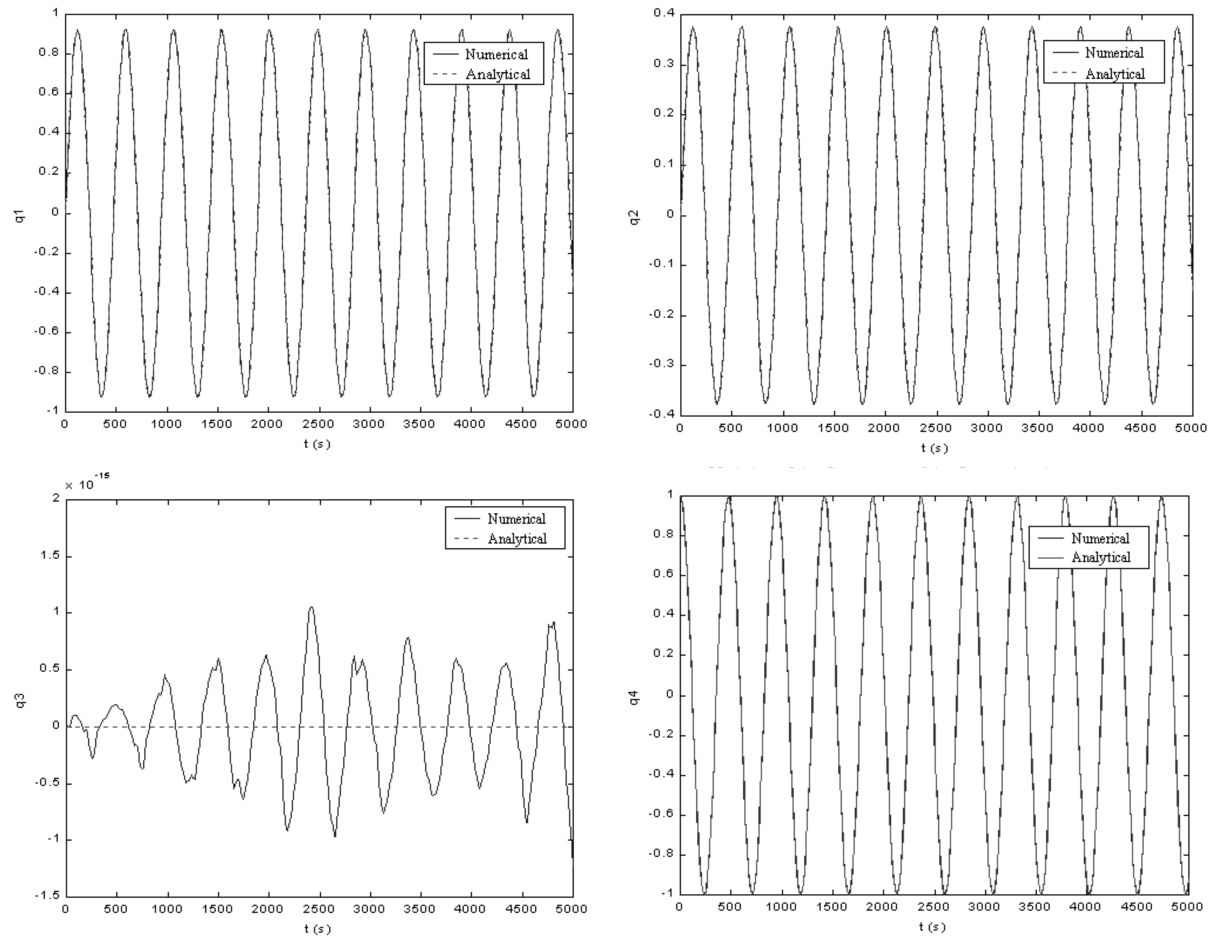
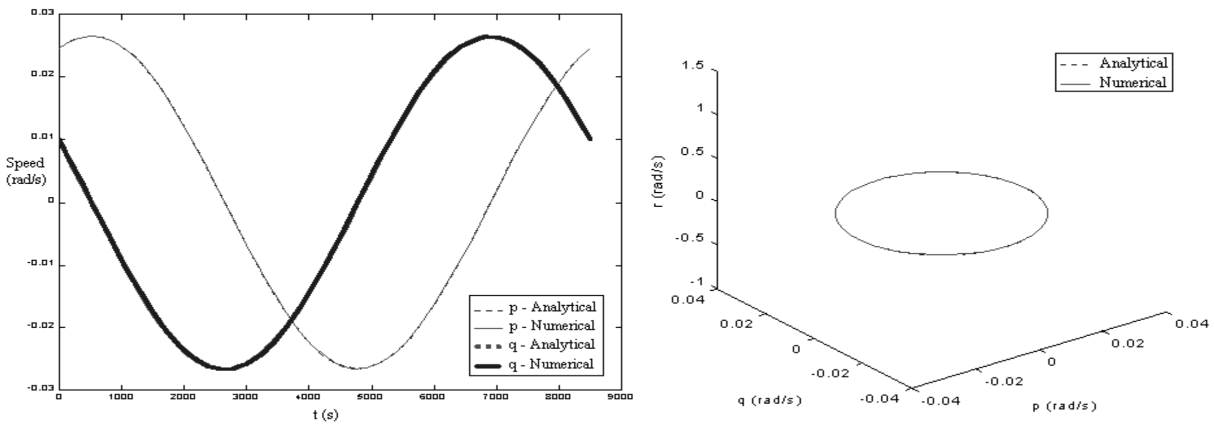


FIGURE 1 – Evolution of the quaternion components for a symmetrical satellite.



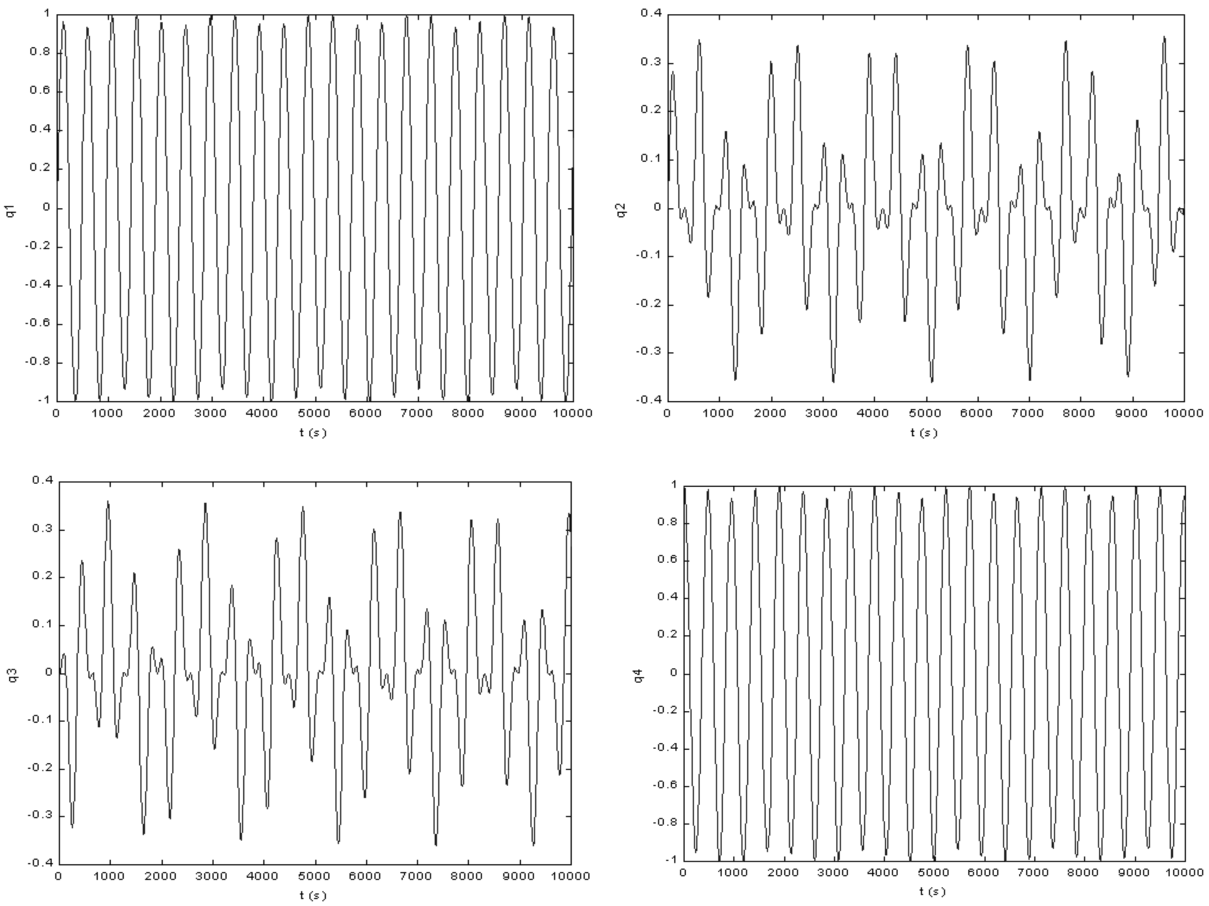


FIGURE 2 – Evolution of the spin velocity for a symmetrical satellite.

FIGURE 3 – Evolution for quaternion components for no symmetrical satellite.

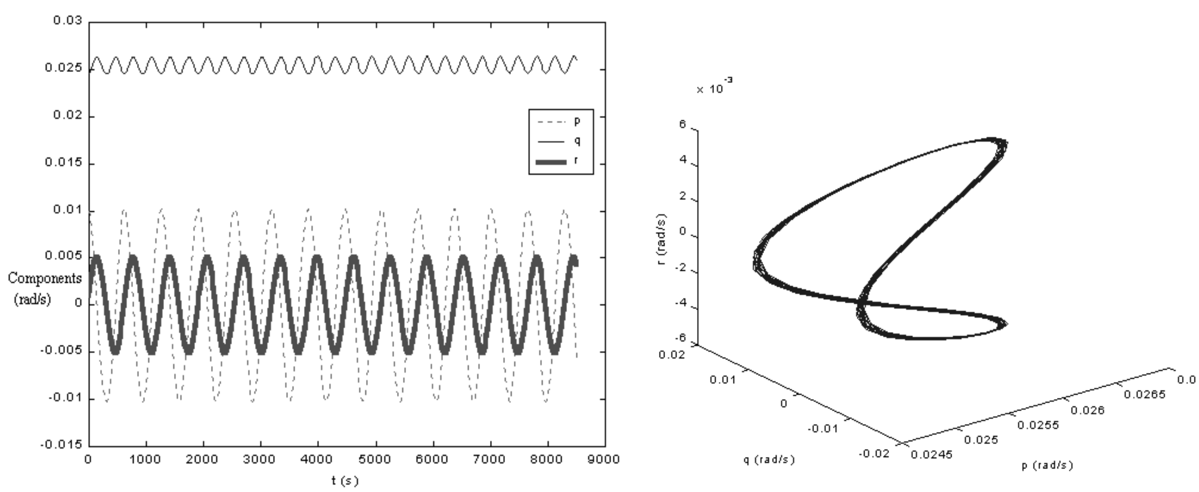


FIGURE 4 - Evolution of the spin velocity for no symmetrical satellite.

SUMMARY

Analyzing the results presented in the previous section and in other simulations (Rodrigues, 2002) it is observed:

- the precession of the spin axis in the Fig. (2) for symmetric satellite;
- for no symmetric satellite, the z-component of the spin velocity (r) has a periodic variation due to the interaction between the dynamic equations (Eq. (1)) and the precession, and nutation of the spin axis are represented in Fig. (4);
- for symmetrical satellite, the Andoyer variables G, H, L and h are constant; for no symmetrical satellite the z-component of the angular moment (L) has periodic variations and the angular variables l and g have linear and periodic variations with the time, while H, G and h remain constant;
- for symmetrical satellite the quaternion component q_3 is constant (with a numerical error of order 10^{-15}) while the others present periodic variations; the semi-analytical solution and numerical solution agree and it is useful to verify the developed numerical program;
- for no symmetrical satellite, all the quaternion components oscillate with the time. In both cases the magnitude of the quaternion is constant and equal to 1.

The analysis developed will be useful when the external torques will be included in the equation of the rotational motion for no symmetrical satellites.

REFERENCES

Kinoshina, H. “First order perturbations of the two finite body problem”, Publ. Astron. Soc. Japan, 24, p. 423, 1972.

Pisacane, V. L., Moore, R. C. “Fundamentals of Space Systems”, New York: Oxford University Press, 1994.

Rodrigues, D. S. S. “Propagação da Atitude de Satélites Artificiais com Diferentes Representações”, Guaratinguetá,. 65 p. Technical Report, CNPq/PIBIC, FEG/UNESP, 2002.

Zanardi, M. C., Vilhena de Moraes, R. “Analytical and Semi-Analytical Analysis an Artificial Satellite’s Rotational Motion*”, Cel. Mech. And Dyn. Astr.*, v.75, 4, 227, 1999.

Zanardi, M. C. “Movimento Rotacional Livre de Torques Externos”, Proceeding of Summer School on the Orbital Dynamic and Planetology, pp. 94 , UNESP – Guaratinguetá, 2000.