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CONTINUOS TRAJECTORIES UNDER SUPERPOSITION OF THE THRUST VECTOR DEVIATIONS

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ABSTRACT

Optima orbital trajectories of spatial vehicle can be realized with continuos or impulsive burn regime of the propulsion system. The optimality can be explored in the problem variables, depending of the missions aims. In this paper we studied “theoretical” and “practical” (brazilian satellite EUTELSAT II-F2) continuos transfers under influence of superposition of the gaussian, uniforms and systematic deviations in the thrust vector. The “pitch” and “yaw” angles were taken like control variables, providing in each instant the minimum fuel consumption direction. Monte-Carlo analysis of the “theoretical” and “practical” orbits was realized, presenting results to one exact keplerian dynamic, which suggests one cause/effect relation between the semi-major axis of the orbits and the vector thrust direction deviations. This relation shows that the final orbit is so deformed w.r.t. the ideal cases (without deviations) and without deviations superposition. The deformation found in the final orbit shows too the loss of optimality of the system.

INTRODUCTION

The optimum orbital transfer problem of the spatial vehicle was formulated initially by Goddard (1919), with his pioneer work about the maximization of the final altitude of one rocket under the gravitation field and atmospheric drag. After it, many searchers analyzed this problem under others ways, using several mathematics methods to found the exact or not exact solution of the associated problems. resolve. The advance of the digital computers and the development of the Optimum Control Theory allowed the study of the problem under more realistic approaches. However, only after almost one century the study about the spatial vehicles propulsion systems errors effects worried a few scientists. We presume that the most cause this nerveless investigation results from the many searchers did not believe in the cause/effect relation existence between the deviations provided by propulsion system of the spatial vehicle and theirs final orbits. This fact is easily verified at the absence this study through the accessible scientific literature.

Many papers studied numerically the orbital transfer problem through deterministic approach to the deviations (not those thrust vector deviations). Porcelli and Vogel (1980) presented an algorithm for the determination of the orbit insertion errors in bi-impulsive noncoplanar orbital transfers (perigee and appogee), using the covariance matrices of the sources of errors and time fixed between the both impulses. Adams and Melton (1986) extended such algorithm to ascent transfers under a finite thrust, modeled as a sequence of impulsive burns. They developed an algorithm to compute the propagation of the navigation and direction errors among the nominal trajectory, with finite perigee burns. These two papers developed semi-analytical algorithms, with one covariance errors analysis. Howell and Gordon (1994) also applied covariance analysis to the orbit determination errors and they develop a station-keeping strategy of Sun-Earth L1 libration point orbits. Junkins (1997) discussed the matrix covariance errors method precision through the transformation of the nonlinear coordinates. Rios-Neto and Pinto (1986) studied maneuvers under errors in the constraints. They proposed one stochastic version of the gradient projection method. They concluded that the stochastic version provide fuel economy in the constraints flexibility range. Carlton-Wippern (1997) used the Lagevin equation approach and first order perturbation theory to develop one differential equations set to the variances increase in the positions errors to directions different of the movie in polar coordinates. This paper suggest one approximated analytical solution to the maneuver mean without thrust vector in-plane, only with initial errors in radial and angular direction, gravitational and random perturbations forces. Recently, we studied the orbital transfer problem under thrust vector deviations to spatial vehicles. We found the cause/effect relations (numerical and algebraic) between these deviations and the final semi-major axis values and to others orbital keplerian elements (Jesus, 1999).

In this paper we developed one numerical investigation about the orbital transfer problem with superposition of the thrust vector deviations, through spatial vehicle mission, due the non-ideal propulsion system. This approach is more realistic than the individual deviation application. That is, during the burns along the transfer trajectory, the action of the “pitch” and “yaw” deviations are considered simultaneous. This phenomenum is more closed of the actual and non-ideal case. Our numerical analysis was done for two transfers: the first, a low thrust transfer between high coplanar orbits (we call it "theoretical transfer"), used by Biggs (1978,1979) and Prado (1989); the second, a high thrust transfer between middle non-coplanar orbits (the first transfer of the EUTELSAT II-F2 satellite, we call it "practical transfer") implemented by Kuga (1991) et alli. The simulations were done for both transfers with minimum fuel consumption. The "pitch" and "yaw" angles were taken as control variables such that the overall minimum fuel consumption defines each burn of the thrusters. We computed the total “pitch” and “yaw” effects deviations over the final orbit of the spatial vehicle. We analyzed these effects in the keplerian element, final semi-major axis of the transfer trajectory.

THE THRUST VECTOR APPLIED TO THE SATELLITE WITH DEVIATIONS SUPERPOSITON

The Figure 1 shows the coordinate system localized in the satellite (TRN system) and the thrust vector applied to this vehicle.

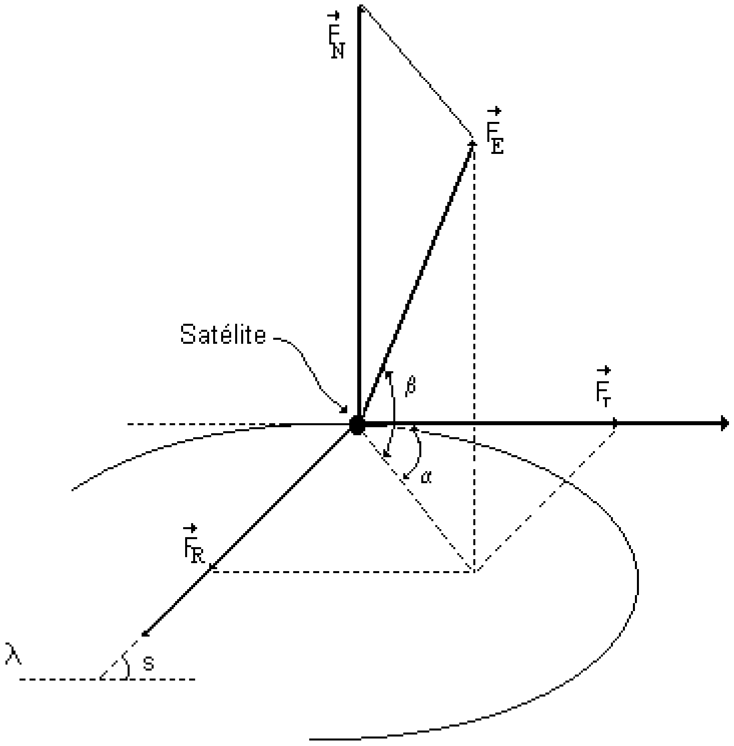


Fig. 1 – The thrust vector applied to the satellite

The thrust components are affected by "pitch" and "yaw" during the burn. The thrust vector is given,

$$\vec{F}_E = \vec{F} + \Delta\vec{F} \tag{1}$$

$$\vec{F}_E = \vec{F}_R + \vec{F}_T + \vec{F}_N \tag{2}$$

$$\left|\vec{F}_E\right| = F_E, \quad \left|\vec{F}\right| = F \tag{3}$$

and their components are,

$$F_R = (F + \Delta F)\cos(\beta + \Delta\beta)\mathrm{sen}(\alpha + \Delta\alpha) \tag{4}$$

$$F_T = (F + \Delta F)\cos(\beta + \Delta\beta)\cos(\alpha + \Delta\alpha) \qquad (5)$$

$$F_N = (F + \Delta F)\mathrm{sen}(\beta + \Delta\beta) \qquad (6)$$

with,

F, F_T and ΔF (DES1) are the vector without errors modulus, the vector with errors, and the vector thrust error, respectively; $\Delta\alpha$ (DES2) e $\Delta\beta$ (DES3) are the “pitch” and “yaw” errors, respectively; F_R, F_T and F_N are the thrust vector components with errors in the transversal, radial and normal directions, respectively.

THEORETICAL AND PRACTICAL MANEUVERS WITH EQUALS SUPERPOSED DEVIATIONS

We simulated (Monte-Carlo, 1000 runs) the both maneuvers ("theoretical" - T, "practical" - P), for the random bias (systematic – S) and white noise (operational – O) equals deviations in “pitch” and “yaw” directions. Figures 2 and 3 show $E\{a(t_f)\}$ for cases TS, and the Figures 4 and 5 show $E\{a(t_f)\}$ for PO orbits.

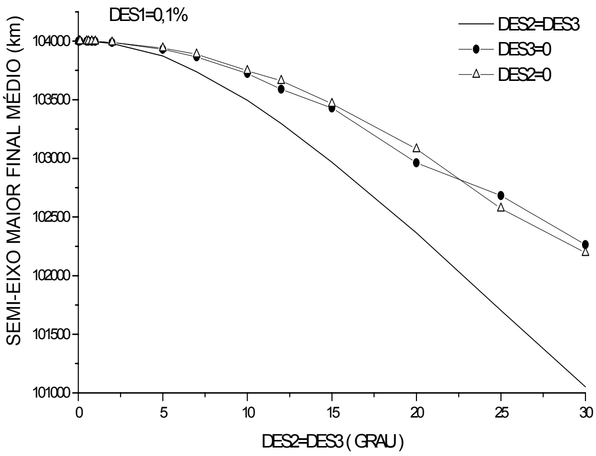


Fig. 2- Mean semi-major axis vs. DES2=DES3, DES1=0,1%, TS

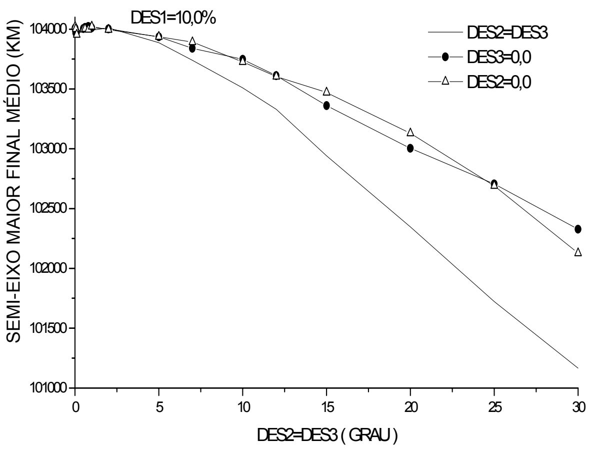
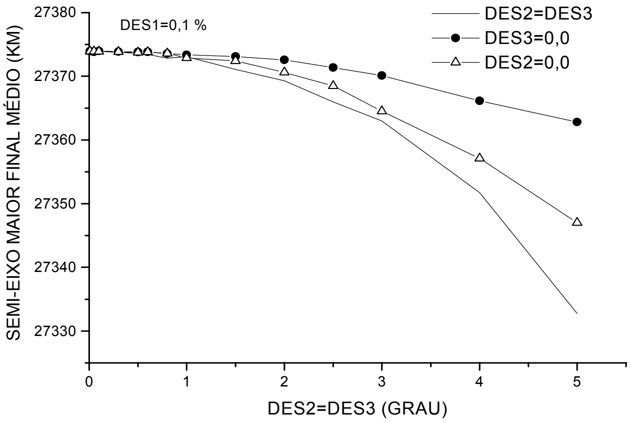
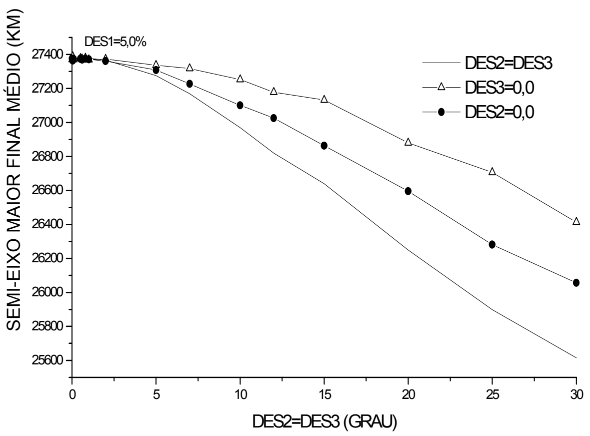


Fig. 3 – Mean semi-major axis vs. DES2=DES3, DES1=10,0%, TS

The first important result we found is the nonexistence of the cause/effect relation between the mean semi-major axis and DES1, that is, the modulus thrust vector deviations. Besides this, we can observe that inside practical interest range of the direction deviations ($0{,}0^0$ to $2{,}0^0$), the behavior of the semi-major axis mean values is the same for the three cases: 1) superposed deviations; 2) non-superposed and only “pitch” deviations and; 3) non-superposed and only “yaw” deviations. The superposition effect of these deviations is evidenced only outside practical interest range. In this range the semi-major axis admits more strong decay. These results show that the cause/effect between directions deviations and the semi-major axis, found by Jesus (1999) is valid too for the superposition deviations inside the practical interest range. These figures show too that the qualitative behavior for the direction deviations non-superposed are, approximately, the same. In the follow we show the results of the "practical" orbit.

Fig. 4- Mean semi-major axis vs. DES2=DES3, Fig. 5 – Mean semi-major axis vs. DES2=DES3,

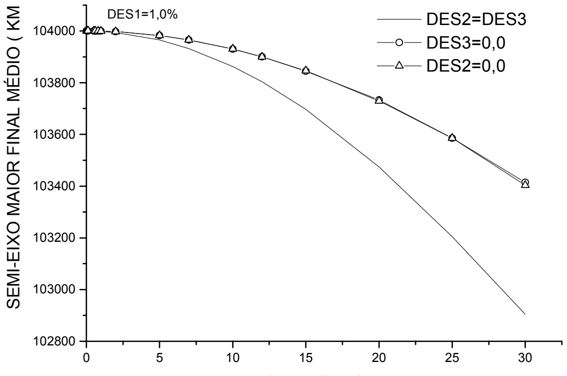




DES1=0,1%, PO

DES1=3,0%, PO

We can verified that the loss of optimality in the final mean semi-major axis values occurs with the increasing of the equals superposed directions deviations, for the "practical". This loss is more strong w.r.t. the individual "pitch" or "yaw" angles deviations cases. This transfer includes changes in the orbit inclination and thrust stronger than for the "theoretical" orbit. It is evident that one more energetic transfer regime is needed in this case, because it occurs change plane. The "yaw" deviations are out-plane deviations and affects more than the "pitch" deviations in the semi-major axis values. In the Figures 6 and 7 we show the results of the "theoretical" operational (TO) orbits.



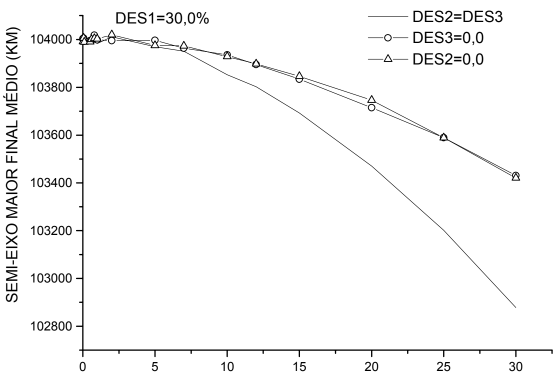
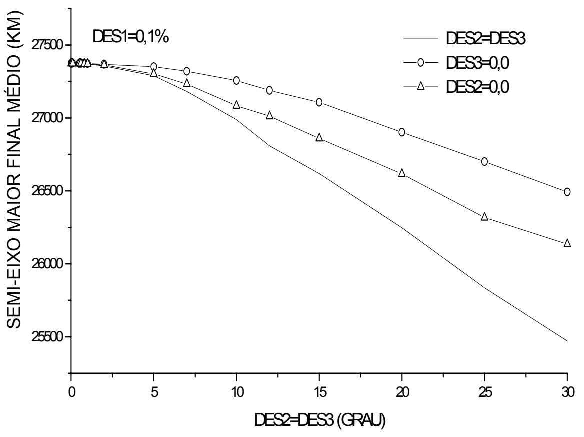


Fig. 6 - Mean semi-major axis vs. DES2=DES3, DES1=1,0%, TO

Fig. 7 – Mean semi-major axis vs. DES=DES3, DES1=30,0%, TO

These curves for the “theoretical” operational orbits show similar behaviors w.r.t. that for the “theoretical” systematic. We can observe inside practical interest deviations range, the effects of the superposition equal directions deviations do not appear. In the off-range, the decay in the final mean semi-major axis is more accentuated. These results do not depend of the DES1 inside the spatial missions range. For DES1 > 20,0%, we verified final mean semi-major axis up-values fluctuations.

In the Figures 8 and 9 we show the results of the “practical” systematic (PS) orbits.



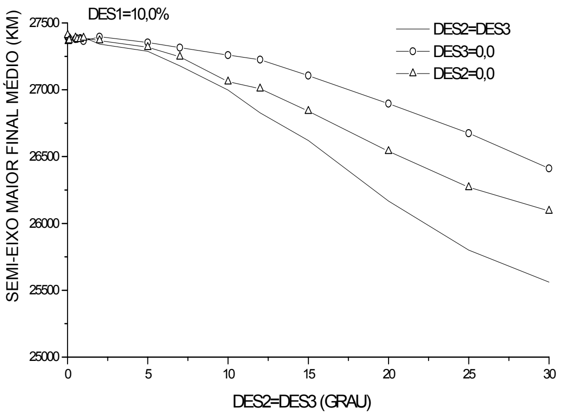
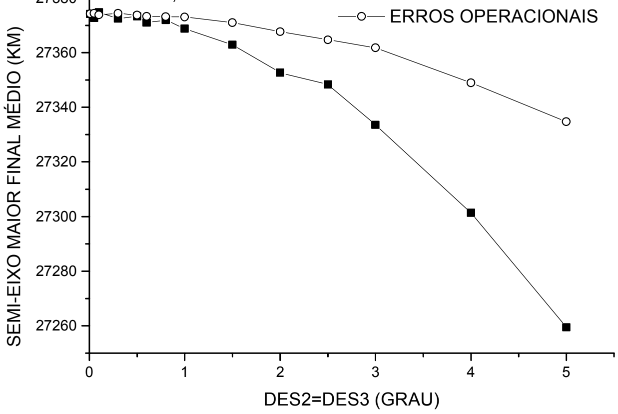


Fig. 8 – Mean semi-major axis vs. DES2=DES3, DES1=0,1%, PS

Fig. 9 – Mean semi-major axis vs. DES2=DES3, DES1=10,0%, PS

These maneuvers under systematic equals directions deviations keep the general behavior, observed in the “practical” operational orbits. But, the effects of the directions equals superposed deviations are more strong for the PS orbits w.r.t. PO. Once more, the effects from DES1 are seen only for the off-range deviations inside the final mean semi-major axis up-values. In the follow, we show the comparison between the systematic and operational deviations to the both orbits (Figures 10 and 11).



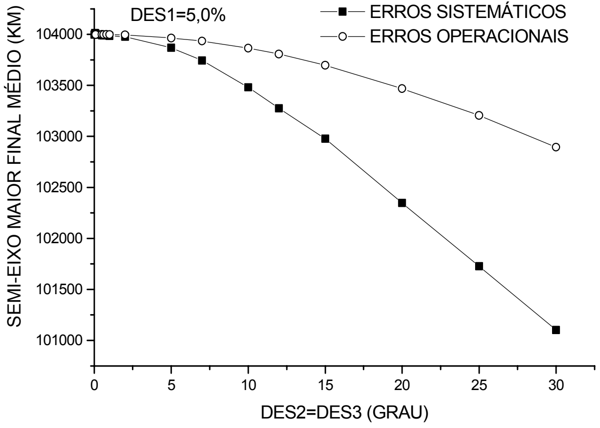


Fig. 10 - Comparison between PS and PO orbits, DES2=DES3, DES1=1,0%

Fig. 11 - Comparison between TS and TO orbits, DES2=DES3, DES1=5,0%

We observe, clearly, that the systematic effects are more strong than operational effects for the both orbits. They deform more the final mean semi-major axis, because they are equals along the burn arcs, during the transfers orbits. The operational deviations occur randomly in each burn and this fact provides not strong effect in mean. So, the final mean semi-major axis values decay, but with the minor “velocity”.

THEORETICAL AND PRACTICAL MANEUVERS WITH DIFFERENTS SUPERPOSED DEVIATIONS

In this section we present the orbital transfers under different superposed thrust deviations. These results are more general and more realists, because model the physical reality with more precision. We know that during the propulsion (or burn arcs) the fuel flux is ejected and the thrust vector deviations occur in just time, that is, occur the burn with superposition different deviations in “pitch”, “yaw” directions and in modulus vector. Therefore, this approach is more realist w.r.t. the last with individual action deviations.

The Figures 12 to 19 show the several results of these effects to the final mean semi-major axis with different combinations for the directions deviations. The Figures 12 to 15 show the TO and TS results.

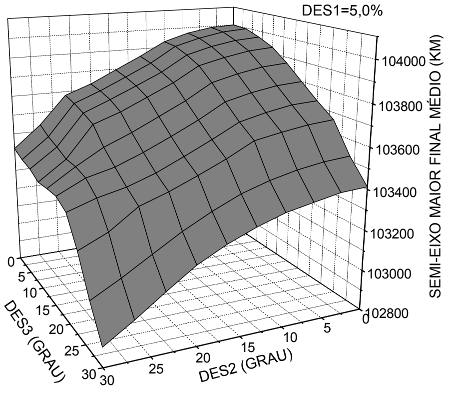


Fig. 12 – Mean semi-major axis vs. DES2≠ DES3, DES1=5,0%, TO

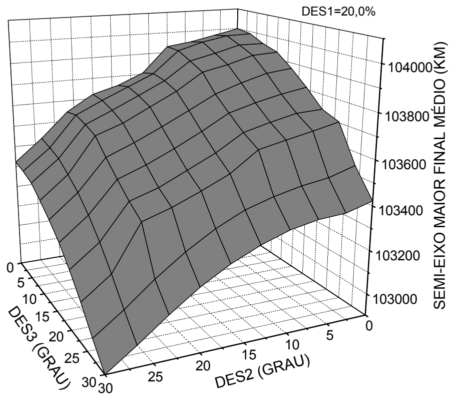
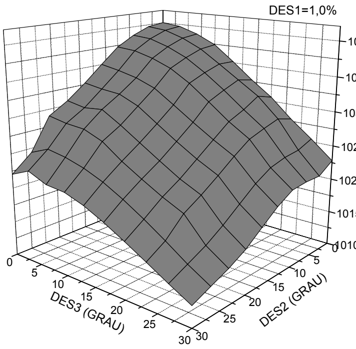


Fig. 13 – Mean semi-major axis vs. DES2≠ DES3, DES1=20,0%, TO



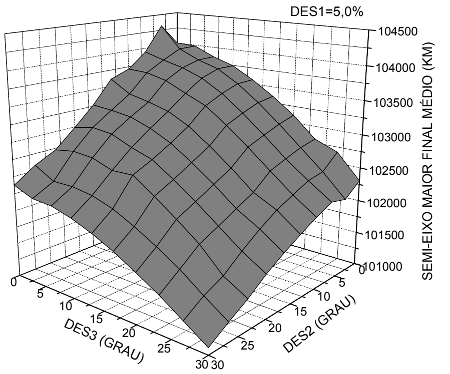
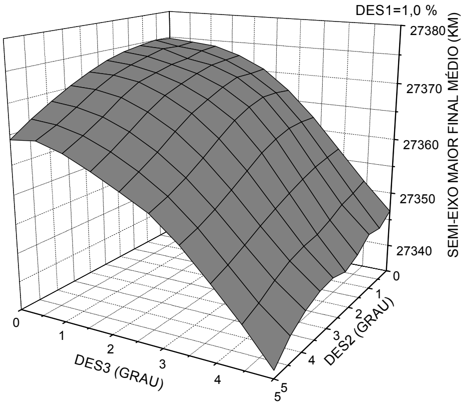


Fig. 14 – Mean semi-major axis vs. DES2≠ DES3, DES1=1,0%, TS

Fig. 15 – Mean semi-major axis vs. DES2≠ DES3, DES1=5,0%, TS

The Figures 12 to 15 show the TS results for the different superposed thrust vector deviations. The surface general is one revolution paraboloid for the final mean semi-major axis values, inside the practical interest range. It occurs loss of this symmetry in the off-range deviations values (deviations > $2,0^0$), that is, the revolution paraboloid turns deformed for great directions deviations. With relation to the equals superposed deviations case, this results present considerably fall in the semi-major axis values. It means, the final orbit is so different of the nominal orbit, therefore, it demands corrections maneuvers with more fuel consumption w.r.t. the equals deviations. These results are expected in the real and practical missions and this shows the importance of ours numerical results. In the follow, we present the results of the "practical" orbits (Figures 16 to 19).



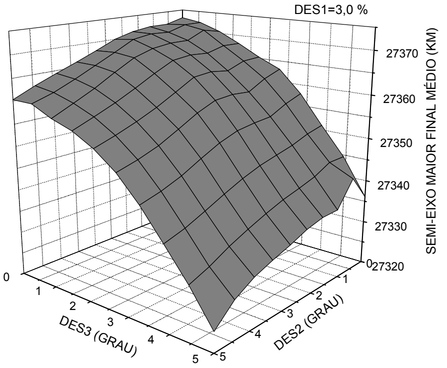
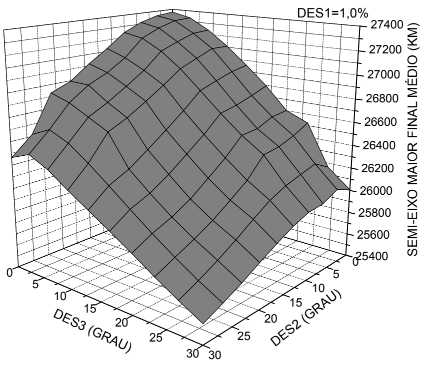


Fig. 16 – Mean semi-major axis vs. DES2≠DES3, DES1=1,0%, PO

Fig. 17 – Mean semi-major axis vs. DES2≠DES3,DES1=3,0%,PO



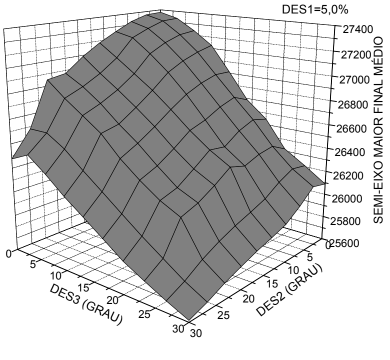


Fig. 18 – Mean semi-major axis vs. DES2≠DES3, DES1=1,0%, PS

Fig. 17 – Mean semi-major axis vs. DES2≠DES3,DES1=5,0%,PS

These last Figures (16 to 19) confirm the expected results. The surface deformation is evident in off-range practical interest. In this particular orbit (out-plane) the effect of the plane changing increase the decay of the final mean semi-major axis. Only for the small directions deviations it occurs one regular surface, characterizing superposition deviations negligible effect. The corrections maneuvers out-plane are very expensed and the orbit-target will require more them (Jesus, 1999). Besides this, the PS orbits will demand more corrections than the PO orbits.

CONCLUSIONS

We obtained from the numerical analysis (Monte-Carlo simulation) of the non-impulsive under superposition deviations, many practical missions results. All the simulations realized in this paper were done in 1000 runs. In general, the results showed that the orbital transfers under thrust vector superposed directions deviations effects are more realist and more damaged in the final orbit w.r.t. those orbital transfers without superposition of the thrust vector deviations. The superposition of the different directions deviations are more strong, according with the expected and practical missions results, than the superposition of the equals directions deviations. In the practical interest range, the revolution paraboloide of the cause/effect is regular. In off-range this surface is deformed, due loss the symmetry. The out-plane orbits are more affected through the "yaw" superposed deviations. The surface presents the propagation of the thrust vector deviations inside the real transfer trajectory in the final mean semi-major axis values, turning them minor than those of the nominal orbit, along the fuel burn. This results are general and inside the practical interest they confirm the results by Jesus (1999) for the transfers without superposition in direction deviations. In other hand, all the results show that the systematic superposed deviations are more strong w.r.t. operational superposed deviations. The superposition of the deviations in the orbital transfers, particularly, different deviations, models many real situations to attend space missions interests.

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