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ABSTRACT

The thrust vector deviations effects in the eccentricity of the spatial vehicles trajectories were analyzed in this paper. The trajectories were subjected to the nonlinear and stochastic keplerian dynamic with fuel economy. We found the algebraic relations of the effect this dynamic along the transfers maneuvers and in the final trajectory, through the eccentricity of the orbits and of the deviations provided in the thrust vector, which show nonlinear and quadratic relation for the first approximation. Numerically, with Monte-Carlo analysis, the results confirmed this dependence to out-plane orbits. In the coplanar orbits, the eccentricity values do not affected. The relations show these effects in the final orbit eccentricity are proportional to the even powers of the causes deviations. We showed the JPS2 family curves.

INTRODUCTION

Generally, we divide the perturbations in the transfers orbits in 1) natural perturbations and 2) non-natural perturbations. These perturbations deviate the spatial vehicles from their nominal trajectories, causing loss of energy and optimality. The natural perturbations can occur under several forms, through natural forces action. The non-natural perturbations in spatial vehicles occur due the technologic limitations or non-idealistic model of the vehicle systems. All these perturbations act in several orbital elements, causing them many deviations effects. These effects provide non-ideal (actual) orbits, which need direction corrections, for example. The analytical solution of the transfer problems under perturbations is very difficult, presenting many mathematical obstacles. In the most cases, the success is possible only for one problem approximated version. In this paper, we analyzed the non-natural perturbations effects through the orbit eccentricity of spatial vehicle under thrust "pitch" and "yaw" direction deviations.

The deviations effects over eccentricity due to several causes have been studied for many authors. Garfinkel (1958), Perkins (1958) and Roberson (1958) presented important results about the drag atmospheric and irregularities of the Earth influences. They showed approximated expressions for the eccentricity variation. Others authors studied the eccentricity variations under others ways, for example, King-Helle et all (1960). In Jesus et all (2002a) we can found one survey on the perturbations orbital problems. In this paper were considered many results with forces (non-rotational and non-sphericity atmosphere, solar radiation pressure, infrared, etc.) influences that were studied over the orbital maneuvers for an artificial satellite. In general, we can say the exact analytical solution for this problem with several forces (natural or non-natural) do not exist.

In this paper, we present the numerical and algebraic results of the eccentricity effects due the thrust direction misalignments in the coplanar transfer mission. We used the approach probabilistic for the uniform and gaussian function distribution deviations (causes). The results were obtained for two transfers: the first, a low thrust transfer between high coplanar orbits (we call it "theoretical transfer"), used by Biggs (1978,1979) and Prado (1989); the second, a high thrust transfer between middle noncoplanar orbits (the first transfer of the EUTELSAT II-F2 satellite, we call it "practical transfer") implemented by Kuga (1991) et alli. The simulations were done for both transfers with minimum fuel consumption. The "pitch" and "yaw" angles were taken as control variables such that the overall minimum fuel consumption defines each burn of the thrusters. The geometric development of these coordinates systems can be found in Jesus et all (2002b).

ALGEBRAIC ANALYSIS FOR THE CAUSE EFFECT RELATIONS TO ECCENTRICITY

To begin this algebraic analysis we use the relation between angular momentum and the eccentricity:

$$H = m.\sqrt{\mu.a.\left(1-e^{2}\right)} \tag{1}$$

with,

H = angular momentum;
m = satellite mass;
a = semi-major axis;
e = eccentricity;

The thrust force acting over the satellite, due the propulsion system, is in the transversal and radial directions. Associated to this force are torque in the same directions. The time variation rate of the angular momentum is equal to the external torque and, in this case, only their transversal component is different of zero. So, the transversal torque is:

$$\frac{dH(t)}{dt} = \Gamma_{t} = T.\cos\alpha(t).r(t) \tag{2}$$

We can compute the integral the equation (2) to orbit without “pitch”, in the time interval $[t_1,t_2]$:

$$\Delta H(t_1,t_2) = H(t_2) - H(t_1) = \int_{t_1}^{t_2} T.\cos\alpha(t).r(t).dt \tag{3}$$

Equivalently, to orbit with "pitch" error, we have,

$$\Delta^{'} H(t_1,t_2) = H^{'}(t_2) - H^{'}(t_1) = \int_{t_1}^{t_2} T.\cos[\alpha(t)+\Delta\alpha(t)].r^{'}(t).dt \tag{4}$$

If we take the difference between these two last angular momentum variations, we have,

$$\Delta_2 H(t_1,t_2) \equiv \Delta^{'} H(t_1,t_2) - \Delta H(t_1,t_2) = \int_{t_1}^{t_2} T.\{\cos[\alpha(t)+\Delta\alpha(t)].r^{'}(t) - \cos\alpha(t).r(t)\}.dt \tag{5}$$

The integral of the right size the equation (2) can be written, after some algebraic manipulation,

$$\Delta_2 H(t_1,t_2) = \int_{t_1}^{t_2} T.\cos\alpha(t).[\cos[\Delta\alpha(t)]-1].r^{'}(t)dt \text{ -}$$

$$\int_{t_1}^{t_2} T.\text{sen}\,\alpha(t).\text{sen}[\Delta\alpha(t)].r^{'}(t)dt \text{ - } \int_{t_1}^{t_2} T.\cos\alpha(t)\{r^{'}(t)-r(t)\}dt \tag{6}$$

If we apply the expectation operator through the both parts of the equation (6), considering the ergotic hypothesis and the fact the functions T, sine and cosine are time-deterministic, we have,

$$\text{E}\{\,\Delta_2 H(t_1,t_2)\,\} = \int_{t_1}^{t_2} T.\cos\alpha(t)\,\text{E}\{[\cos[\Delta\alpha(t)]-1].r^{'}(t)\,\}.\text{dt} -$$

$$\int_{t_1}^{t_2} T.\text{sen}\,\alpha(t)\,\text{E}\{\,\text{sen}[\Delta\alpha(t)].r^{'}(t)\,\}.\text{dt - } \int_{t_1}^{t_2} T.\cos\alpha(t)\,\text{E}\{\,r^{'}(t)-r(t)\,\}.\text{dt} \tag{7}$$

The expression (7) is general and, we can consider some cases of the practical interests. For example, the $\Delta\alpha(t)$ non-correlated with the vector ratio r'(t). In this case, the computation of the expectation in (7) reduces to the computation of the trigonometric functions expectations of the "pitch" error. So,

$$\text{E}\{\,\Delta_2 H(t_1,t_2)\,\} = \int_{t_1}^{t_2} T.\cos\alpha(t)\,\text{E}\{[\cos[\Delta\alpha(t)]-1]\}.\text{E}\{\,r^{'}(t)\,\}.\text{dt} -$$

$$\int_{t_1}^{t_2} T.\text{sen}\,\alpha(t)\,\text{E}\{\,\text{sen}[\Delta\alpha(t)]\}.\text{E}\{\,r^{'}(t)\,\}.\text{dt} - \int_{t_1}^{t_2} T.\cos\alpha(t)\,[\text{E}\{\,r^{'}(t)\,\}\text{-E}\{\,r(t)\,\}].\text{dt} \tag{8}$$

We can consider the expectance of the vector ratio with "pitch" error, approximately, equal to that the vector ratio without error, because in the mean zero process these values turn very closed, when each error is inserted symmetrically and with equal occurrence probability. With this condition, the last integral in (8) turns null and this expression turns function only of the known expectations to simple trigonometric functions (sine and co-sine). Besides this, in this distribution process of the symmetric probability, the expectation of the odd functions is null. So, the second integral is null too. Therefore, we have,

$$E\{\Delta_2 H(t_1,t_2)\} = Q_{13}.\left(-\frac{1}{3!}.\Delta^2\alpha_{máx} + \frac{1}{5!}.\Delta^4\alpha_{máx} - \frac{1}{7!}.\Delta^6\alpha_{máx} + \ldots\right) \tag{9}$$

for the uniform case and

$$E\{\Delta_2 H(t_1,t_2)\} = Q_{13}.\left(-\frac{\sigma_\alpha^2}{2} + \frac{\sigma_\alpha^4}{8} - \frac{\sigma_\alpha^6}{48} + \ldots\right) \tag{10}$$

for the gaussian case. The quadrature is,

$$Q_{13} = \int_{t_1}^{t_2} T.\cos\alpha(t).r(t).dt \tag{11}$$

The right size of the equations (10) and (11) can be computed. We have,

$$E\{\Delta_2 H(t_1,t_2)\} = E\{m.\sqrt{\mu.a'(t_2).\left(1-e'^2(t_2)\right)} - m.\sqrt{\mu.a'(t_1).\left(1-e'^2(t_1)\right)} +$$

$$m.\sqrt{\mu.a(t_1).\left(1-e^2(t_1)\right)} - m.\sqrt{\mu.a(t_2).\left(1-e^2(t_2)\right)}\} \tag{12}$$

The second and third terms of the expression (12) are symmetric, because the values of the semi-major axis and of the eccentricity to the departure orbits with "pitch" error and without it are equals in the initial instant. Using this fact,

$$E\{\Delta_2 H(t_1,t_2)\} = E\{m.\sqrt{\mu.a'(t_2).\left(1-e'^2(t_2)\right)} - m.\sqrt{\mu.a(t_2).\left(1-e^2(t_2)\right)}\} \tag{13}$$

or,

$$E\{\Delta_2 H(t_1,t_2)\} = m.\sqrt{\mu}\ .E\{\sqrt{a'(t_2)}.\sqrt{1-e'^2(t_2)} - \sqrt{a(t_2)}.\sqrt{1-e^2(t_2)}\} \tag{14}$$

We can expand the both terms of the expectation in binomial series, when the eccentricity values are between 0 and 1,

$$\left(1-e'^2\right)^{1/2} = 1 + \frac{1}{2}.\left(-e'^2\right) - \frac{1}{2}.\frac{1}{4}.\left(-e'^2\right)^2 + \frac{1}{2}.\frac{1}{4}.\frac{3}{6}\left(-e'^2\right)^3 - \frac{1}{2}.\frac{1}{4}.\frac{3}{6}.\frac{5}{8}\left(-e'^2\right)^4 + \ldots \tag{15}$$

or

$$\left(1-e'^2\right)^{1/2} = 1 - \frac{1}{2}.\left(e'^2\right) - \frac{1}{2}.\frac{1}{4}.\left(e'^4\right) - \frac{1}{2}.\frac{1}{4}.\frac{3}{6}\left(e'^6\right) - \frac{1}{2}.\frac{1}{4}.\frac{3}{6}.\frac{5}{8}\left(e'^8\right) + \ldots \tag{16}$$

and

$$\left(1-e^{2}\right)^{1/2}=1+\frac{1}{2}.\left(-e^{2}\right)-\frac{1}{2}.\frac{1}{4}.\left(-e^{2}\right)^{2}+\frac{1}{2}.\frac{1}{4}.\frac{3}{6}\left(-e^{2}\right)^{3}-\frac{1}{2}.\frac{1}{4}.\frac{3}{6}.\frac{5}{8}\left(-e^{2}\right)^{4}+\ldots \tag{17}$$

or

$$\left(1-e^{2}\right)^{1/2}=1-\frac{1}{2}.\left(e^{2}\right)-\frac{1}{2}.\frac{1}{4}.\left(e^{4}\right)-\frac{1}{2}.\frac{1}{4}.\frac{3}{6}\left(e^{6}\right)-\frac{1}{2}.\frac{1}{4}.\frac{3}{6}.\frac{5}{8}\left(e^{8}\right)+\ldots \tag{18}$$

Taking these last results and putting them in (13), we have,

$$\mathrm{E}\{\Delta_{2}H(t_{1},t_{2})\}= m.\sqrt{\mu}\ .\mathrm{E}\{\frac{\Delta a}{\sqrt{a'}+\sqrt{a}}-\frac{\Delta a}{\sqrt{a'}+\sqrt{a}}.\frac{e'^{2}}{2}-\sqrt{a}.\frac{e'^{2}}{2}+\sqrt{a}.\frac{e^{2}}{2}-$$

$$\frac{\Delta a}{\sqrt{a'}+\sqrt{a}}.\frac{e'^{4}}{8}-\sqrt{a}.\frac{e'^{4}}{8}+\sqrt{a}.\frac{e^{4}}{8}-\frac{\Delta a}{\sqrt{a'}+\sqrt{a}}.\frac{3.e'^{6}}{48}-\sqrt{a}.\frac{3.e'^{6}}{48}+\sqrt{a}.\frac{3.e^{6}}{48}-$$

$$\frac{\Delta a}{\sqrt{a'}+\sqrt{a}}.\frac{15.e'^{8}}{384}-\sqrt{a}.\frac{15.e'^{8}}{384}+\sqrt{a}.\frac{15.e^{4}}{384}-\ldots\} \tag{19}$$

We used the result,

$$\sqrt{a'}=\frac{\Delta a}{\sqrt{a'}+\sqrt{a}}+\sqrt{a} \tag{20}$$

The expressions of the even powers differences in (18) assume the form,

$$\mathrm{E}\{\Delta_{2}H(t_{1},t_{2})\}= m.\sqrt{\mu}\ .\mathrm{E}\{\frac{\Delta a}{\sqrt{a'}+\sqrt{a}}.[1-\frac{e'^{2}}{2}-\frac{e'^{4}}{8}-\frac{3.e'^{6}}{48}-\frac{15.e'^{8}}{384}-\ldots]-$$

$$\frac{\sqrt{a}}{2}.\left(e'^{2}+e^{2}\right)\Delta e-\frac{\sqrt{a}}{8}.\left(e'^{2}+e^{2}\right).\left(e'+e\right)\Delta e-\frac{3.\sqrt{a}}{48}.\left(e'^{4}+e^{4}+e'^{2}.e^{2}\right).\left(e'+e\right)\Delta e-$$

$$\frac{15.\sqrt{a}}{384}.\left(e'^{4}+e^{4}\right).\left(e'^{2}+e^{2}\right)\left(e'+e\right)\Delta e-\ldots\} \tag{21}$$

with

$$\Delta e = e'(t_2) - e(t_2) \equiv e' - e \tag{22}$$

To terms with order superior to two in the eccentricity, the expression (21) can be simplified (that is valid in the interval of the eccentricity, [0,1]), that is, the equation (21) turns,

$$E\{\Delta_2 H(t_1,t_2)\} \cong m.\sqrt{\mu}\ .E\{\frac{\Delta a}{\sqrt{a'}+\sqrt{a}}.[1-\frac{e'^2}{2}] - \frac{\sqrt{a}}{2}.\left(e'^2+e^2\right)\Delta e\} =$$

$$m.\sqrt{\mu}\ .[E\{\frac{\Delta a}{\sqrt{a'}+\sqrt{a}}\} - E\{\frac{\Delta a}{\sqrt{a'}+\sqrt{a}}.[\frac{e'^2}{2}]\}\ - E\{\frac{\sqrt{a}}{2}.\left(e'^2+e^2\right)\Delta e\}] \tag{23}$$

If we assume that in the final instant the semi-major axis with and without “pitch” error are so closed,

$$\sqrt{a'(t_2)} \cong \sqrt{a(t_2)} \tag{24}$$

then, the expression in (23), turns,

$$E\{\Delta_2 H(t_1,t_2)\} \cong E\{\Delta a\}.g_1 - E\{\Delta e\}.g_2 \tag{25}$$

with,

$$g_1 = \frac{m.\sqrt{\mu}}{\sqrt{a(t_2)}}.\left(\frac{1}{2}-e^2\right) \tag{26}$$

$$g_2 = (m\sqrt{\mu}.\sqrt{a(t_2)}.e^2) \tag{27}$$

Returning to the equations (9) and (10), we obtain,

$$E\{\Delta e\} = \frac{g_1}{g_2}.E\{\Delta a\} - \frac{Q_{13}}{g_2}.\left(-\frac{1}{3!}.\Delta^2\alpha_{máx} + \frac{1}{5!}.\Delta^4\alpha_{máx} - \frac{1}{7!}.\Delta^6\alpha_{máx} + \ldots\right) \tag{28}$$

for the uniform case and,

$$E\{\Delta e\} = \frac{g_1}{g_2}.E\{\Delta a\} - \frac{Q_{13}}{g_2}.\left(-\frac{\sigma_\alpha^2}{2} + \frac{\sigma_\alpha^4}{8} - \frac{\sigma_\alpha^6}{48} + \ldots\right) \tag{29}$$

for the gaussian case.

The curve form that we look for can be found, to our approach, using the values of the semi-major axis expectations computed by Jesus (1999) in the equations (28) and (29). After this, we have to the non-correlated case,

$$\mathrm{E}\{\Delta e\} = g_3.\left(-\frac{1}{3!}.\Delta^2\alpha_{máx} + \frac{1}{5!}.\Delta^4\alpha_{máx} - \frac{1}{7!}.\Delta^6\alpha_{máx} + \ldots\right) \quad (30)$$

for the uniform case and,

$$\mathrm{E}\{\Delta e\} = g_3.\left(-\frac{\sigma_\alpha^2}{2} + \frac{\sigma_\alpha^4}{8} - \frac{\sigma_\alpha^6}{48} + \ldots\right) \quad (31)$$

for the gaussian case.

with,

$$g_3 = \left(\frac{g_1}{g_2}.K_1 - \frac{Q_{13}}{g_2}\right) \quad (32)$$

The expressions (30) and (31) show the curve form. The coefficients g_i's depend of the quadratures and of constants values.

THE JPS2 CURVES OF NONIMPULSIVE AND NONCOPLANAR ORBITAL MANEUVERS. NUMERICAL RESULTS - ECCENTRICITY

The expressions (30) and (31) show the nonlinear relation between the expectation of the eccentricity deviation and the maximum "pitch" deviation. These results were obtained only for the "practical" transfer orbit, that is, for the noncoplanar orbit of the EUTELSAT II-F2 satellite realized with change of inclination. For the "theoretical" orbit, coplanar, the values eccentricity keep constants with "pitch" deviation (Jesus, 1999). The numerical results confirm the curves JPS2 forms, previewed in (30) and (31). The Figures 1 show these results. We simulated (Monte-Carlo) 1000 ensembles of the transfer trajectories for the both kind of deviations (uniform -U, gaussian - G), for the maneuver "practical" - P, for the random bias (S) and white noise (O) deviations.

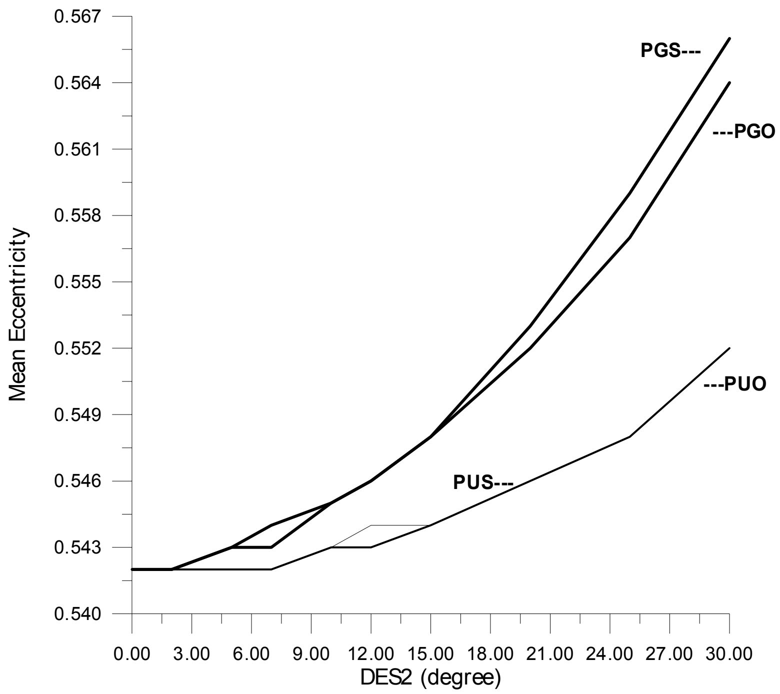

Fig. 1 – Mean Eccentricity vs. DES2, Practical Transfer Orbit

This graphic shows, clearly, the nonlinear dependence between the mean eccentricity values and the "pitch" deviations, during the transfers maneuvers. This fact, do not depends of the kind of deviations distribution or of their dynamic, if systematic deviations or if operational deviations. Qualitatively the results are the same for the transfer phenomena, that is, the JPS2 curves present the equivalent form.

CONCLUSIONS

We studied the orbital transfer maneuvers of the satellite under thrust direction "pitch" deviations. We found the cause/effect relation between the expectation eccentricity and the "pitch" deviations for the "practical" transfer orbit, those noncoplanar orbits. In the coplanar orbits, the "theoretical" orbits, the eccentricity values do not affected. This relation showed one composition of terms of even powers deviations order. The first term presented one quadratic relation, exactly in the deviation region of the practical missions interests. The results of the numerical simulation (Monte-Carlo) confirmed the algebraic relations found in this paper. The JPS2 curves found presents the contributions of the all the even powers terms of the "pitch" deviations. The out-plane effects in the "practical" transfers orbits, affect in the trajectories forms, because depend of the energies changes during the noncoplanar dynamic along the orbit transfer. We conclude that occur one dilatation of the elliptical orbits.

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