CONTROL OF A NONLINEAR SLEWING FLEXIBLE BEAM

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Summary The system under investigation here comprises a flexible beam connected to an actuator (dc motor) in such a way that the angular displacement of the motor axis and the beam deflection develops in the same plan perpendicular to the before quoted axis (this movement is named *slewing*). A control law is designed and tested for the linear and ideal (no influence of the structure dynamics over the energy source) and for the linear and nonideal (the structure dynamics interacts with the energy source dynamics) mathematical model. The beam curvature is considered linear and the only nonlinear term present here comes from centripetal effects. Although the gain of the controller is obtained using the linear model, its performance is tested also for the non-linear model with good results. Low and high velocities are investigated in order to verify the robustness of the proposed control law when dealing with the not initially considered term.

INTRODUCTION

Nowadays a great number of space missions involve Large Space Structure (LSS) using attitude control systems with high degree of pointing accuracy and at same time urging for quick time of response. Examples of mission with that requirement are the Hubble Space Telescope and the International Space Station. Other general examples are: lightweight robotic manipulators in industry or space applications and solar panels or antennas in satellites (to quote just a few).

In the modeling of flexible rotating structures, the interaction between the angular displacement, θ , and the deflection of a beam like structure, *v* (x,t) (or q(t) after discretization), can be very important, mainly when the maneuvers involve high angular velocities [1],[2]. Assuming linear curvature for the beam, this kind of interaction involves a nonlinear term in the governing equations. This term is of order three and is associated with centripetal stiffening. Figure 1 presents a schematic of the problem under considerations here. The elastic displacement of the beam, v(x,t), is excited by the angular motion θ (t) (and its derivatives).

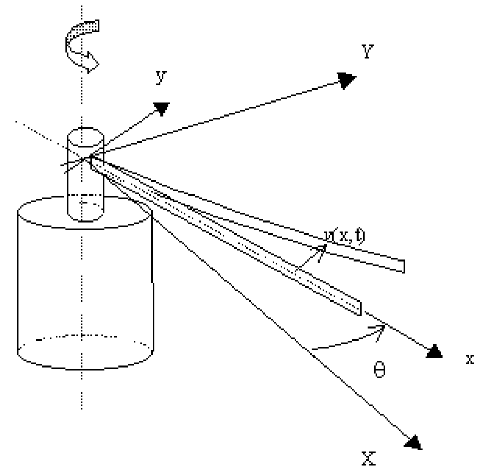

Figure 1 – The rotating flexible beam (XY is the inertial frame and
xy is the rotating frame)

The actuator and energy source investigated here is a dc motor, which is also used for noncolocated control of the beam. Two different approaches used in this work to analyze the behaviour of the nonlinear system is related to the concepts of *ideal system* and *nonideal system*. When the dynamics of the driven system (the beam-like structure here) interacts with the dynamics of a limited energy source (the dc motor, for instance) it is very important to take into account the effects of this interaction (exchange of energy) in the control system design. The complete system is then said to be *nonideal.* For these systems, the excitation profile is not known previously (since it depends on the structure unknown response). Otherwise the system is said to be *ideal* [3]. The governing equations of motion for the system depicted in Figure 1 are derived in [4]. An investigation about the influences of the nonlinearties in the dynamical behavior of this system can be found in [5] and [6].

MATHEMATICAL MODELING AND RESULTS

The governing equations of motion for the flexible beam are derived through the lagrangian formalism [4]. For the system composed by the linear curvature beam together with a dc motor the governing equations are given by:

$$L_m \dot{i}_a + R_a i_a + K_b N_g \dot{\theta} = U \tag{1}$$

$$(I_{eixo} + I_{motor} N_g^2)\ddot{\theta} + (c_m N_g^2)\dot{\theta} - (N_g K_t) i_a - \beta\left(EI \sum_{i=1}^{n} q_j(t)\phi_j''(0)\right) = 0 \tag{2}$$

$$\ddot{q}_j + \mu\dot{q}_j + \mathbf{w}_j^2 q_j + \alpha_j\ddot{\theta} - \dot{\theta}^2 q_j = 0 \tag{3}$$

plus the boundary conditions: $\phi''(L,t)=0$, $\phi'''(L,t)=0$. In equation (2), if $\beta = 0$ the system is ideal and if $\beta = 1$ the system is nonideal.

The control used here is based on state feedback with pole placement. The idea is to control the beam vibrations after some specified value for the angle θ is reached. This angular displacement must be performed as fast as possible and the control signal (electric tension) is applied in the instant the slewing angle is equal to the desired final value.

The gains of the feedback control law are obtained applying the pole placement technique for the linear system (initially the nonlinear term is neglected). The nonlinear term considered depends on the square of the angular velocity of the slewing axis, which means that the greater this velocity the greater the influence of the nonlinear term. Despite the gain being calculated using a linear model, the control is evaluated considering the linear system and the nonlinear system (both ideal and nonideal).

Carrying out numerical simulations considering different values for the slewing velocity it is possible to evaluate the influence of the nonlinear term in the performance of the controller. As expected, the performance is degraded for high velocities in the nonlinear case. Increasing the angular (or slewing) velocity beyond some value it is noted that the control fails and the system is unstable. Though with this critical velocity the beam deflection is great enough for the linear curvature model adopted to be no more valid. Some numerical results are depicted in Figures 2 and 3. The nonlinear case (ideal and nonideal) is presented. The proposed control law was succesfully (numerically) tested.

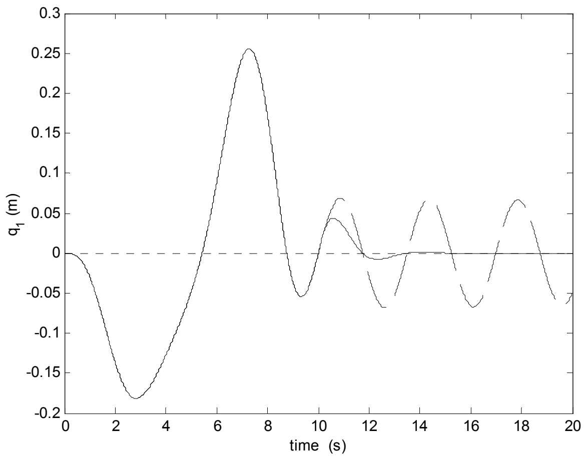

Figure 2 Nonlinear ideal system

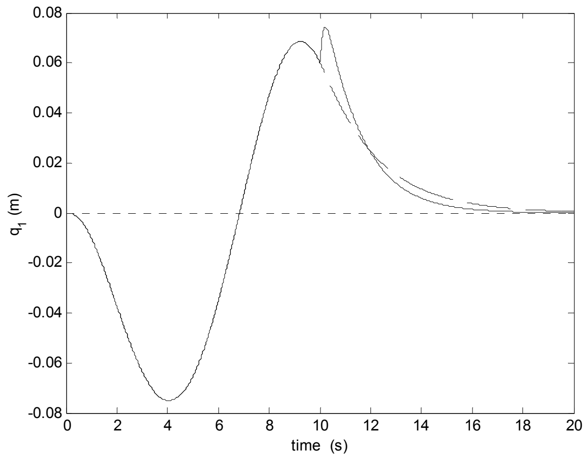

Figure 3 Nonlinear nonideal system

CONCLUSIONS

For a suitable range of beam deflections (associated with suitable angular velocities of the alewing axis), the proposed control law and gains derived based on the linear governing equations of motion works satisfactorily well, even in the presence of the nonlinear term. The beam vibrations in the ideal model is less damped than in the nonideal model and the control influence over the system response is more remarkable in the first case. The nonideal interaction between the beam and the actuator helps to eliminate the beam vibration, even though the control law is able to slightly improve this behaviour. Figures 2 and 3 shows that the linear control law works for both linear and nonlinear mathematical models.

References

[1] Low, K. H., “Vibration analysis of a tip-loaded beam attached to a rotating joint”, Computers and Structures, vol. 52, no. 5, pp 955-968, 1994.

[2] Putter, S., Manor, H., “Natural frequencies of radial rotating beams”, Journal of Sound and Vibrations”, 56(2), pp 175-185, 1978.

[3] Kononenko, V. O., “Vibrating systems with a limited power supply”, Iliffe Books Ltd., 1969.

[4] Fenili, A., “Mathematical modeling of nonlinear slewing structures and applications on mechanical engineering”, PhD Thesis, Univ. Campinas, Campinas, São Paulo, Brazil, 2000.

[5] Fenili, A; Souza, L.C.G.; Góes, L.C.S.; Balthazar, J.M.; “On the Damping Effect of No ideal Interaction in Beam-Like Structures in Slewing Motion”. In 6th Biennial Conference on Engineering Systems Design and Analysis – ESDA 2002. Istanbul, Turkey, July 8-11, 2002.

[6] Nayfeh, A. H., Mook, D. T., “Nonlinear oscillations”, John Wiley and Sons, Inc., New York, 1979.