# RECONSTRUCTION OF CHLOROPHYLL VERTICAL PROFILES FROM *IN SITU* RADIANCES USING THE ANT COLONY META-HEURISTIC

**Roberto P. Souto**
**Haroldo F. de Campos Velho**
**Stephan Stephany**
*[roberto,haroldo,stephan]@lac.inpe.br*
Laboratory for Computing and Applied Mathematics (LAC)
National Institute for Space Research (INPE)
P.O. Box 515 – 12245-970 – São José dos Campos (SP), BRAZIL

***Abstract.*** *It is proposed a methodology to reconstruct vertical profiles of the absorption ($a$) and scattering ($b$) coefficients in natural waters from* in situ *radiance measurements in several depths and single wavelength. A multi-region ($R = 10$) approach is employed and these coefficients are assumed as being constant in each region. The inverse problem is iteratively solved using the radiative transfer equation as direct model. A former work employed a step-by-step reconstruction methodology, estimating $a$ and $b$ in an alternate manner (Stephany et al., 2000) ($R = 1$). In the current work, bio-optical models (Mobley, 1994) are employed to correlate the chlorophyll concentration to $a$ and $b$, and thus to the single scattering albedo $\omega_0$. At every iteration, the inverse solver generates a candidate solution that is a set of discrete chlorophyll concentration values for each region. The radiative transfer equation is solved using these values by the Laplace transform discrete ordinate (LTS$_N$) method (Barichello and Vilhena, 1993; Segatto and Vilhena, 1994; Segatto et al., 1999) for 50 polar angles and 174 azimuthal modes. An objective function is given by the square difference between reconstructed and experimental radiances at every iteration. In order to compensate the nearly exponential radiance decay with depth, that unbalances the influence of radiances of different depths, a depth correction factor is used to adjust radiance values at each level. The referred objective function is minimized by an Ant Colony System (ACS) (Dorigo et al., 1996) implementation. A new regularization scheme pre-selects candidate solutions based on their smoothness, in addition to the classical Tikhonov regularization. This scheme was proposed in a crystal growth inverse problem to reconstruct the diffusion coefficient (Preto et al., 2004). Another chlorophyll candidate profile is generated and iterations proceed. Test results show the suitability of the proposed method. As hundreds of iterations are typically demanded, a parallel implementation of the LTS$_N$ method (Souto et al., 2003) is used and executed in a distributed memory machine.*

***Keywords:*** *Radiative Transfer, Inverse Problems, Parallel Computing, Ant Colony System*

## 1. INTRODUCTION

In the last decades, the development of inversion methodologies for radiative transfer problems has been an important research topic in many branches of science and engineering (McCormick, 1992; Gordon, 2002). The direct or forward radiative transfer problem in hydrologic optics, in the steady state, involves the determination of the radiance distribution in a body of water, given the boundary conditions, source term, inherent optical properties (IOPs), as the absorption $a$ and scattering $b$ coefficients, and the scattering phase function. The corresponding inverse radiative transfer problem arises when physical properties, internal light sources and/or boundary conditions must be estimated from radiometric measurements of the underwater light field. In previous works, we tried to establish a general methodology to separately tackle the reconstrution of internal sources (Stephany et al., 1998), IOP estimation (Stephany et al., 2000; Chalhoub and CamposVelho, 2000; Chalhoub et al., 2000; Chalhoub and CamposVelho, 2002), the identification of boundary conditions (CamposVelho et al., 2002, 2003b; Retamoso et al., 2002), or even a joint inversion scheme (Stephany et al., 2000). In these works, the inverse model is an implicit technique for parameter and/or function estimation from *in situ* sintetic radiometric measurements. The algorithm is formulated as a constrained nonlinear optimization problem, in which the direct problem is iteratively solved for successive approximations of the unknown parameters. Iteration proceeds until an objective-function, representing the least-square fit of model results and experimental data added to a regularization term, converges to a specified small value. An overview of this technique, as well as a survey of the results, can be found in CamposVelho et al. (2001) or in CamposVelho et al. (2003a).

This work presents a methodology to reconstruct vertical profiles of the absorption and scattering coefficients in natural waters from *in situ* radiance measurements in several depths and single wavelength. The inverse problem is iteratively solved using the radiative transfer equation (RTE) as direct model. A former work (Stephany et al., 2000) employed a step-by-step reconstruction methodology, estimating $a$ and $b$ in an alternate manner. A deterministic optimizer was employed to solve the associated inverse problem. In the current work, bio-optical models (Mobley, 1994) are employed to correlate the chlorophyll concentration to $a$ and $b$. At every iteration, the inverse solver generates a candidate solution that is a set of discrete chlorophyll concentration values. The spatial domain (geometrical depth) is discretized in 10 regions ($R = 10$). These coefficients and thus the chlorophyll concentration are assumed as being constant in each region. The dicrete chlorophyll profile is then defined by $10$ points.

The inverse problem is formulated as an optimization problem and iteratively solved using the radiative transfer equation as direct model. An objective function is given by the square difference between computed and experimental radiances at every iteration. However, the radiance values near the surface are much higher than those in deeper water, since radiance value decays nearly exponentially with depth. A depth correction factor for the radiances was first proposed by Tao et al. (1994). This work employs a new correction factor based on the mean radiance at each depth level/region.

At each iteration, the RTE is solved using the candidate set of $a$ and $b$ values by the Laplace transform discrete ordinate (LTS$_N$) method (Barichello and Vilhena, 1993; Segatto and Vilhena, 1994; Segatto et al., 1999). The RTE variables are discretized in both the polar and azimuthal dimensions: 50 polar angles and 174 azimuthal angles were considered. The associated optimization problem is solved by an Ant Colony System (ACS) (Dorigo et al., 1996) implementation. A recent intrinsic regularization scheme that pre-selects candidate solutions based on their smoothness is applied, quantified by a Tikhonov norm. This scheme was proposed in a crystal growth inverse problem to reconstruct the diffusion coefficient (Preto et al., 2004). A subsequent chlorophyll candidate profile is generated and iterations proceed. As hundreds of iterations are typically demanded and the direct model demands most of the processing time, a parallel implementation of the LTS$_N$ is used and executed in a distributed memory machine. The code was parallelized using the Message Passing Interface (MPI) communication library (Forum, 1994). The parallelization scheme distributes the pre-selected candidate solutions of the current iteration among the processors. Test results show the suitability of the proposed method using 5% and 30% noisy data. Reconstructions were performed using the proposed methodology with no depth correction factor for the radiances and with the new factor. The reconstructed profiles with the new depth correction factor have good agreement to the exact curves. Parallel performance was also evaluated.

## 2. RADIATIVE TRANSFER EQUATION IN HYDROLOGIC OPTICS

The Radiative Transfer Equation (RTE) models the transport of photons through a medium (Sobolev, 1962). Light intensity is given by a directional quantity, the radiance *I*, that measures the rate of energy being transported at a given point and in a given direction. Considering a horizontal plane, this direction is defined by a polar angle $\mu$ (relative to the normal of the plane) and a azimuthal angle $\varphi$ (a possible direction in that plane). At any point of the medium, light can be absorbed, scattered or transmitted, according to the absorption ($a$) and scattering ($b$) coefficients and to a scattering phase function that models how light is scattered in any direction. An attenuation coefficient $c$ is defined as $c = a + b$ and the geometrical depth is mapped to a optical depth $\tau$ that imbeds $c$. Assuming a plane-parallel geometry, and a single wavelength, the unidimensional integral-differential RTE, can be written as:

$$\mu \frac{\partial}{\partial \tau} I(\tau, \mu, \varphi) + I(\tau, \mu, \varphi) = \frac{\varpi_0(\tau)}{4\pi} \int_{-1}^{1} \int_{0}^{2\pi} \beta(\mu, \varphi; \mu', \varphi') I(\tau, \mu', \varphi') d\varphi' d\mu' + S(\tau, \mu, \varphi) \tag{1}$$

where $\mu \in [-1, 1]$ and $\varphi \in [0, 2\pi]$ are the cosine of the incident polar angle $\theta$ and the incident azimuthal angle, respectively. $\varpi_0(\tau) = b(\tau)/c(\tau)$ is the single scattering albedo. The scattering phase function $\beta(\mu, \varphi; \mu', \varphi')$, gives the scattering beam angular distribution, mapping the incident beam direction $(\mu, \varphi)$ to the scattered direction $(\mu', \varphi')$, and the source term is $S(\tau, \mu, \varphi)$. The heterogeneous medium, in this case offshore ocean water is then modeled as a set of $R$ homogeneous finite layers. Boundary conditions are defined between regions, at the surface (incident light) and the bottom of the water. Each layer is denoted as being a region $r$ of the multiregion domain:

$$\varpi_0(\tau) = \varpi_r \quad r = 1, 2, \ldots, R \tag{2}$$

There are several resolution methods, most of them adopting the Chandrasekhar's decomposition on the azimuthal angle (Chandrasekhar, 1960) that generates $L+1$ integral-differential equations, each one with no dependence on $\varphi$. For the discrete ordinate method, the above equations are approximated by a colocation method, where the $\mu$ integral is computed by the Gauss-Legendre quadrature formula. This yields a set of $N$ differential equations for each azimuthal mode. Each set (discretized RTE) is solved by the LTS$_N$ method (Barichello and Vilhena, 1993; Segatto and Vilhena, 1994; Segatto et al., 1999), that generates a system of equations of order $R \times N$. For the considered test cases, it was assumed $R = 10$, $N = 50$ and $L = 0, 1, \ldots, 173$ (no azimuthal simmetry on $\varphi$).

This work employs a bio-optical models to correlate the absorption and scattering coefficients of each region to the chlorophyll concentration. These coefficients are assumed to be constant in each region. Therefore discrete values $a_r$ and $b_r$ can be estimated for each region from the discrete values $C_r$.

Usually, chlorophyll profiles can be represented according Gaussian distributions (Mobley, 1994). A particular profile, corresponding to the Celtic Sea was considered:

$$C(z) = 0.2 + \frac{144}{9\sqrt{2\,\pi}} \exp\left[-\frac{1}{2}\left(\frac{z-17}{9}\right)^2\right] \tag{3}$$

where $z$ is the depth in meters and $C$ is given in $mg/m^3$. This profile can be shown in section 6. of this work, termed *exact profile*. A bio-optical model was formulated by Morel (1991) for the absorption coefficient,

$$a_r = \left[a^w + 0.06\; a^c\; C_r^{0.65}\right]\;\left[1 + 0.2\; e^{-0.014(\lambda-440)}\right] \tag{4}$$

where $a^w$ is the pure water absorption and $a^c$ is a nondimensional, statistically derived chlorophyll-specific absorption coefficient, and $\lambda$ is the considered wavelength, while another was $a_g^w$ formulated by Gordon and Morel (1983) for the scattering coefficient,

$$b_r = \left(\frac{550}{\lambda}\right)\; 0.30\; C_r^{0.62} \tag{5}$$

The considered Celtic Sea profile refers to a type of water that present a high concentration of phytoplankton in comparison to organic particles (Morel and Prieur, 1977). The values of $a^w$ and $a^c$ depend on the wavelength and can be found in tables (Mobley, 1994).

### 2.1 The discrete ordinate equations

Using the addition theorem of the spherical harmonics (Chandrasekhar, 1960), the phase function can be expressed as

$$\beta(\mu, \varphi; \mu', \varphi') = \sum_{m=0}^{L} (2-\delta_{0,m}) \left[\sum_{l=m}^{L} \beta_l^m P_l^m(\mu) P_l^m(\mu')\right] \cos m(\varphi - \varphi') \tag{6}$$

the radiance and the source term are also expanded as a Fourier decomposition (Chandrasekhar, 1960),

$$I(\tau, \mu, \varphi) = \sum_{m=0}^{L} I^m(\tau, \mu) \cos m\varphi \tag{7a}$$

$$S(\tau, \mu, \phi) = \sum_{m=0}^{L} S^m(\tau, \mu) \cos m\varphi \tag{7b}$$

The substitution of Equations (6)-(7)-(2) in Equation (1) yields

$$\mu \frac{\partial}{\partial \tau} I_r^m(\tau, \mu) + I_r^m(\tau, \mu) = \frac{\varpi_r}{2} \sum_{l=m}^{L} \beta_l^m P_l^m(\mu) \int_{-1}^{1} P_l^m(\mu') I_r^m(\tau, \mu') d\mu' + S_r^m(\tau, \mu) \tag{8}$$

subjected to the following boundary conditions, for $\mu \in (0,1]$

$$I_1^m(0,\mu) = f^m \tag{9a}$$

and

$$I_R^m(\tau_R,-\mu) = g^m \tag{9b}$$

and to the interface conditions, for $r = 1,2,\ldots,R-1$

$$I_r^m(\tau_r,\pm\mu) = I_{r+1}^m(\tau_r,\pm\mu) \tag{10}$$

Radiance was decomposed into azimuthal modes, while the phase function was replaced by the associated Legendre function expansion with $L$-order of anisotropy.

An approximation of the integral equation (1) is obtained using a quadrature of order $N = 2n$, with nodes $\{\mu_j\}$ and weights $\{\eta_j\}$. The value of $\mu$ is then discretized in $\mu_j$, with $j = 1,2,\ldots,N$, that are the discrete ordinate directions. The radiate transfer equation is then expressed as the related discrete ordinate equations, also known as $S_N$ equations, given by

$$\begin{aligned} &\mu_j \frac{d}{d\tau} I_r^m(\tau,\mu_j) + I_r^m(\tau,\mu_j) = \\ &\frac{\varpi_r}{2}\sum_{l=m}^{L}\beta_l^m P_l^m(\mu_j)\sum_{i=1}^{N}\eta_i P_l^m(\mu_i) I_r^m(\tau,\mu_i) \\ &+ S_r^m(\tau,\mu_j), \\ &j = 1,2,...,N \end{aligned} \tag{11}$$

The boundary conditions are

$$I_1^m(0,\mu_j) = f_j^m, \qquad j = 1,2,\ldots,n \tag{12a}$$

$$I_R^m(\tau_R,-\mu_j) = g_j^m, \qquad j = n+1,n+2,\ldots,N. \tag{12b}$$

The scattering angle was then discretized into $(L+1)$ azimuthal modes and $N$ polar angles, while the domain was split into $R$ homogeneous regions. The integral-differential equation (1) was rewritten as a set of $(L+1)\times N \times R$ ordinary differential equations.

The LTS$_N$ method (Barichello and Vilhena, 1993; Segatto and Vilhena, 1994) applies the Laplace transform on the radiative transfer discrete ordinates equations, given by (11) and (12)

## 3. INVERSION SCHEME

This work formulates the inverse problem according to an implicit approach, leading to an optimization problem (Lamm, 1993). the set of parameters to be estimated, is given by $\boldsymbol{p}$, in this case, the $R$ discrete values of the chlorophyll concentration $C$ at optical depths $\tau$ taken at the interface of the regions. Thus $p_r = C(\tau_r)$ for $r = 0,1,...,R-1$.

Experimental data are the discrete radiances $I(\tau_r,\mu_i)$ for $r = 0,1,...,R$ and $i = 1,2,...,N$. The objective function $J(\boldsymbol{p})$ is given by the square difference between experimental and model radiances plus a regularization term:

$$J(\boldsymbol{p}) = \sum_{i=1}^{N}\sum_{r=0}^{R}\left[I^{exp}(\tau_r,\mu_i) - I_{\boldsymbol{p}}(\tau_r,\mu_i)\right]^2 + \gamma\,\Omega(\boldsymbol{p}) \tag{13}$$

The $R$ discrete values of the concentration are estimated from $(R+1)\times N$ radiance values. $\Omega(\boldsymbol{p})$ is the regularization function, that is weighted by a regularization parameter $\gamma$. For instance, the 2nd order Tikhonov regularization (Tikhonov and Arsenin, 1977) is defined by

$$\Omega[\boldsymbol{p}] \;=\; \sum_{i=2}^{R-1} (p_{i+1} - 2p_i + p_{i-1})^2 \tag{14}$$

The regularization term is required for noisy data due to the ill-posedness nature of inverse problems. Then, small changes in radiance data cause big changes in the concentration profile. There are some criteria for the choice of $\gamma$, but an optimal value can be difficult to adjust, as it requires a choice criteria (Morozov discrepancy principle, L-curve, etc. (CamposVelho et al., 2002)) that may demand many executions of the inverse solver. A value too small may yield a profile with fluctuations, while the opposite makes the profile flat.

The influence of radiance data of higher depths can be underestimated since radiances decrease is nearly exponential with depth. Therefore, a depth correction factor may be used in the objective function.

It is proposed a new depth correction factor $(CF_r)$, given by the ratio between the mean radiance $\overline{I_r}$ related to the polar angle $\mu$ at each level $r$ and the mean radiance at the surface $\overline{I_1}$. This is done separately for negative/upward ($u$) and positive/downward ($d$) polar directions. The depth correction factor for the regions $r = 0, 2, ..., R-1$ is given by:

$$\overline{I_r^u} = \sum_{i=1}^{N/2} I^{exp}(\tau_r, \mu_i)/(N/2) \tag{15}$$

$$CF_{r+1}^u = \left(\overline{I_1^u}/\overline{I_r^u}\right)^2 \tag{16}$$

For regions $r = 1, 2, ..., R$ the depth correction factor is given:

$$\overline{I_r^d} = \sum_{i=N/2+1}^{N} I^{exp}(\tau_r, \mu_i)/(N/2) \tag{17}$$

$$CF_r^d = \left(\overline{I_1^d}/\overline{I_r^d}\right)^2 \tag{18}$$

Thus, the objective function can be written as:

$$\begin{aligned} J(\boldsymbol{p}) = &\sum_{i=1}^{N/2}\sum_{r=0}^{R-1} CF_{r+1}^u \left[I^{exp}(\tau_r, \mu_i) - I_{\boldsymbol{p}}(\tau_r, \mu_i)\right]^2 + \\ &\sum_{i=N/2+1}^{N}\sum_{r=1}^{R} CF_r^d \left[I^{exp}(\tau_r, \mu_i) - I_{\boldsymbol{p}}(\tau_r, \mu_i)\right]^2 + \;\gamma\,\Omega(\boldsymbol{p}) \end{aligned} \tag{19}$$

A previous correction factor was proposed by Tao et al. (1994), where the radiance values at each level are corrected by $\sqrt{e^{Z_r}}$. In this expression, $Z_r = z_r/\mathcal{L}$ (r=0,1,...,R), where $z_r$ is the depth in meters and $\mathcal{L}$ is a scaling factor (in this work, $\mathcal{L} = 1.5$ m). Considering the adopted plane-parallel geometry, Fig. 1 compares the values of the upward depth-corrected radiances, while Fig. 2 compares the values of the downward depth-corrected radiances. In these figures, the radiance decay is shown in function of depth ($z$) and of the polar angle ($\mu$), for two cases: (a) no depth correction factor, and (b) using the proposed factor. Section 6. presents the reconstructed profiles for these two alternatives in a particular test case.

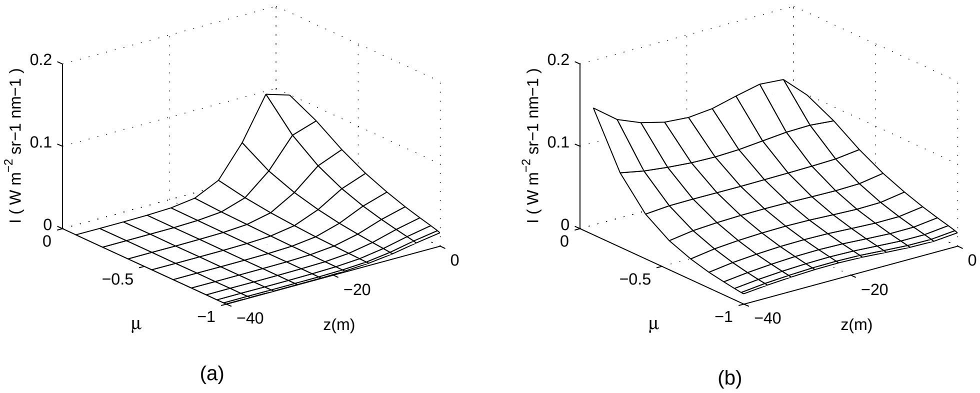


**Figure 1:** Upward radiance $I(z, \mu)$ decay: (a) no depth correction factor, and (b) with the proposed factor.

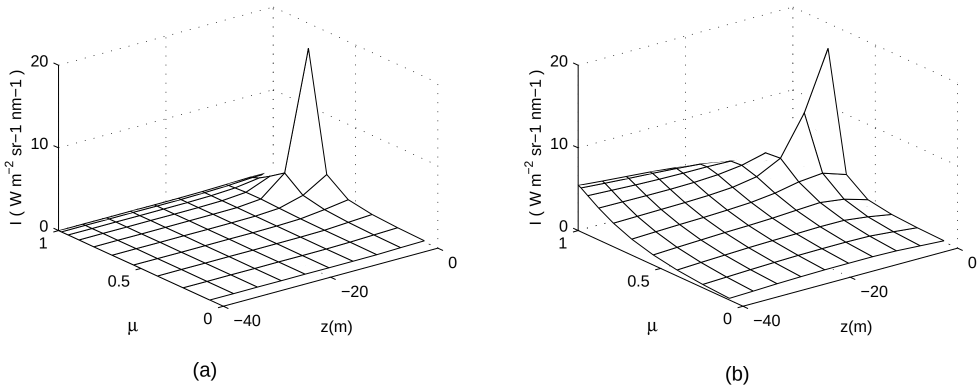


**Figure 2:** Downward radiance $I(z, \mu)$ decay: (a) no depth correction factor, and (b) with the proposed factor.

## 4. ANT COLONY SYSTEM

The Ant Colony System (ACS) is a method that employs a meta-heuristic based on the collective behaviour of ants chosing a path between the nest and the food source (Dorigo et al., 1996). Each ant marks its path with an amount of pheromone and the marked path is further employed by other ants as a reference. As an example of this, the sequence in Fig. 3 shows how ants, trying to go from point A to point E (a), behave when an obstacle is put in the middle of the original path, blocking the flow of the ants between points B and D (b). Two new paths are then possible, either going to the left of the obstacle (point H) or to the right (point C). The shortest path causes a greater amount of pheromone to be deposited by the preceding ants and thus more and more ants choose this path (c).

In the ACS optimization method, several generations of ants are produced. For each generation, a fixed amount of ants ($na$) is evaluated. Each ant is associated to a feasible path and this path represents a candidate solution, being composed of a particular set of edges of the graph that contains all possible solutions. Each ant is generated by choosing these edges on a probabilistic basis. This approach was succesfully used for the Traveling Salesman Problem (TSP) and other graph-like problems (Becceneri and Zinober, 2001). The best ant of each generation is then choosed and it is allowed to mark with pheromone its path. This will influence the creation of ants in the further generations. The pheromone put by the ants decays due to an evaporation rate. Finally, at the end of all generations, the best solution is assumed to be achieved.

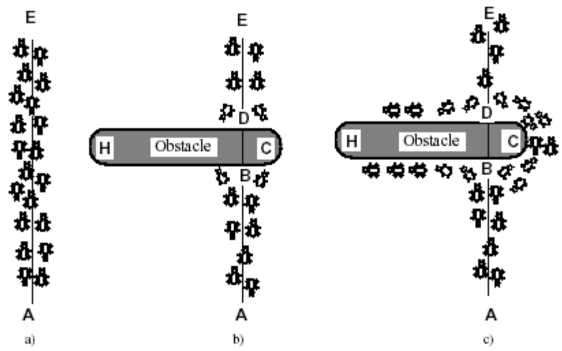


**Figure 3:** Ants overcoming an obstacle in the trail - from Dorigo et al. (1996).

A solution is composed of linking $ns$ nodes and in order to connect each pair of nodes, $np$ discrete values can be choosen. This approach was used to deal with a continuous domain. Therefore, there are $ns \times np$ possible paths $[i, j]$ available. Denoting by $\rho$ the pheromone decay rate and $\tau_0$ the initial amount of pheromone, the amount of pheromone $\tau_{ij}$ at generation $t$ is given by:

$$\tau_{ij}(t) = (1 - \rho)\tau_{ij}(t) + \tau_0, \tag{20}$$

In this work, $\tau_0$ is calculated as sugested by Bonabeau et al. (1999) using an evalutation $Q$ of the function to be optimized obtained with a greedy heuristics:

$$\tau_0 = 1/(ns * Q), \tag{21}$$

The probability of a given path $[i, j]$ be choosed is then

$$P_{ij}(t) = \frac{[\tau_{ij}(t)]^{\alpha} [\eta_{ij}]^{\beta}}{\sum_l \{[\tau_{il}(t)]^{\alpha} [\eta_{il}]^{\beta}\}} \tag{22}$$

where $l \in [1, np]$ and $\eta_{ij}$ is the visibility/cost of each path, a concept that arises from the TSP, where the cost is the inverse of the distance of a particular path. The above equation assumes that all paths are possible for any ant, but the TSP does not allow this assumption. The parameters $\alpha$ and $\beta$ are weights that establish the trade-off between the influence of the pheromone and the visibility in the probability of each path.

However, there is a further scheme for the choice of a path for a new ant. According to a roulette, a random number in the range $[0, 1]$ is generated for the new ant and it is compared with a parameter $q_0$ chosen for the problem. If the random number is greater than this parameter, the path is taken according to $P_{ij}$. If not, the most marked path is assigned.

In the current work, a ACS based inverse solver with a recent implicit regularization scheme (Preto et al., 2004) is proposed and employed without the explicit regularization ($\gamma = 0$). Since here is an *a priori* information about the smothness of the solution profile, such knowledge is included in the generation of the candidate solutions, by means of pre-selecting the smoother ants.

This information can be used as an alternative for the visibility $\eta_{ij}$ of each path. First, an overpopulation of ants is generated, making $\alpha = 1$ and $\beta = 0$ in Equation (22) and after, a certain fraction containing the smoother ants/paths is selected. Thus the visibility is assumed to be associated to the smoothness of the path. The criteria choosen to select the paths according to the their smoothness was precisely the 2nd order Tikhonov norm, that is normally used as a regularization function. Actually, a kind of pre-regularization is performed, and the usual regularization term is not required.

It was difficult to find an ACS visibility/cost criteria in this problem since it does not belong to the TSP class and this issue lead to choice of the smoothness for the path of each ant. It can be shown that the ACS has poor performance compared to other stochastic optimization algorithms when no visibility information can be defined. In addition, the proposed strategy leads to a reduction of the number of evaluations of the objective function and therefore improves the performance since the direct model demands a significant amount of processing time.

## 5. PARALLEL IMPLEMENTATION

Since the radiative transfer solver accounts for most of the total processing time, the adopted parallelization strategy was to use a sequential ACS-based inverse solver and a parallel implementation of the radiative transfer solver, the $\mathrm{LTS}_N$.

A preliminary study of the $\mathrm{LTS}_N$ solver has led to the chosen parallelization strategy. The method discretizes polar ($N$ discrete values) and azimuthal angles ($L + 1$ discrete values) for $R$ homogeneous regions and employs the non-discrete optical thickness $\tau$, that is the one-dimensional spatial variable. An immediate analysis would consider parallelization based on the distribution of these quantities among processors:

- polar angles
- azimuthal angles
- regions
- range of optical thickness inside a region
- a combination of the above in a multidimensional arrangement

The discrete ordinates associated to the polar angles are strongly coupled, as can be observed in the Equation (11). This precludes a parallel implementation that distributes these values among processors. On the other hand, the azimuthal modes are independent and a parallelization based on assigning different modes to different processors is straightforward and was employed in this work. Therefore, each $m$ azimuthal-mode system of equations of order $R \times N$ is solved on a different processor. The third option was not considered as continuity of boundary conditions between adjacent regions forces a sequential computation for the regions. The same restriction applies to the spatial decomposition of the domain inside each region, related to the optical thickness $\tau$. Finally, a combination of the above decompositions would suffer these implied limitations.

The related code was optimized and parallelized using calls to the message passing communication library MPI and executed on a distributed memory parallel machine, a cluster composed of 17 microcomputers with AMD Athlon 1.67 GHz processors, connected by a standard Fast Ethernet network.

An important issue is to maximize the amount of computation done by each processor and to minimize the amount of communication due to the MPI calls in order to achieve good performance. The speed-up is defined as the ratio between the sequential and parallel execution times. A linear speed-up denotes that processing time was decreased by a factor of $p$ when using

$p$ processors and can be thought as a kind of nominal limit. Efficiency is defined as the ration of the speed-up by $p$ and thus it is $1$ for a linear speed-up. Usually, communication penalties lead to efficiencies lesser than $1$.

## 6. NUMERICAL RESULTS

The inverse solver, based on a ACS implementation, was tested for a multi-region ($R = 10$) offshore ocean water radiative transfer problem considering $N = 50$ polar angles and $L = 0, 1, \ldots, 173$ (no azimuthal symmetry). Two sets of radiance data were used, corrupted with 5% and 30% gaussian noise. A parallel implementation of the Laplace transform discrete ordinate ($LTS_N$) method was used as direct model.

In the test cases, as mentioned in section 2., synthetic data was used to simulate the experimental values of the Celtic sea. In the case of noisy data, no "classical" regularization was used ($\gamma = 0$). Instead, the smoothness-based pre-selection was employed in the generation of the ants. The 2nd order Tikhonov norm was used as smoothness criteria.

Since radiance decay is nearly exponential with depth, the use of a depth correction factor may be required: radiances near the surface have a greater influence in the objective function than those at higher depths. A new depth correction factor, shown in Section 3, is proposed to balance the radiance values.

Similarly to other stochastic optimization algorithms, the tuning of parameters in the ACS has a big influence in the results. This implementation required adjustment of parameters like $\rho$, the pheromone decay rate and $q_0$, used in the roulette scheme. Other parameters may influence the quality of the solution, like the the number of $np$ possible paths between each pair of the $ns$ nodes, the number of ants $na$, or the maximum number of iterations $mit$. The adopted values for these parameters are shown in Table 1. In each generation, of the $90$ ($na$) ants, $1/6$ were pre-selected according to their smoothness.

**Table 1:** Ant Colony System parameters

| seed | ns | np | na | mit | $\rho$ | $q_0$ |
|---|---|---|---|---|---|---|
| 33 | 10 | 3000 | 90 | 500 | 0.03 | 0.0 |

Figs. 4 and 5 shows the reconstructed profile of the chlorophyll concentration, using 5% noisy data. The exact solution is compared to the estimations performed using the ACS-based solver employing no depth correction factor, and the proposed factor for the reconstruction using 5% noisy data. It can be seen that the use of the proposed depth correction factor was decisive for the quality of the solution.

Table 2 compares the processing time of the sequential version to the parallel version for different number of processors and considering 500 iteractions/generations of the ACS. The proposed parallelization strategy, i.e. to use a parallel version of the $LTS_N$ method that distributes azimuthal modes among processors, presented good performance and good scaling up to 16 processors.

## 7. FINAL REMARKS

This work presented a new methodology for the estimation of the chlorophyll concentration vertical profiles in natural waters using radiance data in several depths. Vertical profiles of the absorption and scattering coefficients can be inferred from the chlorophyll profile by means of bio-optical models.

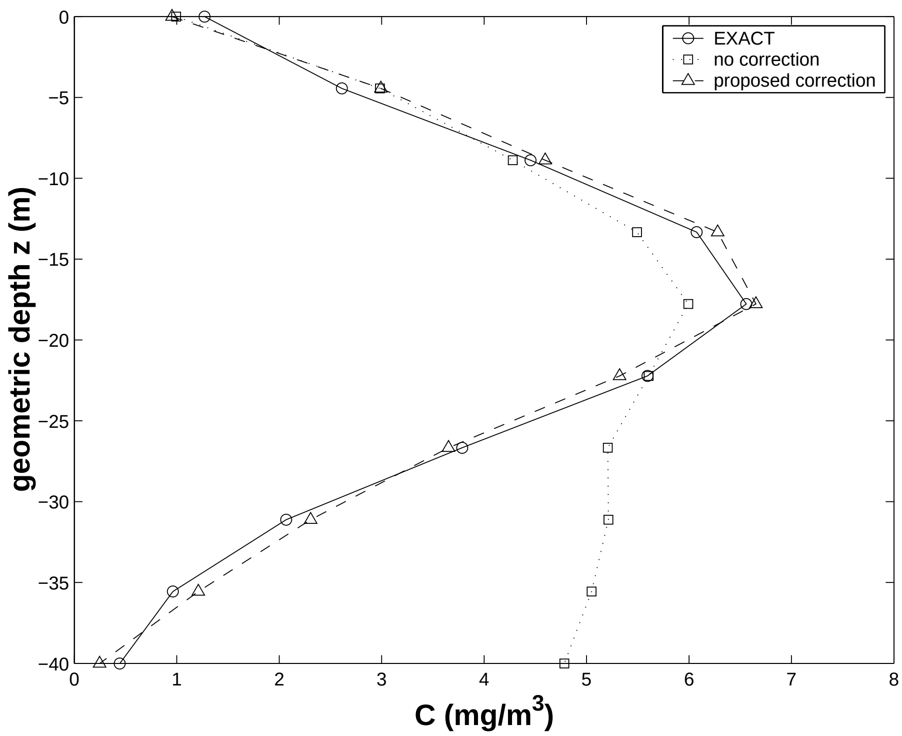


**Figure 4:** Chlorophyll profile reconstruction using 5% gaussian noisy radiance data with no depth correction factor and with the proposed correction factor.

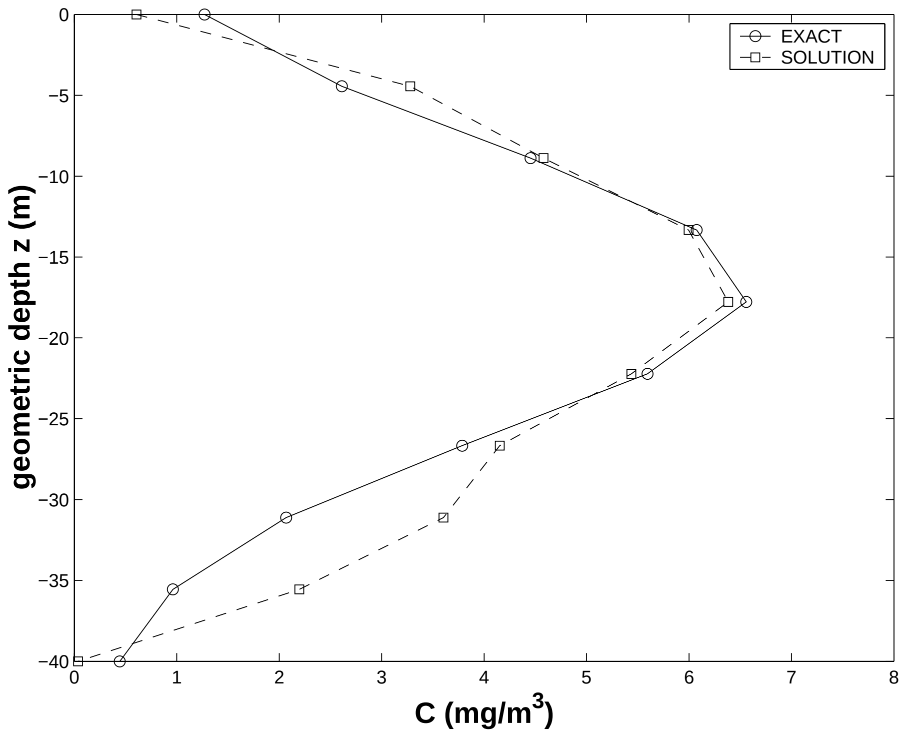


**Figure 5:** Chlorophyll profile reconstruction using 30% gaussian noisy radiance data with the proposed depth correction factor.

**Table 2:** Execution time (hours), speed-up and efficiency of the ACS inverse solver for 500 iterations.

| $p$ | *runtime* | *speed-up* | *efficiency* |
|---|---|---|---|
| 1 | 56.05 | | |
| 2 | 28.31 | 1.98 | 0.99 |
| 4 | 14.54 | 3.86 | 0.96 |
| 8 | 7.80 | 7.19 | 0.90 |
| 16 | 4.42 | 12.68 | 0.79 |

A parallel implementation of the Laplace transform discrete ordinate ($LTS_N$) method was used as direct model and the objective function is evaluated by the square difference between experimental and model radiance data at each level/region. A new depth correction factor is used for the radiances and a recently proposed pre-regularization scheme filters candidate solutions according to their smoothness. The inverse solver is an implementation of the Ant Colony System meta-heuristics.

The proposed ACS-based inverse solver yield a good estimation of chlorophyll vertical profiles using the new depth correction factor for radiance data for a given test case using radiance data with 5% and 30% noise. No standard regularization was required in the objective function. Quality of the reconstructed profiles and performance results are shown.


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