THE DYNAMICS OF THE GRAVITATIONAL CAPTURE PROBLEM

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ABSTRACT

Many authors use the gravitational capture phenomenon to explain the first process for the natural satellite captures. The gravitational capture is a temporary phenomenon, so it can not explain the existence of the natural satellites alone. It is necessary other process to complete the capture. But the process of the gravitational capture can be used in astrodynamics as a method to reduce the fuel consumption in a transfer maneuver between two celestial bodies. With this aim, and because the gravitational capture is still not completely known, it is proposed in this paper the study of some particularities of the gravitational capture. Discontinuities that appear in the limit between the capture and non-capture trajectories are discussed and explained. Some trajectories are shown as examples. It is also explained why some discontinuities occur in the entry position angle when the velocity of the capture is reduced. Finally, the forces involved in this problem are studied and more explanations about the phenomenon appear. This study will be performed using the model given by the restricted three-body problem and the relation of the capture velocity with the two-body energy is used as the initial condition.

INTRODUCTION

The gravitational capture concept depends on the problem studied. In the general three-body problem the gravitational capture occurs when two bodies (of the three involved), comes from a long distance and became near each other for the rest of the time. In the restricted three-body problem, the gravitational capture occurs when the massless particle come close enough to one of the primaries and stay near it for some time. Sizova (1952) and Merman (1953) showed that the gravitational capture is permanent in the hyperbolic restricted problem. Merman (1954) showed that the permanent capture is also possible for the parabolic case. Many authors showed that in the elliptic and circular case this phenomenon is temporary (see for example Tanikawa, 1983). This means that, after some time after the approximation, the massless particle escapes from the gravitational attraction of the primary.

Considering the temporary gravitational capture, there are two main approaches to the problem: the

Lagrangian points capture and the phase space. In this work the phase space approach is used. In this approach, conditions that cause the gravitational capture are searched in the phase space. In the computational procedure is used a negative step of time for the integration of the dynamics. In this way, the final conditions of the integration are the start condition for the gravitational capture. If the object goes away from near the primary in a limited time without collision, this object is in a gravitational capture trajectory in the forward time.

To consider that if a trajectory is a gravitational capture or not, it is used the Yamakawa (1992) approach. It is defined a parameter $C3$ as the energy of the two-body problem:

$$C3 = V^2 - \frac{2\mu}{r}, \qquad (1)$$

where V is the velocity of the object relative to the primary, r is the distance from the primary and μ is the dimensionless mass of the primary. In this approach, the variation of $C3$ is verified all the time. If the value of the parameter change the sign from negative (closed trajectory) to positive (open trajectory) it is considered that the trajectory escaped, so, in the positive sense of time, there is a gravitational capture.

To understand the behavior of the gravitational capture trajectories, some parameters are numerically analyzed. The analysis made here explain some results obtained by Yamakawa (1992), Vieira Neto (1999), Vieira Neto and Prado (1995, 1996, 1998) and Prado *et. all.* (1997).

Finally, it is made the numerical study of the forces involved in the dynamical system. This analysis reinforces the first results.

THE MODEL

The model used is the planar restricted three-body problem, with the parameters of the Earth-Moon system. This study is made using dimensionless units, in such way that the distance from Earth to Moon, the total mass of the system, and the angular velocity of the system are equals to one. The frame of reference is rotating with the Earth-Moon system. For the integration of the trajectories the initial conditions are calculated with a certain negative value of $C3$ with null relative radial velocity at the perilune distance. Then the value of $C3$ is monitored until the negative value turn into positive. A trajectory is considered a gravitational capture when the change of the sign occurs in a time smaller than 50 days. This happens in a distance of approximately 100000 km of the Moon. This distance is considered the gravitational sphere of influence of the Moon (Yamakawa, 1992; Vieira Neto, 1999). The parameters of the trajectory are shown in Figure 1. In the figure $\mathbf{r_p}$ is the perilune distance (1838 km), $\mathbf{V}$ is the velocity relative to the Moon, α is the perilune position angle and β is the entry position angle. In this figure occurs a direct capture (counter-clockwise), but it is also possible to happen a retrograde capture (clockwise).

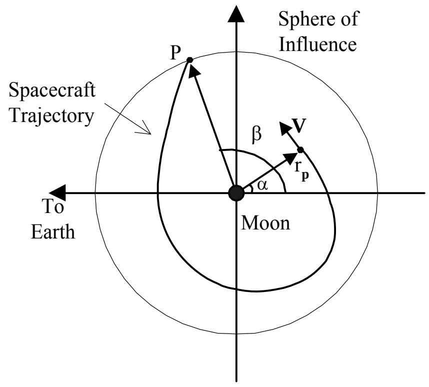

Fig. 1 – Parameters of the gravitational capture.

THE MINIMUM *C*3

The value of *C*3 is related to the velocity variation (ΔV) needed to change a spacecraft from the elliptical to circular motion. Lower values of *C*3 gives lower values of ΔV, and the fuel consumption to perform this maneuver is reduced. So the search for the minimum values of *C*3 is very important. Figure 2 show the minimum *C*3 achieved for the Earth-Moon system in the direct and retrograde captures. The radial variable is the absolute value of *C*3 and the angular variable is α.

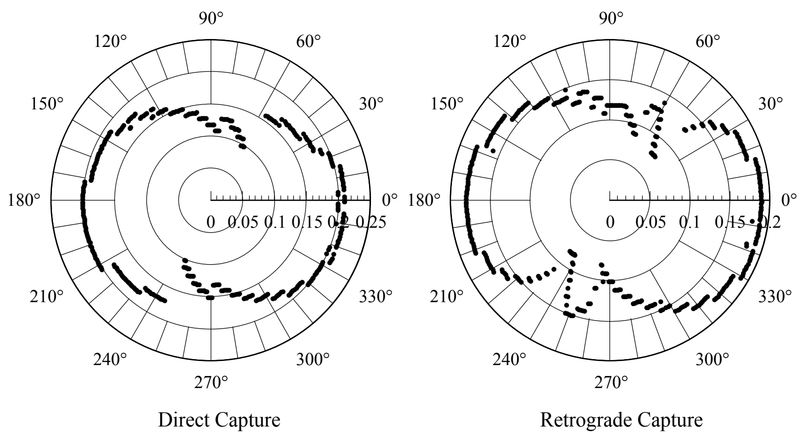

Fig. 2 – Regions of minimum C3.

It is possible to take some conclusion from this picture. The first one is that some regions are better for the capture than others. The region near $\alpha = 0^o$, that is, in the anti-Earth side, gives the most negative values for *C*3. The second conclusion is that the direct capture is better than the retrograde one.

Other thing that it is seen in the figure is the existence of some discontinuities in the boundary of minimum *C*3. To investigate these discontinuities the region near $\alpha = 60^o$ is used. In the passage from α = 57.5^o to α = 57.6^o, the minimum values for *C*3 goes from –0.16 to –0.1. Figure 3 shows the trajectories for *C*3 = –0.16 in the cases α = 57.5^o and α = 57.6^o. In this figure is possible to see that the modification of 0.1^o in the perilune position angle starts a series of trajectories that ends in collisions with the Moon. This process continues with the raising of $C3$ until it reaches the value of –0.1, making the discontinuity.

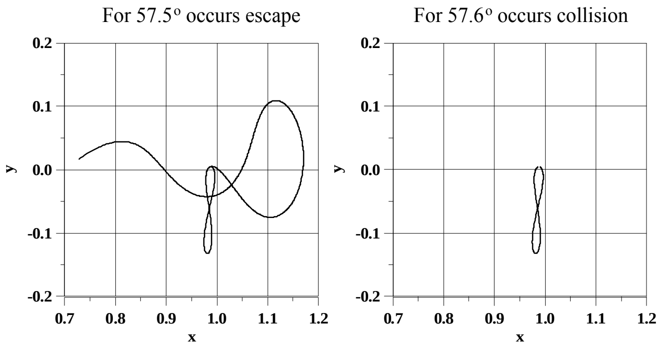

Fig. 3 – Trajectories of the discontinuity.

ENTRY POSITION ANGLE

Another topic of interest in the study of the gravitational capture is the behavior of the capture trajectory in the gravitational sphere of influence of the Moon. Figure 4 shows the behavior of the entry position angle (β) for α fixed at 180° for the direct and retrograde capture. This picture was first made by Yamakawa (1992) and the version shown here is from Vieira Neto (1999). Due to comparison of the two results it is taken the dimensional value of $C3$ for this figure. The energy $C3$ varies from 0 to -0.25 km^2/s^2. The process starts with a continuous behavior until $C3$ approximately -0.19 km^2/s^2 for the retrograde capture and approximately -0.21 km^2/s^2 for the direct one.

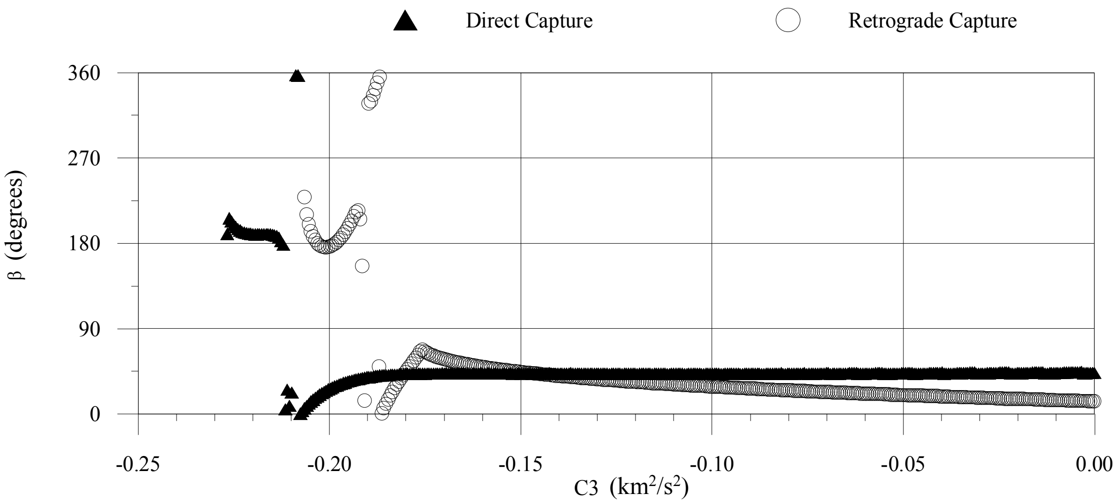

Fig. 4 - Entry position angle for $\alpha = 180^{\circ}$. Direct and retrograde captures.

The studies for these discontinuities were done for the direct capture case. The escapes begun to occurs close to $\beta = 45^{\circ}$ at $C3 = 0$ km^2/s^2, reach 0° close to $C3 = –0.21$ and continue on the above side of the graphic until 357.3°, after that there is a jump of 25.3° to $\beta = 22.6^{\circ}$. Figure 5 shows two sets of trajectories for the discontinuities in β from 357.3° to 22.6° and from 5° to 178.8°. These discontinuities occur in the transition for $C3$ from –0.2087593 km^2/s^2 to –0.2098816 km^2/s^2 for the first case and from -02115652 km^2/s^2 to –0.2121263 km^2/s^2 for the second case. These discontinuities happen because of the decreasing of the energy, that is, the lower velocity. So, for a certain limit, the trajectories begun to change more rapidly the entry position angle due to low velocity at the capture position. This shows that the velocity is very relevant for the capture problem. It is interesting to see in Figure 4 that after the large transition showed there is some stability in the region between –0.21 to –0.22 km^2/s^2.

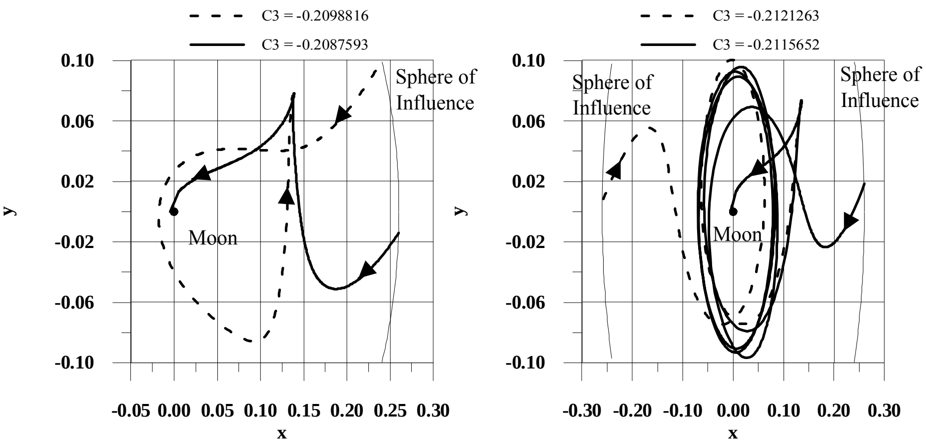

Fig. 5 - Trajectories for the discontinuities in the entry position angle.

FORCES INVOLVED IN THE DYNAMICS

To obtain the forces acting on the massless particle it is used the Newtonian method. In this method the forces are seem geometrically, and the process to obtain the equations of motion is token by the Newton's law. After some considerations, the equation of motion of the restricted three-body problem in the fixed system, can be taken as:

$$\frac{d^2\mathbf{r}_{23}}{dt^2} = -\frac{1-\mu}{\left(1+r_{23}^2+2r_{23}\cos\alpha\right)^{3/2}}\left(\mathbf{r}_{23}+\mathbf{h}\right)-\frac{\mu}{r_{23}^3}\mathbf{r}_{23}\,, \tag{2}$$

where $\mathbf{h}$ is the vector joining the two primaries, t is the time and r_{23} is the distance from the massless particle to M_2, the central body in this case. The first term of the equation above is due to M_1, and the second is due to M_2.

In the fixed system only the gravitational forces can be taken. To see the other forces it is necessary to use the rotating frame of reference. A new equation of motion can be find after the use of the Coriollis theorem, that is:

$$\frac{d^2\mathbf{r}_{23}}{dt^2} = \frac{d^{*2}\mathbf{r}_{23}}{dt^2} + \vec{\omega}\times\left(\vec{\omega}\times\mathbf{r}_{23}\right) + 2\vec{\omega}\times\frac{d^*\mathbf{r}_{23}}{dt} + \frac{d\vec{\omega}}{dt}\times\mathbf{r}_{23}\,. \tag{3}$$

The vector $\vec{\omega}$ is the angular velocity of the system (unitary for this problem). The star system is the rotating system. In the Coriollis equation, the second term is the centripetal force, the third is the Coriollis force, and the last term has no defined name and it has a null value in this problem because the constancy of the angular velocity. With this equation is possible to write each force described above and the gravitational. The gravitational force due the largest body in polar coordinates centered in the lowest primary (M_2) is:

$$\mathbf{F}_1 = -\frac{(1-\mu)(r_{23}+\cos\alpha)}{\left(1+r_{23}^2+2r_{23}\cos\alpha\right)^{3/2}}\hat{\mathbf{r}}_{23} + \frac{(1-\mu)\operatorname{sen}\alpha}{\left(1+r_{23}^2+2r_{23}\cos\alpha\right)^{3/2}}\hat{\boldsymbol{\alpha}}\,; \tag{4}$$

the gravitational force due to the less massive primary is:

$$\mathbf{F}_2 = -\frac{\mu}{r_{23}^2}\hat{\mathbf{r}}_{23}\,; \tag{5}$$

the centripetal force is:

$$\mathbf{F}_{ce} = \left[r_{23} + (1-\mu)\cos\alpha\right]\hat{\mathbf{r}}_{23} - (1-\mu)\operatorname{sen}\alpha\,\hat{\boldsymbol{\alpha}}\,; \tag{6}$$

and the Coriollis force is:

$$\mathbf{F}_{co} = 2v_\alpha\,\hat{\mathbf{r}}_{23} - 2v_r\,\hat{\boldsymbol{\alpha}}\,. \tag{7}$$

In the equations showed before:

$$v_r = \frac{d^* r_{23}}{dt} \text{ and } v_\alpha = r_{23}\frac{d^*\alpha}{dt}\,. \tag{8}$$

NUMERICAL ANALYSES OF THE FORCES

In the gravitational capture the main force is the gravitational force due to the central body, in this case, the Moon. The others forces are perturbations on the movement of the massless particle. So, to understand the behavior of the perturbing forces, an analysis was made by measuring the components of each force. The chosen components are in the radial, transversal and in the direction of motion of the massless particle. In the radial direction, the positive sign means that the force is acting opposite to direction of the body. In the transversal direction, the positive sign indicates that the force is acting in the counter-clockwise direction. In the direction of motion, the force is positive when it is being applied in the direction of the movement of the particle.

Two trajectories were chosen. In the first one occurs a fast capture. In the second one some revolutions occurs before the particle arrives at the capture position. The time step used for the numerical integration is negative, so the zero time means the moment that the particle arrives at the desired perilune. Every trajectory begins with a perilune at 100 km from the surface of the Moon, and all plots are in dimensionless units. The first trajectory enters in the sphere of influence in the same side of the Earth. It arrives at the perilune with $C3 = -0.15$ at $\alpha = 0^{\circ}$. Figure 6 shows this trajectory.

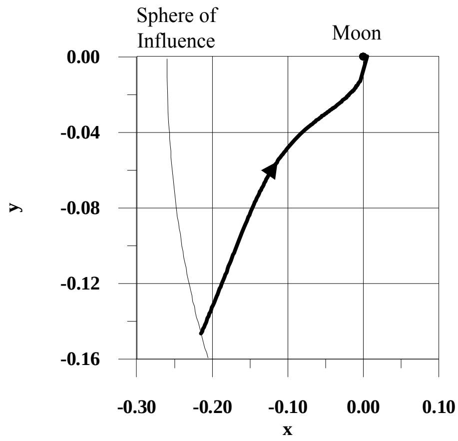

Fig. 6 – Trajectory with $C3 = -0.15$ and $\alpha = 0^{o}$ at perilune.

Figure 7 shows the behavior of the gravity force due to Earth over the spacecraft. In the radial direction the force began with a positive sign. This means that the force is pushing the spacecraft to an opposite direction to the Moon in this moment. There is a change in sign in the last instant, close to the Moon. So, the majority of the time, the gravity force due the Earth in the radial direction is slowing down the object. In the transversal direction, the force is always negative. This decreases the transversal velocity. In the direction of motion the sign is also negative, breaking the spacecraft all the time. In magnitude the force is decreasing. This happens because the spacecraft is moving away from Earth.

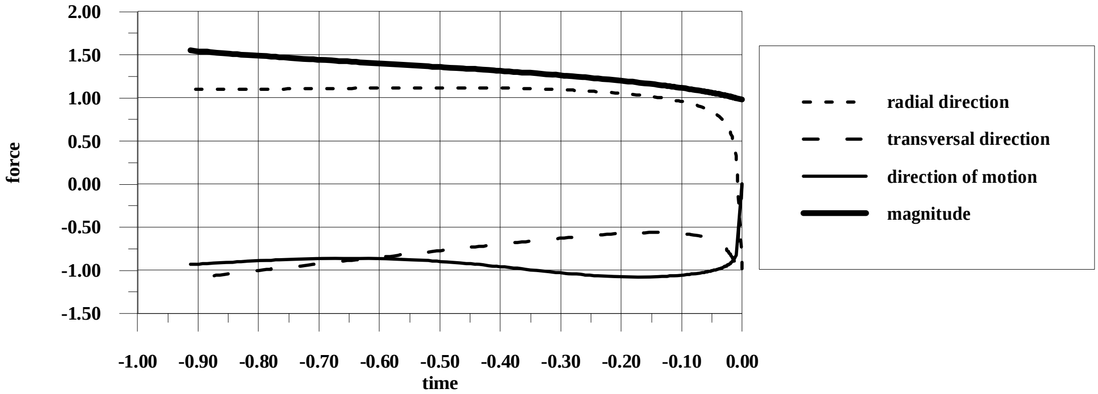

Fig. 7 – Gravitational force due to Earth for the first trajectory.

The centripetal force is presented in Figure 8. It acts in an opposite direction from the gravity force due to Earth, but with absolute values smaller than the centripetal one. So the net result is due to gravity force of the Earth, which makes the vehicle to reduce its velocity.

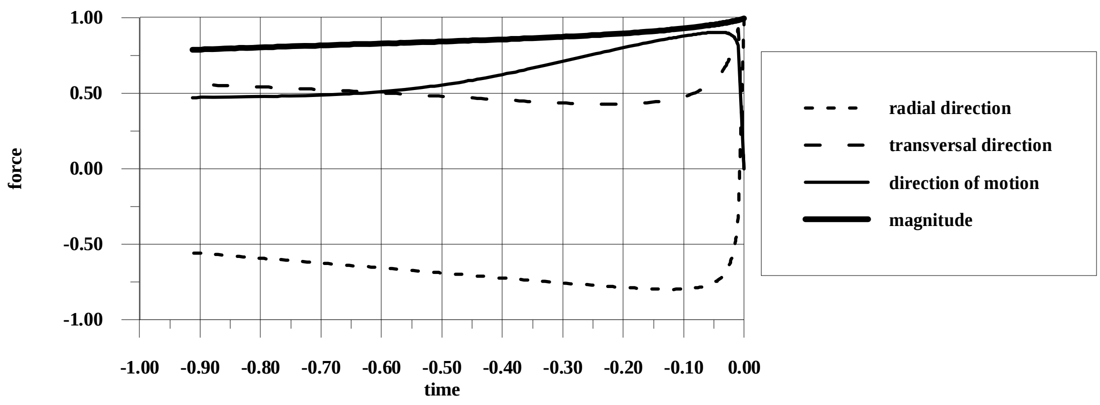

Fig. 8 – Centripetal force for the first trajectory.

Figure 9 shows the resultant force including the gravity force due to Earth and the centripetal force. Now it is possible to see that the net result works to slow down the spacecraft. The sum of these forces works on the vehicle to reduce its velocity working in opposite direction of the gravitational force due to Moon, as can be seen in Figure 9 for the resultant in the direction of motion. The resultant on the transversal direction works to pull the massless particle to Earth-Moon axis.

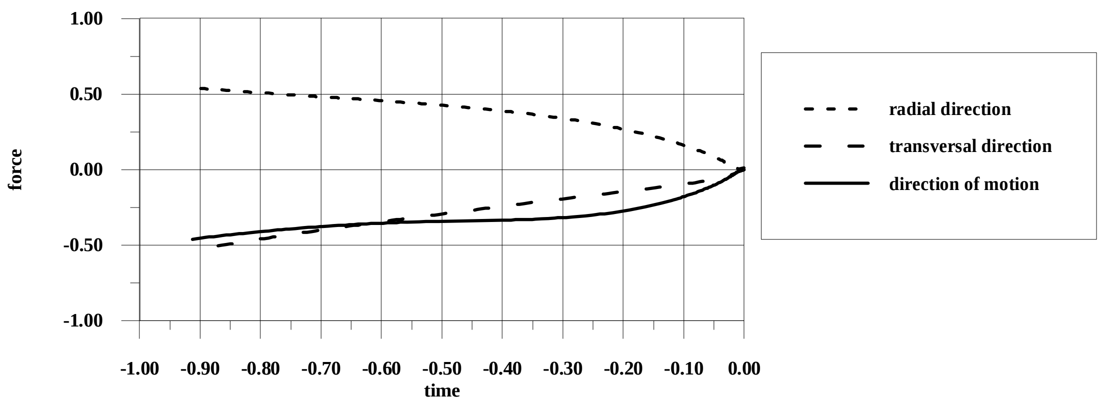

Fig. 9 – Resultant force between the gravity force due to Earth and the centripetal force for the first trajectory.

The Coriollis force acts like the gravity force due to Moon, but with much less effect. It causes no perturbation in the direction of the motion of the spacecraft, because of its definitions (see equations 3 and 7).

The second trajectory enters in the sphere of influence of the Moon in the anti-Earth side, and it arrives at a perilune with $C3 = -0.15$ at $\alpha = 45^{o}$. See it in Figure 10.

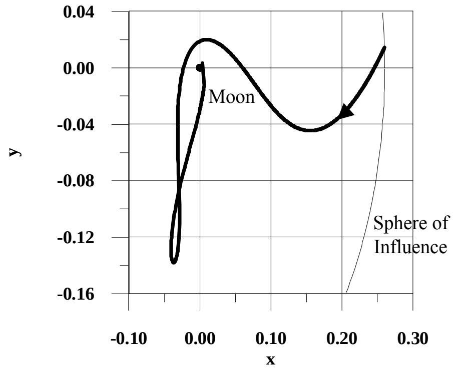

Fig. 10 – Trajectory which ends in a perilune at $C3 = -0.15$ e $\alpha = 45^o$.

For this case there is one revolution of the spacecraft over the Moon. This makes the forces to change the sign as can be seen in the next figures. But the left side of the figures behaves like the first case showed. Due to this trajectory enters on the other side of the gravity sphere of influence, related of the first case, the gravity force due to Earth behaves in an opposite way from the first case, as can be seen in the left side of Figure 11.

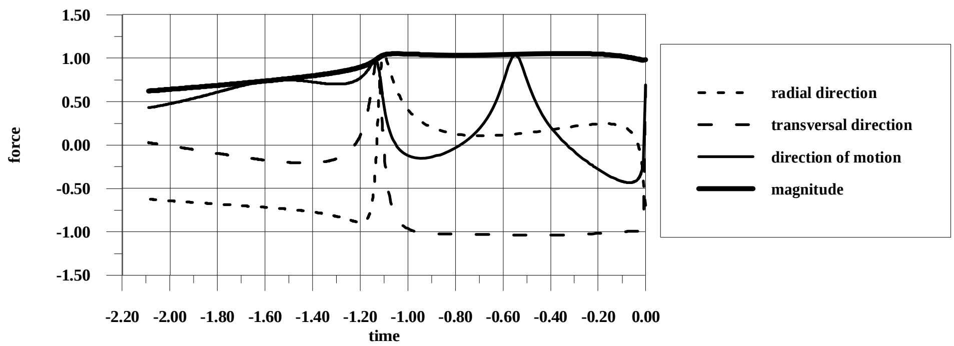

Fig. 11 – Gravity force due to Earth for the second trajectory.

Figure 12 shows the behavior of the centripetal force. Like the gravitational force due to Earth, the left side of the figure has an opposite behavior from the first case. Here this force is decreasing. Due to the rotation of the system and because the particle is more distant from the center, the centripetal force is stronger than the gravitation force due to Earth.

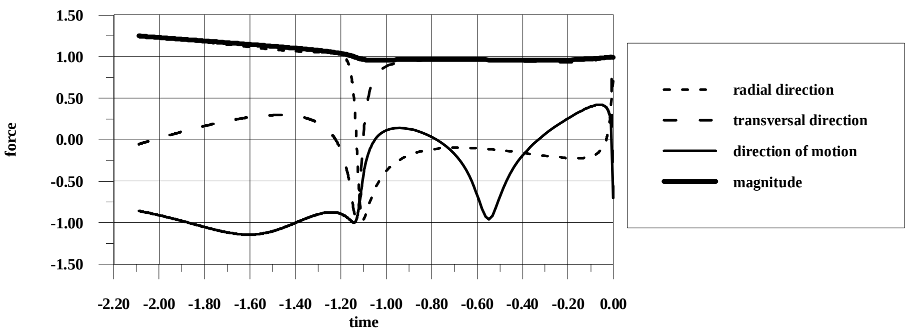

Fig. 12 – Centripetal force for the second trajectory.

Figure 13 shows the behavior of the resultant between the centripetal and the gravitational force due to Earth. Although the forces for this case has an opposite behavior in the first half of the figure compared with the first case, the left side of Figure 13 is very similar to Figure 9. There is an exception for the transversal direction, it happens because the particle is rotating the central body in an opposite way from the first case. This two cases show that the gravitational force due to Earth and the centripetal force work to reduce the velocity of the object.

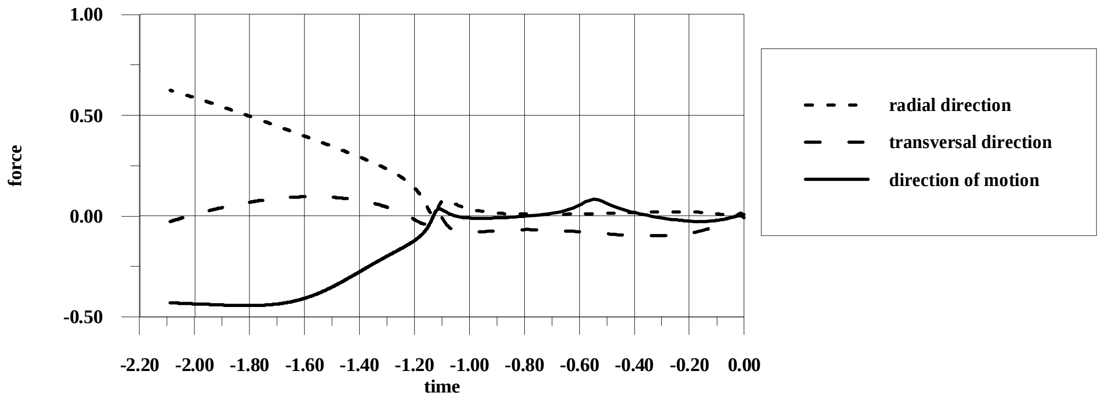

Fig. 13 – Resultant force between the gravity force due to Earth and centripetal force for the second trajectory.

Analyzing the resultant force of the two cases it is possible to get some conclusions.

First, the most relevant component is the direction of motion. This component shows that, independent of where the particle enters the sphere of influence, it is slowing down the particle all the time.

The transversal component ever tries to pull the particle to Earth-Moon axis. This component may explain why the best region of capture, in Figure 2, is near the Earth-Moon axis. The transversal component of the force is stronger when the particle is passing over 90^o and 270^o (because of the sin function in equations 2 and 4). This reduces the transversal velocity, making more difficult the minimum value of $C3$ to achieve the same result as for the Earth-Moon axis.

The radial component of the resultant force has the same effect of the component in the direction of motion. It also shows that the particle is loosing radial velocity when approaching the central body.

CONCLUSIONS

In this paper are studied some particularities of the gravitational capture problem. The forces acting on the gravitational capture problem is also studied. This study answers some questions about the phenomena. Two of the forces are relatively weak, and they act as disturbing forces: the gravitational force due to Earth and the centrifugal force. These forces working together slow down the spacecraft with a force in the opposite direction of spacecraft's movement. This is equivalent of applying a continuous propulsion force against the motion of the spacecraft. In the radial direction the gravitational force due to Earth and the centripetal force tends to equilibrium, but ever rest some work against the gravitation of the central body. It is also true for the components in the direction of movement for these forces. In the transversal direction, the forces pull the particle to Earth-Moon axis. The understanding of these behaviors explain why a particle with a velocity slower than the escape velocity can escape from the Moon. It is the opposite case for the capture and it happens for the cases studied. The negative *C*3 means that the velocity of the vehicle is lower than the escape velocity.

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